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# Non-asymptotic Weibull tails explain the statistics of extreme daily precipitation

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A B S T R A C T

The exceedance probability of extreme daily precipitation is usually quantified assuming asymptotic behaviours. Non-asymptotic statistics, however, would allow us to describe extremes with reduced uncertainty and to establish relations between physical processes and emerging extremes. These approaches are still mistrusted by part of the community as they rely on assumptions on the tail behaviour of the daily precipitation distribution. This paper addresses this gap. We use global quality-controlled long rain gauge records to show that daily precipitation annual maxima are samples likely emerging from Weibull tails in most of the stations worldwide. These non-asymptotic tails can explain the statistics of observed extremes better than asymptotic approximations from extreme value theory. We call for a renewed consideration of non-asymptotic statistics for the description of extremes.

## 1. Introduction

The statistical analysis of hydro-meteorological extremes is of critical importance for risk assessment and management, early warning systems, insurance and reinsurance, and climate change impact studies (Gumbel, 1958; Katz et al., 2002). Although issues with the asymptotic assumption of extreme value theory were pointed out (e.g., Makkonen, 2008; Papalexiou and Koutsoyiannis, 2013; Serinaldi and Kilsby, 2014; De Michele, 2019), the statistics of hydro-meteorological extremes have been mostly explored actively assuming asymptotic behaviours. Reality however is not asymptotic, and to what extent it can be approximated by asymptotic theory remains rather unexplored.

According to the extreme value theorem (EVT – Fréchet, 1927; Fischer and Tipett, 1928; Gnedenko, 1943), the distribution of maxima from asymptotically large blocks (i.e., block size tending to infinity) of independent and identically-distributed variables can only converge to three limiting types: Gumbel; Fréchet; reversed Weibull. The same applies to the distribution of Poissonian exceedances of asymptotically high thresholds (i.e., thresholds that tend to the upper limit of the parent distribution domain – usually infinity). The practical advantage of EVT is clear as it allows to describe extremes sampled from unknown parent distributions using known distributions. Indeed, general distributions which include all the three limiting types are available, that is, the Generalized Extreme Value (GEV) distribution for block maxima and the Generalized Pareto (GP) distribution for threshold exceedances (von Mises, 1954; Jenkinson, 1955). Since decades, EVT represents a major theoretical background for the statistical analysis of hydro-meteorological extremes (Gumbel, 1958; Katz et al., 2002).

Determining the tail behaviour of a variable of interest – that is the rate at which the probability of exceeding increasingly high values decreases – is crucial, because it drastically influences the design values used in applications. Two recent studies summarize the state of the art for what concerns daily precipitation. Papalexiou and Koutsoyiannis (2013) examined more than 15,000 records of annual maxima globally, with the aim of identifying which of the three limiting types could better describe reality and at what conditions. The length of the available record strongly affects the estimation of the GEV shape parameter, which ultimately determines the limiting type. This has important implications for the estimation of design values in regions where short data records are available, such as many countries of the global South. Correcting for the effect of record length Papalexiou and Koutsoyiannis (2013) could narrow the range in which this parameter varies, revealing that the Fréchet law, characterised by tails heavier than exponential (i.e., the exceedance probability of extremely large values decreases more slowly than a negative exponential function), is the most likely limiting type globally. Similarly, Serinaldi and Kilsby (2014) examined approximately

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1900 complete records of daily precipitation to investigate the case of threshold exceedances. Again, the mean shape parameter tends to positive values, i.e. Pareto law (unbounded power-type tail), and its estimation variance decreases with record length. Additionally, they showed that when the threshold is decreased the EVT asymptotic assumption becomes less realistic and the GP asymptote is replaced by a Weibull penultimate asymptote.

When the tail of the parent distribution $F(x)$ is known, however, the extreme value distribution $\zeta(x)$ of the maxima sampled from $n$-sized blocks can be derived analytically (e.g., see De Michele, 2019):

$$\zeta(x) = F(x)^n \tag{1}$$

In real cases the block size $n$ may vary across the blocks $j = 1 \ldots m$, and Eq. (1) can be replaced by the expression $\zeta(x) \simeq \sum_{j=1}^{m} F(x)^{n_j}$, where $m$ is the number of blocks, e.g. years (Marani and Ignaccolo, 2015). Notably, when estimating rare extremes this inter-block variability can be neglected and Eq. (1) can still be adopted using, for $n$, the expected value of the block size $n = \frac{1}{m}\sum_{j=1}^{m} n_j$ (Marra et al., 2019; see also Serinaldi et al., 2020).

While these non-asymptotic solutions require a confident a-priori assumption about the tail behaviour of the parent distribution $F(x)$, they allow us to use ordinary statistics to describe extremes. This brings important advantages over EVT: (1) we could reduce estimation uncertainty, because the parameters of $F(x)$ can be inferred from many independent events while the parameters of the limiting distributions can only be inferred from extremes; (2) we could establish direct relations between ordinary and extreme events, that is between the physics of the processes and the statistics of the emerging extremes. Similarly to extreme value mixture models, non-asymptotic methods also allow to handle the statistics of extremes that emerge from multiple processes and thus violate the identical-distribution assumption of traditional EVT using $\zeta(x) = \prod_{i=1}^{S} F_i(x)^{n_i}$, where $i$=1…$S$ represents a process of interest (Marra et al., 2019). These advantages are very appealing, for example if one wants to consider the diverse impact that climate change may have on different synoptic meteorological systems and precipitation processes. This brings us to a natural question: provided a robust hypothesis on the tail of $F(x)$ is available, why we do not routinely use these non-asymptotic solutions?

Indeed, non-asymptotic approaches are adopted and yield rather encouraging results (e.g., Zorzetto et al., 2016; Vidrio-Sahagún and He, 2022), but are still mistrusted by part of the community. A common objection paradoxically stems from a limitation of EVT itself, that is the stochastic uncertainty that characterizes the observed extremes (e.g. Serinaldi and Kilsby, 2015; Fatichi et al., 2016; Tabari, 2021). In fact, goodness of fit tests based on the observed extremes will rarely prefer distributions derived from many ordinary events (such as Eq. (1)) over distributions directly derived from the observed extremes (such as EVT). This spreads concern about the ability of non-asymptotic methods to reproduce extremes. But is a goodness of fit with respect to a largely uncertain sample a proper measure of the quality of a model? A second common objection claims that "extremes are different and we cannot infer their statistics from the ordinary events". While apparently reasonable, this objection contrasts EVT in the same way it contrasts non-asymptotic approaches. In fact, even neglecting identical-distribution issues, the asymptotic assumption of EVT ultimately requires in each block an infinitely large sample of all the extremes we want to describe. How is this possible if extremes are so 'different' that we only have a few of them in our entire record? If we want to say anything at all about the problem, we need to accept EVT and Eq. (1) to the same extent because they ultimately build on a common practical necessity: the information we have about extremes is contained in our sample. The alternative would be to wait a million years and derive our statistics empirically. From a non-asymptotic perspective, the problem then becomes: can we identify and recognize the tail of a parent distribution $F(x)$ that contains at least the same information as the extremes used in EVT?

In this study, we make a significant step toward the use of non-asymptotic approaches for the analysis of extreme daily precipitation, a case for which a physics-backed model for the tail of the parent distribution is available: the Weibull distribution. We perform a global analysis on long rain gauge records to test whether the observed extremes are likely samples from Weibull tails, and to evaluate how well non-asymptotic samples from Weibull tails explain the statistics of observed extremes and the emerging asymptotic limiting types.

## 2. Methods

### 2.1. Data

We integrate (1) the global daily precipitation dataset created by Papalexiou and Montanari (2019), based on the Global Historical Climatology Network-Daily database (Menne et al., 2012a), with data covering (2) tropical Sub-Saharan Africa (Ghana; Amponsah et al., 2022) and (2) Mediterranean, semiarid and arid regions in the Levant (Israel; Marra et al., 2021). Although only accounting for a small fraction of the entire dataset these two additional sources represent the sub-sampled climate of Sub-Saharan Africa, typically characterized by fairly homogeneous climate stretching from the East coast of the Atlantic Ocean in West Africa through East Africa (Peel et al., 2007; Panthou et al., 2012), and the sharp transition between Mediterranean and arid climates of the southern Mediterranean (Goldreich, 2003). Both the additional datasets were quality-controlled by the Ghana Meteorological Agency (using the CLIDATA software; www.clidata.cz), the Israel Meteorological Service and by the authors of the studies. For the dataset at (3) only data from Israel, for which the quality control history was known, have been retained. The global dataset at (1) was carefully screened by Papalexiou and Montanari (2019) to only retain stations with at least 50 years with less than 1% of the days assigned with quality flags. Similarly, we screened the datasets at (2) and (3) to only retain stations with less than 1% of the days assigned with quality flags. We further screen the data to remove possible duplicate stations and to extract long and complete records of daily precipitation. Only years with less than 10% missing data are considered complete, and only stations with at least 50 complete calendar years are retained. The final dataset consists of 8254 stations (see Fig. S1 in the Supporting Information for the exact locations).

### 2.2. Non-asymptotic approach

#### 2.2.1. A parent distribution for heavy daily precipitation

Following theoretical reasoning (e.g., Wilson and Toumi, 2005; Porporato et al., 2006) and empirical results (e.g., Papalexiou and Koutsoyiannis, 2012; Serinaldi and Kilsby, 2014; Zorzetto et al., 2016), we focus on parent distributions with Weibull tails. The Weibull distribution should not be confused with the third limiting type of the EVT, which is the *reversed* Weibull law and is upper-bounded. The Weibull distribution belongs to the powered-exponential family and its cumulative distribution function has a scale parameter $\lambda$ and a shape parameter $\kappa$:

$$F(x) = 1 - \exp\left(-\left(\frac{x}{\lambda}\right)^{\kappa}\right) \tag{2}$$

Although asymptotically converging to the Gumbel law, Weibull tails are compatible with previous findings showing that extreme precipitation tends to follow the Fréchet law: "*as strange it may seem, annual maxima extracted from a parent distribution that belongs to the domain of attraction of the Gumbel law*" (such as Weibull) "*are better described by the Fréchet law. This occurs for two reasons: first, the convergence rate to the Gumbel law is extremely slow, and second, the shape parameter of the Fréchet law enables the distribution to approximate quite well not only distributions with power-type tails but also other heavy-tailed distributions*" (Papalexiou and Koutsoyiannis, 2013).

Here, we define as wet all the days in which at least 0.1 mm of precipitation is reported, and we assume independence of the wet days. Note that daily precipitation typically has low lag-1 autocorrelation, around 0.1–0.4 (and even wet hours have moderate autocorrelation; Papalexiou, 2022). Monte Carlo simulations show that our results would be essentially unaltered by the presence of such correlations. The parameters of the Weibull distribution are estimated by left-censoring the ordinary events not exceeding a threshold of interest (see below) and using a least-square linear regression in Weibull-transformed coordinates (codes available in Marra, 2020).

### 2.2.2. A test for the tail of the parent distribution

Saying that daily precipitation has a Weibull tail means that there exists a threshold θ above which the parent distribution $F(x)$ can be approximated using the two parameter Weibull distribution in Eq. (2). To verify this assumption, we use a Monte Carlo test based on the idea presented in Marra et al. (2020) and refined by Marra et al. (2022). The test starts from a null-hypothesis about the tail behaviour of $F(x)$ and evaluates whether the observed extremes are compatible with this model. The added value of this approach over goodness of fit methods is twofold. First, we start from a physics-backed hypothesis (in our case the Weibull model by Wilson and Toumi, 2005) and test whether the statistics of the observed extremes are or are not likely samples from this tail model. Second, we explicitly censor all the observed extremes from the tail testing and definition. To do so, we explicitly censor from the parameter estimation both the values below a given left-censoring threshold, and all the annual maxima. This grants an independent evaluation of the ability of our tail model to represent the statistics of the observed extremes, a feature not available using EVT methods.

The test proceeds as follows: (1) estimate the tail model parameters corresponding to a given left-censoring threshold θ by explicitly censoring the observed block maxima (i.e., censoring their magnitude but retaining their weight in probability) – in our case we estimate λ and κ of Eq. (2), but any tail model can be considered and tested; (2) generate a large number of synthetic records according to this tail model and with the same sampling characteristics of observations (number of blocks, number of elements per block); (3) test whether observed maxima are likely samples from these records. A schematic of the test is provided in Fig. 1 and the codes are made available in Marra (2022).

The outcome of the test cannot guarantee a specific tail model is appropriate, but serves as a filter to reject models which contradict observations. Additionally, the sensitivity of the test and its specificity against alternative hypotheses can be evaluated in a controlled environment (see Fig. S2). We generate 500 synthetic records for each case and reject the Weibull model whenever more than $p = 10\%$ of the observed maxima lie outside of the $1 - p = 90\%$ sampling confidence interval of the synthetic records. The choice of $p$ should account for the length of the available record (i.e., how many block maxima are observed). Since its outcomes ultimately depend on the observed maxima, the test is subject to some level of stochasticity. It is advised to run it over multiple records for which a homogeneous left-censoring threshold is expected (e.g., nearby independent stations) to gather robust results. Analyses based on synthetic data show that the test is robust (Fig. S2). The probability of type I errors (wrong rejection of Weibull tail) is of the order of ~5–10%. The probability of Type II errors (wrong not-rejection of Weibull tail) depends on the alternative tail. In presence of Generalized Pareto tails, Type II errors are likely and their probability depends on the characteristics of the data and on the adopted left-censoring thresholds. In presence of power-type tails, a supported heavy-tail alternative, the probability of Type II errors is zero or close to zero.

Running the test over different thresholds and selecting the smallest left-censoring threshold $\theta^*$ for which the test is not rejected (Fig. 1) allows us to identify the tail of the parent distribution for which we cannot reject the assumption that block maxima were sampled. Here, we test thresholds between $\theta = 0$ (i.e., all non-zero daily precipitation amounts are in the tail) to $\theta = 0.95$ (i.e., the 95-th percentile) with steps of 0.05. Our definition of the left-censoring thresholds implies that any threshold $\theta \geq \theta^*$ leads to the same tail model (Marra et al., 2019). When our tail model is rejected also for $\theta = 0.95$, we consider the Weibull tail as rejected but we retain results for $\theta^* = 0.95$ in the subsequent analyses.

## 2.3. Comparison between non-asymptotic Weibull tails and asymptotic tails

Thresholds exceedances to define Generalized Pareto (GP) tails (as in EVT) are extracted using thresholds $\theta_{GP} = 0.95$. This threshold should asymptotically tend to infinity, and our choice comes as a compromise with sample size. Results derived from higher thresholds such as the 98-th percentile used by Serinaldi and Kilsby (2014) are qualitatively

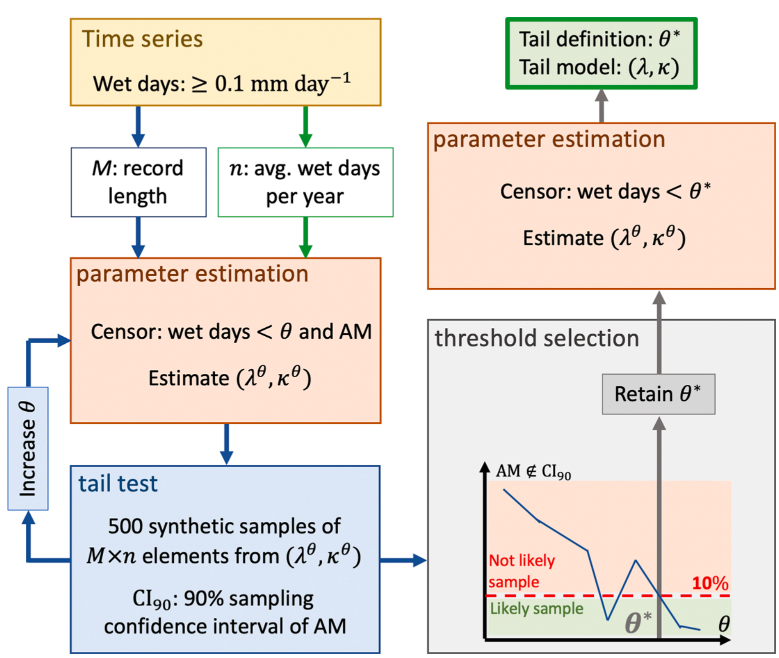


**Fig. 1.** Schematic representation of the tail test.

analogous but characterized by larger uncertainties. Parameters of the GP distribution describing the exceedances are estimated with the L-moments method (Hosking, 1990).

We quantify the statistical properties of extremes using the third and fourth L-moment ratios of the annual maxima: L-skewness and L-kurtosis (Hosking, 1990). These are linear combinations of order statistics of a distribution and are used to quantitatively describe its high moments (shape characteristics). They are independent of the statistical model used to describe extremes, and are robust to stochastic sampling uncertainties. We compute L-moment ratios from observed annual maxima, to serve as a quantification of the statistical properties of the observed extremes, and from stochastic samples derived from Weibull and GP tails. Each stochastic sample is composed of $m$ blocks and $n$ elements in each block (with $n$ rounded to the closest integer, thus reflecting the non-asymptotic characteristics of reality). From each sample, we extract the block maxima and estimate the corresponding L-moment ratios. Weibull tails are generated as Weibull-distributed records with $n$ elements in each block. GP tails are generated as GP-distributed threshold exceedances and are composed of $(1-\theta_{GP}) \cdot n$ elements; the remaining $n \cdot \theta_{GP}$ elements are infilled by randomly resampling the observations

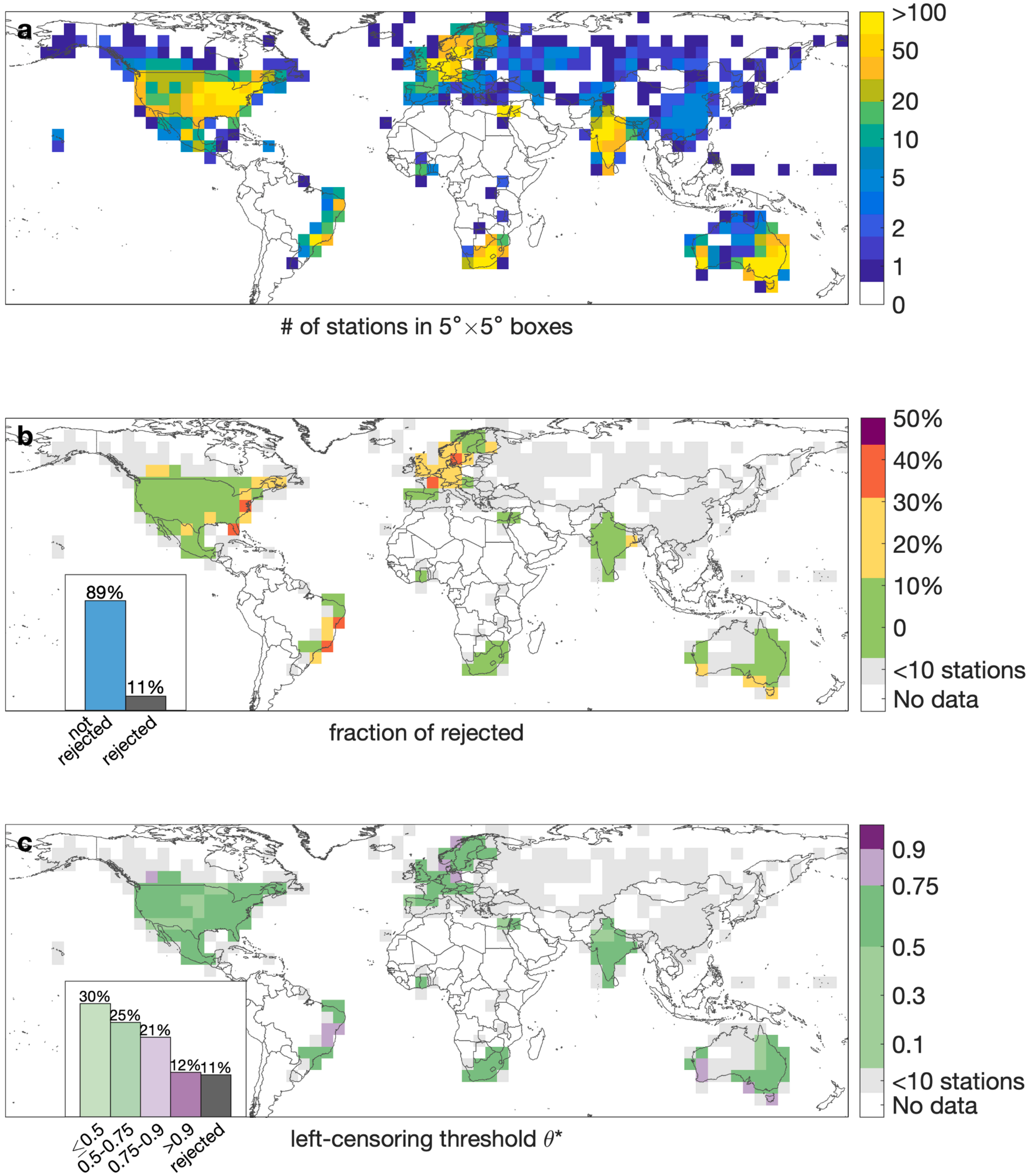


**Fig. 2.** (a) Global density (number of stations in 5°×5° boxes) of the stations used in this study. (b) Fraction of stations within 5°×5° boxes for which the hypothesis of non-asymptotic Weibull tails is rejected. (c) Average left censoring threshold $\theta^*$ used to define the Weibull tails (the value $\theta^* = 0.95$ is used for stations in which Weibull tails are rejected); the inset shows the frequency of the left-censoring thresholds $\theta^*$ across stations. Only boxes in which at least 10 stations are available are shown in (b) and (c).

below threshold. For each station and model, we generate two distinct cases:

1 Synthetic samples with the same number of blocks as the observed records and optimal parameters (i.e., the ones for $\theta = \theta^*$ and $\theta_{GP} = 0.95$). These samples reproduce the characteristics of reality and are used to visually compare reality and models by means of L-moment ratios diagrams.
2 Synthetic samples with $m = 10^3$ and parameters derived using thresholds increasing from 0 to 0.95 with steps of 0.05. These samples represent ideal long-record conditions for which sampling uncertainty is negligible. They are used to quantify the deviations between the statistics of observed maxima and the statistics of maxima sampled from the different tail models. Here we examine a third tail model GP*, which consists of GP tails whose parameters are estimated from the synthetic Weibull samples. This third model represents a situation in which reality has Weibull tails and GP tails are erroneously assumed.

Sample variability related to finite-length observations are quantified as the difference between theoretical L-moments derived from Weibull tails and empirical L-moments computed from the $m$-blocks samples from Weibull tails, where $m$ is the number of available blocks for the station of interest. To this end, we generate 25 synthetic samples for each station and show the median value across these samples.

We derive analytically the shape parameter of a GEV distribution fitting annual maxima from Weibull tails with the same characteristics as the observed data. To do so, we compute the theoretical L-skewness of annual maxima sampled from non-asymptotic Weibull tails by integrating numerically ($du = 10^{-6}$) the relation provided in Zaghloul et al. (2020): $\tau_3 = \frac{\int_0^1 Q(u,1,\kappa,n)\cdot(6u^2-6u+1)du}{\int_0^1 Q(u,1,\kappa,n)\cdot(2u-1)du}$, where $Q(u, 1, \kappa, n) = F(1, \kappa)^n$ is the quantile function of an exponentiated Weibull and $n$ is the average number of ordinary events per year rounded to the closest integer. We then derive the shape parameter $\gamma_{GEV}$ of the GEV distribution corresponding to the computed L-skewness by inverting the relation $\tau_3 = \frac{2(1-3^{-\gamma_{GEV}})}{1-2^{-\gamma_{GEV}}} - 3$ (Hosking, 1990). This parameter can be interpreted as the apparent limiting type for block maxima emerging from non-asymptotic Weibull tails.

## 3. Results

The assumption of having the observed annual maximum daily precipitation emerging from non-asymptotic Weibull tails cannot be rejected in 89% of the stations globally (Fig. 2b). Most (∼74%) of the regions for which sufficient data is available (we report results for $5^\circ \times 5^\circ$ boxes with at least 10 stations, see Fig. 2a) show rejection rates not exceeding ∼10%. Rejection rates never exceed ∼45%, and exceed ∼30% in less than 5% of the examined regions. Most of the stations globally (∼55%) need left censoring thresholds as low as $\theta \leq 0.75$, and ∼76% of the stations globally need left censoring thresholds $\theta \leq 0.9$ (Fig. 2c). Given the sensitivity of our test and its specificity against alternative tails (Fig. S2), these results imply that for a large majority of the examined areas non-asymptotic Weibull tails are to be preferred over heavier-tailed options. Regions with higher rejection rates are clustered in three main areas: central/northern Europe and the Atlantic coasts of north and south America (Fig. 2b). Here, Weibull tails could be either too light or too heavy to fully describe the observed annual maxima. Qualitative analyses (not shown) suggest that Weibull tails could be too light in the two regions in the northern hemisphere, and too heavy in the Atlantic coast of south America. It could be that the assumptions used by Wilson and Toumi (2005) to derive the Weibull tail model are less verified in these regions. For example, temporal trends would lead to apparently heavier tails; this is expected to be more likely in stations with longer records, but we could not find any evidence of such a relation. It should be however noticed that, as shown by Amponsah et al. (2022) (see the supporting information therein), the trends typically reported are not large enough to impact the results of our test. Another possibility is that the assumption of independence of the wet days could be less verified in some areas. Additionally, Monte Carlo simulations show that deviations from the assumption of independence of the wet days may cause the test to reject the Weibull tail hypothesis more often. For example, this could be the case of Northern Europe, where long-lasting stratiform events are frequent. Future works should investigate more in detail the possible physical and/or statistical reasons behind these behaviours.

L-moment ratio diagrams such as the one in Fig. 3 are commonly used to assess the reliability of specific tail models. Fig. 3a shows that GP tails (blue) are able to capture the general statistics of annual maxima (black), although an important tendency to over-estimate tail heaviness is manifested. L-skewness values greater than the largest observed value globally are often reported, even reaching values as high as 0.6 or 0.7. Considering that GEV distributions with shape parameter greater than 0.5 (displayed in the secondary axis of the figure as $\gamma_{GEV}$) have infinite variance, these values are likely unrealistic. These qualitative results support previous findings in which GP tails used to create synthetic data were shown to generate unrealistically high values and the creation of a powered-exponential system of distributions, introduced in Papalexiou (2022), to support the stochastic modelling of precipitation at multiple scales. The statistics emerging from non-asymptotic Weibull tails (in red in Fig. 3b) closely resemble the ones of observations (black) and the ones reported for the global dataset of daily annual maxima (>15,000 stations) by Papalexiou and Koutsoyiannis (2013).

The apparent GEV shape parameters emerging from non-asymptotic Weibull tails depend on the shape parameter of the underlying Weibull distribution and on the average yearly number of wet days (Fig. 4a). Convergence to the limiting distribution (Gumbel law with $\gamma_{GEV} = 0$ in the case of Weibull tails) is extremely slow, and stations with over 250 wet days are still far from this limit (note the logarithmic scale on the colour bar. The apparent limiting type is coherent with the parameter range extrapolated by Papalexiou and Koutsoyiannis (2013). Overall, ∼98% of the stations worldwide appear to follow the Fréchet law, and only 0.2% the reversed Weibull law (Fig. 4b).

Fig. 5a, b shows the error in L-skewness and L-kurtosis of annual maxima emerging from non-asymptotic tails with respect to the ones computed from the observed maxima. We show the case of Weibull tails (red) and GP tails (blue) estimated using different left-censoring thresholds $\theta$. The figure highlights two important aspects. First, within the stochastic uncertainties related to the available records (shaded in grey the 90% confidence interval of the median across stations), the statistics of annual maxima are well reproduced by non-asymptotic Weibull tails even for relatively low thresholds. Conversely, the errors for maxima sampled from GP tails strongly depend on the left-censoring threshold and tend to be too heavy-tailed for $\theta \leq 0.90$. The accuracy of GP tails in reproducing the statistics of observed maxima is comparable to the one of Weibull tails only for thresholds $\theta > 0.9$. Second, GP* tails estimated from synthetic Weibull-distributed data, are virtually indistinguishable from the GP tails estimated from real observations (dashed blue). As predicted by EVT, GP tails tend to provide similar estimates upon asymptotic conditions (here represented by $\theta_{GP} = 0.95$; see also Serinaldi and Kilsby, 2014) and the difference in L-moment ratios between non-asymptotic Weibull tails and GP tails decreases with increasing threshold (Fig. 5c). Crucially, the difference between L-moment ratios of annual maxima emerging from GP and GP* tails (dashed) are virtually indistinguishable also for high thresholds such as $\theta = 0.95$, and smaller than the differences between L-moment ratios of annual maxima emerging from GP and Weibull tails (Fig. 5c). Estimating GP tails from observations is equivalent to estimating GP tails from Weibull data.

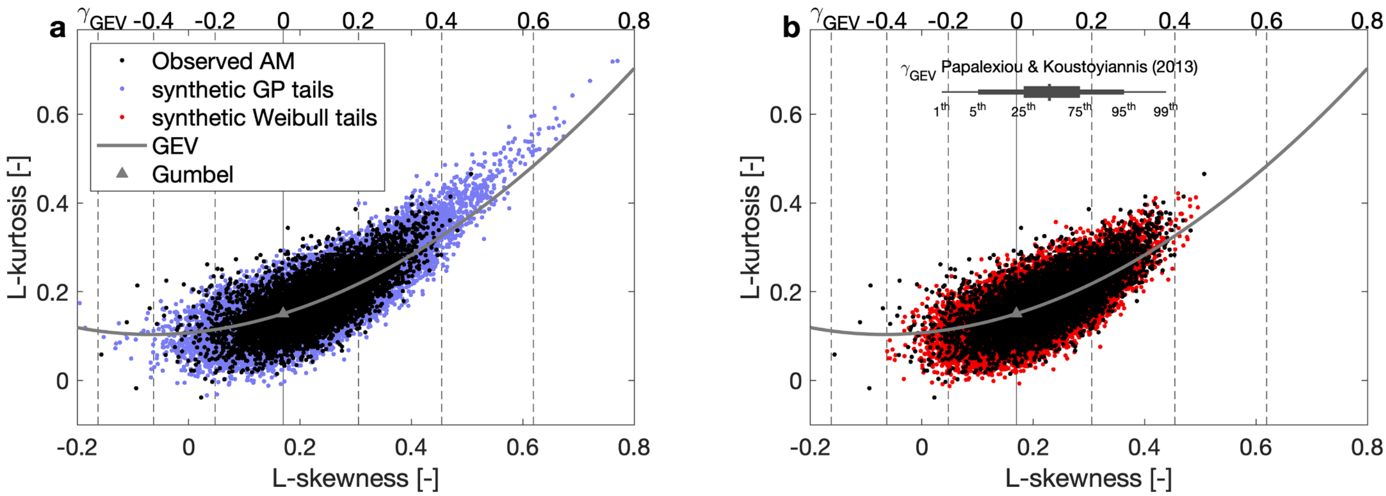


**Fig. 3.** L-moments ratio diagrams for observed annual maxima (AM, black dots – note that they are the same in both panels) and (a) synthetic annual maxima emerging from GP tails (blue dots, $\theta_{GP} = 0.95$) and (b) synthetic annual maxima emerging from non-asymptotic Weibull tails (red dots, $\theta = \theta^{*}$). The secondary x-axes show the corresponding shape parameters of the GEV distribution $\gamma_{GEV}$. The inset in (b) shows the distribution of empirical GEV shape parameters derived by Papalexiou and Koutsoyiannis (2013) (Figure 7 therein).

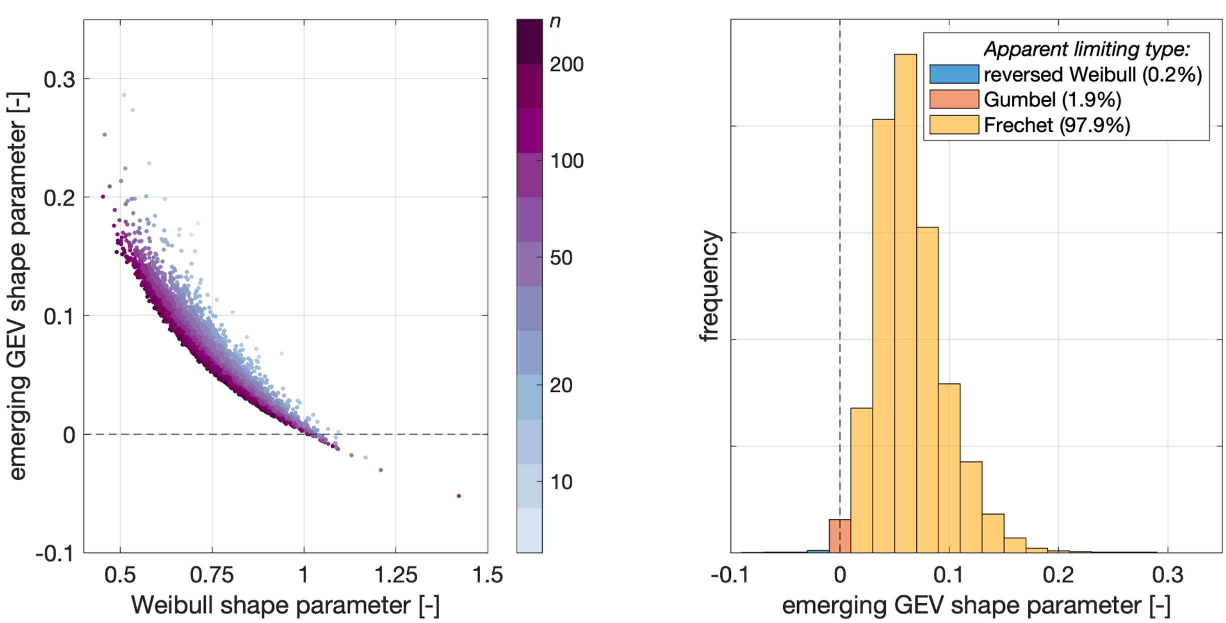


**Fig. 4.** (a) GEV shape parameter (i.e., derived from an analytical estimation of the L-skewness) emerging from non-asymptotic Weibull tails shown as a function of the underlying Weibull distribution shape parameter (x-axis) and of the average yearly number of wet days $n$ (colour). (b) Global frequency of the GEV shape parameters emerging from non-asymptotic Weibull tails. Apparent limiting types of EVT are shown with different colours (Gumbel is associated to $\gamma_{GEV} \in [-0.01, \quad 0.01]$).

