A webMathematica Application for Mathematics Learning

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Abstract. In this talk we would like to present MoMAMath, an on-line interactive software for mathematics learning, based on Mathematica and WebMathematica. The software has been realized by the authors and it has been experimented to support classical courses of basic mathematics for first year engineering students at the University of Salerno. MoMA-Math aims to improve to the didactic potentialities of the traditional textbook. In fact it is completely interactive and based on the learning by doing approach. MoMAMath's distinctive feature is the possibility of on-line interactive exercise sessions, besides other classical services like theory sessions, asynchronous communication with teachers, and so on.

1 Scenario

In this paper we present MoMAMath, that is an on-line interactive software for mathematics learning. At the moment it is in Italian and it is accessible through the following web site www.momanet.it/momamath.

Which is the scenario of the development of MoMAMath? We live in the Information and Communication Society which is more and more a Knowledge society. This has impact on our approach to learn and teach! Moreover it fosters a globalization process in High Education in Europe. This has brought in Italy a reform of the Italian University system, which aims to have two level of degree: a three-year "short degree", more technical; followed by two optional years, ending with a deeper five-year "specialised degree". The programs of the first level degree are very reduced w.r.t. the ones before reform, especially for the theoretical part, that is proofs of theorems, mathematical theoretical background. They aims to "give an idea" and then to solve exercises. Moreover time to teach has been reduced. