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RESEARCH ARTICLE

Pareto and probability distributions

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Abstract
Although familiar with the developments in probability theory of his time, Vilfredo Pareto made little use of this tool in his writings, preferring theoretical constructions based on experimentation and observation. This article attempts to reconstruct Pareto's overall approach to probability by examining his references to the distribution of income, an economic fact that lends itself to probabilistic investigation. The result of this research shows how Pareto alludes to the application of probability to income and social groups, but leaves the task to his followers.
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1 Introduction

The name Vilfredo Pareto is rarely associated with probability. It is true that Pareto wrote at a time - the end of the 19th century and the early decades of the 20th century - when the use of probability in economics was still in its infancy, and that the probabilistic revolution in economics took place in the middle of the 20th century, but Pareto's refusal to introduce probability is fascinating. Understanding the reasons for this deliberate refusal may help us to understand why the probabilistic revolution in economics did not occur while it swept through other disciplines.

That probability is not linked to Pareto's name is also shown by the fact that, with the exception of Vinci (1924), the literature on Pareto and probability is almost non-existent, whereas the literature on Pareto statistician is quite extensive (Mortara 1924, Davis 1949; Jannaccone 1949; Tarascio 1966; among others).

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This explains the aim of this work: to reconstruct Pareto's approach to the use of probability in economic and social research, from the explicit reference to probability distributions in the *Cours d'économie politique* (Pareto 1896-97) to the references to kinetic theory in the *Treatise on General Sociology* (Pareto 1916), via the *Les Systèmes socialistes* (Pareto 1902-03) and the *Manual of Political Economy* (Pareto 1906). These are episodic hints that are nevertheless intriguing, not least because they touch on areas of research in the making at the time, such as statistical mechanics. Pareto's approach to probability must therefore be reconstructed by starting from the cornerstones of Pareto theory, first and foremost the mechanism that takes shape in the attribution of a clear centrality to the exact reconstruction of the 'economic fact' and its logical interpretation. In this perspective, the 'distribution of income' is elevated to an 'empirical law' (Pareto 1896-97, § 959), without proposing any probabilistic interpretation.

It is true that in ignoring rather than rejecting probability in economics, Pareto was in good company; he was part of the majority of economists who shared this rejection until the 1930s. These included economists who made extensive use of the statistical instrument, even in a theoretical key, such as Henry L. Moore, Henry Schultz and Karl Pearsons himself, the founder of the Harvard Business Barometer (see Morgan 1990). It was not until 1944 that Trygve Haavelmo finally introduced probabilities into econometric analysis (Morgan 1990; p. 171).

However, the widespread difficulty of accepting probability does not explain the choice of Pareto, a scientist equipped with the tools, in particular the experimental method, to act autonomously (see Marchionatti and Gambino 2007).

Section 2 focuses on the references to probability in the essays *Considerazioni sui principi fondamentali dell'economia pura* (1892-93) and the *Cours d'économie politique* (1896-97). The references to the empirical approach to probability in the treatment of income distribution in the *Cours* may have indicated Pareto's openness to the use of this tool. Section 3 analyses the references to income and social groups in the *Manuale di economia politica* (1906) and the *Trattato di sociologia generale* (1916), since Pareto's reference to statistical equilibrium seems to allude to a probabilistic analysis of group relations. Section 4 offers an interpretive key to Pareto's non-use of probability, focusing in particular on his conception of experimental economics as the basis for his mechanistic view of economics. Section 5 recalls the use of probability distributions in the Pareto or immediately following years by Italian scholars who took their cue from the distribution of income. Section 6 contains some concluding remarks.

2 Income distribution and probabilities

In the *Considerationi sui principi fondamentali delleconomia politica pura published* in *Il Giornale degli Economisti* (1892-94), after agreeing with Laplace that "many considerations besides mathematical probability are involved in determining human action" (Pareto 1893, p. 8) and expressing his perplexity at Bertrand's reliance on mathematics, Pareto dwells on the probability of causes, a theme developed by Cournot (see Cournot 1851): "Cause, in the calculus of probabilities, means only a conjuncture which, when it follows, gives another a certain probability" (Pareto 1893, p. 27 n1). But if we know nothing about causes, we must assume that the alternatives between two possible ones are equal, although "in practice", Pareto adds, "things are not so arranged" (Pareto 1893, p. 28), and finally, he concludes, "the calculus of probability can only be used to show us some errors of reasoning" (Pareto 1893, p. 29). Referring to the application of probability to the study of changes in the value of gold, Pareto argued that it was impossible to apply probability to the causes of this problem because knowledge at the time did not allow it (ibid.). But the same could be said of many other problems. Mornati, commenting the 1893 *Considerazioni*, writes that Pareto considered probability "as an absolutely subjective phenomenon", definable as "the ratio of equally favourable cases to equally possible cases" (cf. Mornati 2018; p. 238). In other words, probability exists only in people's minds and has no scientific value.

Claude Menard, arguing that the probabilistic approach did not lead to a profound transformation of economic thought from Cournot to Keynes, states that "Cournot was in a good position to generate a radical transformation of economic thought" (Ménard 1990; p. 140): the same can be said of Pareto. Like Cournot, Pareto had a good mathematical background and was familiar with the contributions of Laplace, Poisson and others, but it will not have escaped Pareto's notice that Cournot himself, as he well knew, made no use of probability. Menard wrote that "the ambitions of the first model builder in economics [Cournot], led him to seek solutions in rational economics understood to be a counterpart of rational mechanics" (Ménard 1990; p. 141). So economic models have to be mathematical, but above all they have to use the mathematics of classical physics. Was this also the case for Pareto? Pareto, an engineer by training, does not cease to recognise the parallelism between mechanics and economics, but his is a conception of mechanics based on the experimental method, as in physics. Pareto assumed a mechanical rationality rather than a mathematical rationality (see Donzelli 1997).

In 1897, Pareto wrote to Giovanni Vailati: "In the next few holidays, if I have time, I want to write a short treatise on the calculus of probability and interpolation applied to the social sciences" (Vailati 1971). An intention that can be understood if we remember that Pareto included probability in the courses he taught in Lausanne. Pareto would not follow up on this intention, however, limiting himself in his writings to episodic references to (objective) probability without using it in a predictive key.

In fact, the text he published at the time, the *Cours d'économie politique* (1896-97), contained references to probability and the probability distribution applied to the distribution of income, but without representing a decisive step towards the calculation of probabilities. However, Pareto's proposal on income distribution needs to be examined in more detail in order to understand his position on probability. In particular, it is important to understand whether Pareto was also interested in an empirical rather than asymptotic approach to probability, also in view of the nature of his income curve.

Given N, the number of income units above a certain income threshold, x, A is a positive scale parameter, α is a parameter representing the slope of a curve expressed in terms of income frequencies, Pareto thus expresses the equation of the income curve: $N = A/x^{\alpha}$ (1896-97, § 958). The graphical representation of the Pareto curve is the familiar asymmetric curve in which the majority of the population has low to medium incomes, while the higher incomes are concentrated in a small proportion of the population. The logarithmic expression of this curve appears as a negatively sloping line with slope determined by the scale parameter α.

An essential step for our purposes is that Pareto assumes in footnote 962 that the distribution of incomes can correspond to an error distribution, in other words that the income curve can be represented by a normal distribution in which the mean and median incomes tend to converge. However, if a normal probability distribution can be assumed in the abstract, it must be ruled out in the concrete because it is not representative of the observed data.

As he reiterated in the *Manual* (1906, p. 194), Pareto could not proceed on the basis of assumptions of normal distributions, because experimentation/observation ruled out the possibility of those incomes having a normal distribution. More precisely, incomes above the most frequent income of value *x* have a distribution that repeats itself in time and space; not so those below income *x*. There would therefore be no symmetry between the two branches of the curve, which precludes the application of the analogy between the observed value and the construction of a curve based on the repeated drawing of black and white balls from an urn. Hence the difficulty of transforming the experimental scheme into a logical one, i.e. starting with a hypothesis of probability distribution and then testing it.

The inapplicability of the error curve would thus follow from the fact that the observed distribution has a cause and cannot be attributed to chance, as the error theory assumes, a conclusion reached by some interpreters of the Pareto distribution.

To invoke the error curve is to open up the asymmetric distribution to chance as a possible determinant. Pareto argues that chance determines the distribution of income only in the presence of highly irregular distributions of wealth (Pareto 1896-97, § 957), a condition that does not correspond to the regularity in time and space of his observations.

By 'testing' the discrepancy between the observed data and the error or normal curve, Pareto finds that the most likely value is not the mean, but a given income, *x*, which occurs most frequently.

Leaving aside the causal nature of income distribution, an interesting question arises about Pareto's position on the cause of the asymmetric curve. To hypothesise a cause is to try to identify it. Pareto speaks of a distribution of wealth that depends mainly on human nature (Pareto 1896-97, § 1012) but, as Persky notes (1992, p. 184), he remains "rather vague about what dictates human law." Pareto himself states that the distribution of income cannot be interpreted as a distribution of talents or abilities, and this is because other factors also contribute to the definition of the distribution. The cause is there, but what is important is that it gives rise to a phenomenon that is repeated in history and space, a uniformity, to be precise. It is a rule of thumb, we could say a 'stylised fact' that Pareto extracts in line with the Galilean method. As an interpreter of general economic equilibrium, for Pareto it is evidently not a priority to focus on the cause of the phenomenon, but rather on its uniformity.

Pareto seems to overthrow the relationship between cause and event. He focuses on the fact, on the stability that characterises it when it comes to income distribution.

The cause must be consistent with the results of the empirical survey, an aspect not guaranteed by the theory of errors, and with the characteristics of the phenomenon, stability, in the case at hand.

An asymmetrical curve such as the one presented by Pareto could not be the product of individuals with the same characteristics on average, hence the introduction of heterogeneity or, rather, inequality in the distribution of human abilities and skills as a factor that characterises all societies and explains the asymmetry in the distribution of economic success. In other words, human talent is unevenly distributed and access to high incomes is reserved for the most talented few.

Not only that. He shifts attention to the cost of deviating from the distribution which, although asymmetrical, appears stable. In the *Cours*, he writes: “Wealth can be transferred from some individuals to others by changing the conditions given for free competition as well as for capital investment transformations. The transfer is a necessary test of whether it is another form of already known propositions. This principle plays an analogous role in political economy to that of the second principle in thermodynamics” (Pareto 1896-97, § 730, o.t.).

Given that the second principle of thermodynamics introduces irreversibility and an increase in entropy as a result of the transfer of heat from a hot body to a cold body, Pareto assesses wealth transfers as entropic, i.e. actions that increase the entropy of the system by moving it away from equilibrium. In a planned or programmed economy that relies on wealth transfers, the system would move away from equilibrium (competition) by increasing disorder.

Intervening with income redistribution or transfer programmes means undermining a natural order of which heterogeneity is a component, with the consequent departure of society from the stable equilibrium condition achieved naturally.

The asymmetrical distribution corresponds to the equilibrium that would be lost with the adoption of economic policy measures. Thus, he writes in the *Manual* (Pareto 1906; p. 204) “it seems very likely - indeed almost certain - that humanitarian sentiments, legislative measures in favour of the poor, and other improvements in their condition contribute little or nothing to the increase of wealth, and sometimes even tend to reduce it.”

Pareto seems to want to exclude all decision-making from the distribution of income, which would instead be determined by systemic, structural aspects that exclude all individual intervention. In *Les Systèmes Socialistes*, discussing anarchist production systems, he writes: “competition […] replaces a priori choice with a posteriori choice […] in other words, applies the experimental method” (Pareto 1902-03, p. 289). Pareto changes the perspective from which to observe choices, which are useful not to achieve a goal but to maintain a condition. Given these assumptions, the probabilities should concern structural changes such as the market regime. For Pareto, the competitive market is not an object of choice or a choice among possibilities: it is a necessary condition.

This position is perfectly in line with Pareto’s anti-socialist spirit. The asymmetric distribution of income is due to natural factors that are completely independent of social changes (Cirillo 1979; p. 66). We may think, also because of his aversion to redistributive interventions, that Pareto conceives of the economies he studies as systems capable of self-organisation that converge towards a condition of stability.

By focusing on uniformities, Pareto looks at the existence of equilibria that have been determined and are maintained over time or occur given different starting conditions. Causes exist, but what matters is the stability of the end point which, at this point, becomes the starting point for the analysis.

2.1 Pareto and probability distributions

Having recalled the main features of the Pareto approach to income distribution, we can return to the possibility of expressing it in probabilistic terms.

The comparison between empirical and probability distributions is also at the heart of the *Addition* to § 962 inserted at the end of the *Cours* (1896-97, pp. 416–419). In that case, Pareto used the second experimentally derived income equation, the so-called Pareto II Type (less used because it corresponds less to observed data) corresponding to $N = A/(x + a)^{\alpha}$ (Pareto 1896, pp. 6–7) which, compared to the previous equation, only adds the constant *a* expressing inequality in the income distribution. Probabilities were included in a Gamma probability distribution built assuming g_1, g_2, g_3, … income groups each characterised by p_1, p_2, p_3, … probabilities of attaining a given income *x*. By introducing the hypothesis that there is perfect circulation between strata *g* of society and that the distribution of income probabilities within each group or stratum is equal to the distribution of the entire income, Pareto is able to write a Gamma function expressing the probability of attaining a given income *x*, and to show that for those incomes the probability equation is close to the empirically derived one.

However, the unrealism of the hypothesis - perfect circulation between income groups - leads Pareto to conclude that the correspondence between the probabilistic expression of the distribution and the empirical expression can only be valid for sufficiently high incomes, and in any case above income *x* with the highest frequency (1896-97, p. 419). In the absence of individual qualities, nothing can be said about the probability of attaining a given income, even if it is low. Lack or absence of quality and talent prevent the representation of income distribution, even in probabilistic terms. The conclusion that, despite being more complex to determine, it is the empirically determined equation that represents the income distribution more completely (Pareto 1896-97, p. 418) is not concealed.

3 Income and social groups

As the debate on the use of probability in the social and biological sciences grew, Pareto's interest in probability seems to have waned. Some evidence of this can be found in the changing attitude toward the *Prolusione* given by the mathematician Vito Volterra at the University of Turin in 1901, entitled 'On the Attempts to Apply Mathematics to the Biological and Social Sciences'. It is a text that caused quite a stir at the time and was promptly published by the *Giornale degli Economisti* in November 1901 due to the hope it contained of extending the use of mathematical tools to fields such as economics and biology. In the last part of his speech, Volterra also discusses probability, which he considers to be "the most singular and curious branch of mathematics" (Volterra 1901; p. 23). While acknowledging that "the science of probability is the only part of mathematics whose principles are not rigorously established and are still open to criticism and discussion", Volterra concludes that "whatever confidence we place in its foundations, it is undoubtedly the case that probability theory has rendered and continues to render incalculable and indisputable benefits to all sciences" (1901, p. 24).