## 4. Conclusions

A large dataset of long-recording quality-controlled rain gauges is used to explore the statistics of extreme daily precipitation from a non-asymptotic perspective. We start from a physics-backed hypothesis and we consider the specificity of our test with respect to alternative hypotheses. The null-hypothesis of having daily precipitation annual maxima emerging from parent distributions with Weibull tails cannot be rejected in ~89% of the stations analysed all over the globe. We identify specific regions where this assumption could be less robust and that require further investigations of the underlying physical reasons.

We show that daily precipitation with non-asymptotic Weibull tails can explain: (1) the L-moment ratios of observed annual maxima, (2) the apparent asymptotic limiting behaviour of annual maxima, and (3) the characteristics of asymptotic tails estimated from observations. These results support the use of non-asymptotic Weibull tails to estimate the statistics of extreme daily precipitation in many regions of the Earth. Similar analyses based on sub-daily and multi-day durations are solicited to extend our knowledge of heavy precipitation across scales. Reality is not asymptotic, we call for a renewed consideration of non-asymptotic statistics for the description of extremes.

## CRediT authorship contribution statement

**Francesco Marra:** Conceptualization, Data curation, Writing – original draft, Writing – review & editing. **William Amponsah:** Data curation, Writing – review & editing. **Simon Michael Papalexiou:** Conceptualization, Data curation, Writing – review & editing.

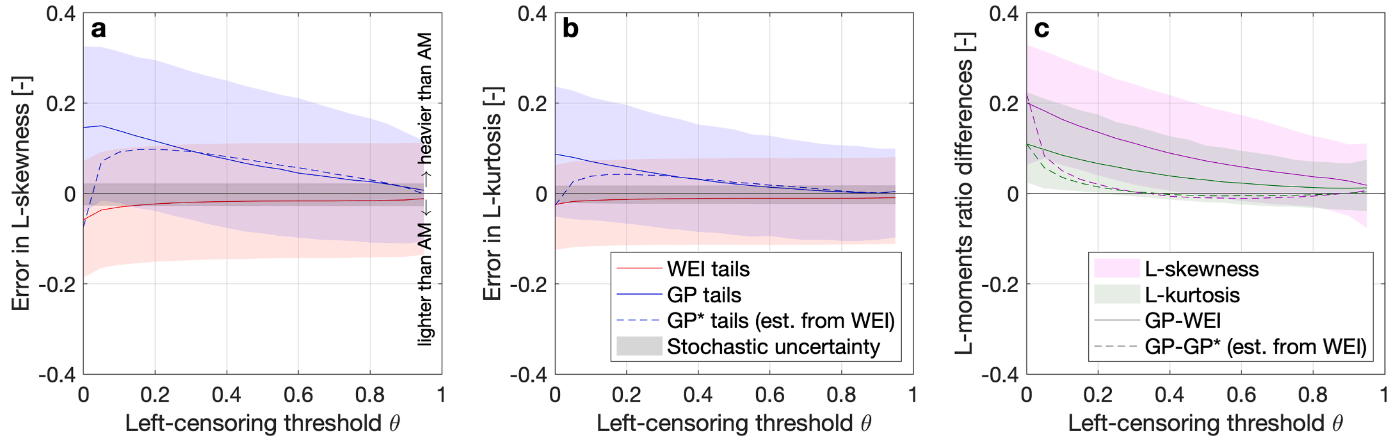


**Fig. 5.** Error in L-skewness (a) and L-kurtosis (b) of annual maxima estimated from MC samples of $10^3$ years of non-asymptotic Weibull tails (WEI, red) and GP tails (blue) with respect to observed annual maxima; solid lines and shaded areas represent, respectively, median and 90% confidence interval across the stations; dashed blue lines show the median for the case of GP tail model estimated from the synthetic Weibull tails (GP*); shaded grey areas in (a) and (b) quantify the stochastic uncertainty due to the available data record in presence of non-asymptotic Weibull tails. (c) Difference between L-skewness (purple) and L-kurtosis (green) derived from GP and Weibull tails (solid lines for the median, shaded areas for the 90% confidence interval) and from GP tails and GP* estimated from the synthetic Weibull tails (dashed, only the median is shown).

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data Availability

Codes used for the study are available in public repositories: 10.5281/zenodo.3971558 (Marra, 2020), 10.5281/zenodo.7234708 (Marra, 2022)

## Acknowledgements

Codes used for the study are available in public repositories: 10.5281/zenodo.3971558 (Marra, 2020), 10.5281/zenodo.7234708 (Marra, 2022). Rain gauge data were obtained combining the dataset from Papalexiou and Montanari (2019) (based on Menne et al., 2012b version 3.22), Marra et al. (2021) (quality-controlled stations only) and Amponsah et al. (2022). We thank the Ghana Meteorological Agency, Yoav Levi from the Israel Meteorological Service and Efrat Morin from the Hebrew University of Jerusalem for providing the data. FM was supported by the CARIPARO Foundation through the Excellence Grant 2021 to the ``Resilience'' Project and by internal projecs of CNR-ISAC.

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