Pareto quoted the *Prolusione* at length in a letter to Meyer on 16 December 1901 (Pareto 1960; III, p. 415) concerning his article to be published in the German Mathematical Encyclopaedia. Pareto asked for a note to accompany the article in which Volterra, Professor of Analytical Mechanics at the University of Rome, observes that the concept of *homo oeconomicus* is perfectly similar to the abstractions of mechanics, and is surprised at the difficulties it may encounter. However, Pareto, in a letter to Pantaleoni of 29 June 1905 (Pareto 1960; II, p. 448) was "delighted that a scientist like Volterra agrees to take over [his] work" (Volterra 1906), but in a subsequent letter to Pantaleoni of 2 February 1906, Pareto judged the article published by Volterra under the title *Les mathématiques dans les sciences biologiques et sociales* in the first issue of *La Revue du Mois* to be "not a great thing" (Parto 1960, II, p. 458). Volterra's article reproduces exactly the *Prolusione* of 1901 in French. Perhaps Pareto's lack of enthusiasm was directed more at the journal in which the article appeared, *La Revue du Mois*, than at Volterra himself. The journal, founded by Emil Borel, was immediately open to reflection on probability and discussion of the dogmas of mechanics (see Durant & Mazliak, 2011).

However, Pareto always seemed to be at a crossroads where one of the directions was the calculation of probabilities. Proof of this is the repetition, first in the *Manual* and then in the *Treatise*, of the division into income groups proposed in the *Addition* to the *Cours*. In the *Manual* (1906, p. 195), Pareto explicitly refers to three income sets: he distinguishes between very low incomes, which would not allow survival; the middle band, the widest, in which incomes on the one hand allow survival, even if they do not guarantee against the risk of ending up at the bottom, and on the other hand are such as to allow talented individuals to aspire to the upper income band; the latter comprises those individuals who manage to use their skills to generate income. As stated in the *Cours*, these are not closed sets, because the circulation of elites, just as it moves individuals from the middle to the upper group, also moves high income earners to the lower groups. Pareto, however, considers the phenomenon of a shift along the distribution curve to be extremely complex.

The distinction between income groups emphasised in the *Manual* becomes in the *Treatise* a focus on a society organised into social groups, an aspect that leads Pareto to consider a possible analogy with kinetic theory, by definition probabilistic. Pareto introduces the analogy with the statistical equilibrium proper to the kinetic theory of gases (1916, Sect. 2074) by admitting that the actions of individuals compensate each other, with the result that the oscillating states of individuals can lead to a general equilibrium and a stable trend.

The analogy with the gas theory is necessitated by the observation that society is made up of more heterogeneous molecules than those that make up the economy, as Pareto states in Sect. 2079 of the *Treatise*:

> The economic system is made up of certain molecules set in motion by tastes and subject to ties (checks) in the form of obstacles to the acquisition of economic values. The social system is much more complicated, and even if we try to simplify it as far as we possibly can without falling into serious errors, we at least have to think of it as made up of certain molecules harboring residues, derivations, interests, and proclivities, and which perform, subject to numerous ties, logical and non-logical actions. (1916, Sect. 2079)

Furthermore, in 1922 Pareto stated:

> Society cannot be depicted as a whole composed of separated molecules, where each one acts following its own logic and the general rules; on the contrary, these molecules orbit around certain centers, grouped in specific collectivities, mainly acting and following the logic of sentiments and interests… Social equilibrium springs from the working of all these groups. (Pareto 1922, p. 1124)

An individual's membership of a group or class would define his or her probability of access to income. Admitting that society can be divided into specific sets introduces the possibility that it is power relations that determine the formation and distribution of groups, as well as an individual's membership of one group rather than another. The very circulation of elites, mentioned above, would be based on relations of this kind. This is Pareto focused almost entirely on sociology, outlining interpretative and research paths without, however, fully developing them. The social equilibrium, regulated by power relations, no longer resembles the stable economic equilibrium, pivoting on the income curve, the distribution of which becomes only one component of a broader and more articulated equilibrium pertaining to social aggregates rather than to individuals.

Although it is limited to a few scattered mentions in his texts, the reference to the probability distribution is sufficient to demonstrate that Pareto had a mastery of the subject and that the choice not to use it was dictated by the explanatory superiority attributed to the empirically derived equations, symmetrically, as Pareto himself explains (1896-97, pp. 416–419), to what Newton did when he used Kepler's observations to construct his theory, and not vice versa.

4 Experimental methods according to Pareto

Pareto was an early pioneer of the experimental method in economics. His income distribution curve is the result of an experiment that crosses history and observation, and the derivation of an empirical law that itself becomes a hypothesis to be tested. The same adoption of mechanics as a model for economics runs through the definition of economics itself as an experimental science, although he is well aware that experimentation in economics is different from experimentation in physics. The important thing is to concentrate on economic facts, because they are representative of the real, not the abstract, economy, and the means of getting at the real is economic experimentation. And probabilities belong to metaphysics, not to real facts (Pareto 1918; p. 112).

Instead of assuming a logical hypothesis to be tested, Pareto prefers the search for uniformities to be translated, when repeatedly confirmed, into ‘empirical laws’ (Pareto 1896-97, § 959). After stating that the deductive method alone may not be sufficient (Pareto 1906, § 35), Pareto repeatedly emphasises that abstraction must proceed from real facts, as this is the only way to keep economics within the *natural* sciences, fully consistent with the method borrowed from classical mechanics: “facts exist previous to laws and ideas and, hence, theories are formulated in order to discover these laws through their conformity to facts (Cirillo 1979; p. 29). Consequently, the application in economics of an experimental and not only logical method is considered by Pareto as an important step in the direction of ‘true’ scientific knowledge. It is thus understandable why, even though Pareto ranged from pure economics to sociology and political science, he did not abandon his experimental method, indeed he made it an authentic “thread running through all [his] research” (Marchionatti et al. 2007, p. xxvi). It is also true that, despite his enduring faith in experimentation, Pareto does not explain what experimental study applied to economics or other social sciences consists of. Concretely, Pareto ties the construction of hypotheses to some form of *experimentation*, of which, however, he does not give a *rigorous* definition when extended to the social sciences. In fact, Pareto makes experimentation coincide with observation, repeating variously that to observe *is to* experiment. In the *Manual he* writes:

> When I speak of the experimental method, I am expressing myself elliptically; I mean the method that uses either experimentation, or observation, or both together if it is possible. (Pareto 1906; p. 8)

Pareto writes that “Comte tries to subordinate facts to his ideas instead of coordinating facts and subordinating his ideas to them” (Pareto 1916, § 1537). For Pareto, fact, which becomes *uniformity* through *observation*, *history* and *statistics*, is the core of analysis:

> In the practice of the social sciences, one must especially be particularly on one’s guard against intrusions of personal sentiments; for a writer is inclined to look not for what is and nothing, but for what *ought* to be in order to fit in with his religious, moral, patriotic, humanitarian or other sentiments. The quest for uniformity is an end in itself (Pareto 1916, § 2411).

It is clear that the observer must not interfere with the observed object, i.e. Pareto excludes all subjective aspects and errors that may be the source of uncertainty with respect to the phenomenon under investigation and that would justify the use of probabilities. Pareto excludes a priori all those aspects - errors of observation, reliability of the sample - that become relevant in the calculus of probability.

Pareto echoes Max Weber’s statement (Weber 1904; p. 122; see also Weber 2023) that the conceptual relationships between problems are important when he goes on to assert that “Science has the almost unique aim of finding relations between facts.

This is not in general possible, and relations are found between artificial facts, which are more or less close to natural facts (physics, chemistry) or between abstractions deduced from those facts (physics, chemistry, astronomy, geology … political economy)" (Pareto 1918; p. 5). In the end, Weber's problems become Pareto's abstractions derived from the uniformity of facts.

Pareto studies past and present realities to distil their equilibrium conditions but does not accept that equilibrium can be the subject of planning or policy. The future is not the temporal dimension in which to place the achievement of plans or projects, but where to project current uniformities (Pareto 1917). He writes in the *Manual*: "If one does not admit that there are uniformities, knowledge of the past and the present is mere curiosity, and nothing can be inferred from it regarding the future" (Pareto 1906; p. 3). It is also evident that such a view does not require any use of probabilities, which are by definition projected into the future. It is, however, a perspective that rests on the possibility of identifying uniformities.

Pareto mentions Laplace, but not Laplacian probability, which is a means of combining observation and calculation (Laplace 1814). "Mathematics, on which calculus is based, is therefore the point of union between theoretical and experimental research" (Fortino 2000; p. 15). Pareto hints at such a role for mathematics, but without any explicit reference to probability. Thus, for Pareto, the calculation of probability does not seem to be a necessary step in the application of mathematics to the world of experience, as it was for Laplace (see Pesenti Cambursano 1967).

5 Pareto followers using the probability distribution

Corrado Gini's 1907 *Contributo alle applicazioni statistiche del calcolo delle probabilità* is remarkable for the clear distinction it makes between deductive and inductive probability calculus. It is interesting to note that Gini can be considered part of a group of economists and statisticians, including Rodolfo Benini, Costantino Bresciani-Turroni and Giorgio Mortara, who, in the early years of the century, discussed both the probability and the Paretian curve of income distribution without, however, arriving at a distribution of income as a probability distribution (see Prévost 2009). A second group of economists/statisticians/mathematicians can also be identified who were also interested in probability and income distribution, but who differed from the previous group in their use of probability distribution curves. This group was ideally led by Paolo Francesco Cantelli and included, among others, Felice Vinci, Luigi Amoroso and Felice D'Addario.

In effect, among the remarkable number of publications that followed the appearance of Pareto's law, two main trends can be identified (see Bagni 1915; Brambilla 1940), although they are not always clearly distinguishable. It is true that all these authors, the first and the second group, recognised the theoretical primacy of Pareto, but although all took from Pareto the income distribution as object of analysis, part of them move autonomously with regard to the methodology.

The first group sought to improve the graphical and mathematical representation of the distribution of income by making the curves fit the data better. The approach of these authors could be described as statistical without probability distributions even though, as said, authors such as Gini had written extensively on probability (Gini 1907; Prévost 2009).

Taking into consideration the second group, those of the mathematicians/economists interpreting the Pareto curve in terms of a probability distribution, it is worth noting that Cantelli and others tried to model probability long before Haavelmo's proposal and the economists and econometricians who, from the 1930s and 1940s onwards, also completed the probabilistic revolution in economics. And it is remarkable that they tried to model the calculus and the distribution of probabilities rather than just presenting them.

As previously mentioned, these economists/mathematicians/statisticians, using the analogy between social phenomena and the behaviour of gas molecules, reinterpreted the income distribution curve as a probability distribution (see also Chipman 1976). Two mathematicians - Castelnuovo (1919) and Francesco Paolo Cantelli (1917, 1920, 1921, 1929) - are credited with initiating this line of research which included, in addition to the aforementioned Vinci (1921, 1924), Amoroso (1925), Bordin (1933), De Pietri-Tonelli (1931), Harold T. Davis (1949), Castellani (1950) and D'Addario (1934, 1936).

Vinci (1924; p. 129) attributes to Pareto the merit of having intuited and anticipated the extension to social phenomena that Cantelli (1921) was to propose regarding Ludwig Boltzmann's demonstration (Boltzmann 1896) stating that a set of gas molecules evolves towards a condition of thermal equilibrium. It is difficult to find a clear confirmation of this interpretation in Pareto, except for the equation between the thermal equilibrium reached by the molecules and the stability of income distribution.

In general terms, the adoption of probability distributions instead of observations forces these authors to question the cause of the asymmetrical distribution, to search for what Castelnuovo termed 'living energy' or 'vis viva' (in short, kinetic energy, Castelnuovo 1919; p. 276), i.e. the cause of the skewed distribution of income. The answers provided include those already foreshadowed by Pareto - the distribution of talent and power - but also factors such as chance and the pursuit of utility under conditions of disequilibrium.

Cantelli's attempt to model the income distribution curve by introducing probabilities (see Regazzini 1987) is worth mentioning. Cantelli's starting point was a multinomial discrete distribution expressing a given group of agents n_y, under a constraint$H = n_1 f(x_1) + n_2 f(x_2) + \cdots + n_y f(x_y)$, where H=constant, and $f(x_y)$ are the "living energy" of Castelnuovo making agents move. Assuming the constant H, Cantelli meant that the sum of utility or the energy of individuals cannot increase. Moreover, each group n is characterized by a given utility or energy. Having stated that the amount of income to distribute is fixed, and the probabilities of each agent belonging in one group rather than another are the same, Cantelli moves by means of Stirling equations to a continuous distribution, obtaining an exponential probability distribution, $n_i = ke^{-bf(x_i)}$,(i=1, 2, …, y; k and b=constant), which maximizes the probability of the above factorial. Equating the above second Pareto-equation to the exponential probability distribution, $\frac{B}{(x+a)^{\alpha+1}} = ke^{-bf(x)}$, with B=constant, Cantelli can obtain $f(x) = c\log(x+a) + h$, where c and h are both constant., and represent the utility of a single income x, or average utility. In this way, Cantelli can conclude that the Pareto II Type $N = A/(x+a)^{\alpha}$ is the more probable among the possible laws of frequence satisfying the condition $\int \phi(x) \log(x+a)\,dx = C,$ where a and C are constant, obtained by generalizing the above $f(x)$ = […].

Cantelli reaches a conclusion of some interest for the interpretation of the Pareto distribution. Having established that the income of an individual depends on a combination of causes (activity, ability, competition, the character of the individual, etc.) in such a way that it appears to depend on chance, the distribution of income in a given country and in a given epoch knows only one constraint: that the sum of the utilities of the incomes received is assigned (1921, p. 90). Assuming an average utility of income holders equal to x, the most probable distribution function, Cantelli concludes, is the Pareto II Type.

Cantelli later (1929) replaced the constraint of total utility, which was too fugitive, with the total wages of a country, which can be compared to the quantity of labour in a country. He shows that the second equation is still the more probable one, but adds a second constraint: that the wage depends on the quantity of labour demanded by the worker (1929, p. 852).

Pareto scholars kept the interpretation of the Pareto curve within the realm of physics, more precisely mechanical statistics, restoring to economic phenomena that aura of uncertainty that Pareto determinism had to some extent erased. It is interesting, in this respect, to observe that the attempts of this group of scholars to identify living energy ended symbolically with the 1953 publication of Champernowne's *A model of income distribution* which, in a way, revived attention to the Pareto curve by taking it back to its origins, to the note to Sect. 962 of the *Cours*. Champernowne (1953), in fact, ties the probability of an income earner moving from one income group to another to entirely stochastic factors.

6 Conclusions

Pareto does not use the calculus of probabilities available in his time by choice, not because he does not know its content, as the notes in the *Cours* testify, but because this would mean adopting an economic method that puts logical construction before experimentation. He does not resort to probabilities because the distribution of income does not depend on chance, but on certain causal, real factors whose effects can be observed.

Probabilities would have obliged Pareto to bring observed facts within average values, as opposed to the need to represent reality as it is observed, even when it shows imbalances, as in the case of the flat tail. Pareto aims to interpret reality, not a probable reality. For Pareto, priority should be given to theorising what is experimental, in other words, what is observable and translatable into uniformity, consistent with classical physics. He shows, however, that he is familiar - as argued by Vinci - with the world of probability, although he proposes sporadic references to it and leaves its application unspecified, as in the case of a society composed of income groups. Implementing it would have meant partially calling into question the rigorous but also artificial experiment-based method pursued up to then.

Can we say that Paretians interpret and develop what Pareto only outlined on the subject of probability and its distributions? In part. The probability distributions that characterise the Castelnuovo, Cantelli, Vinci and Bordin strand deepen the intuitions contained in Note 962 and in the *Addition* to the *Cours*. However, the comparison with empirical detection is lost. What remains to be noted are the Paretian developments in the direction of statistical mechanics hinging on the analogy with kinetic theory associated with Pareto's sporadic references to a society - not to an economy - organised in groups contained in the *Treatise* and in his later writings. As already pointed out, these episodes are more like anticipations.

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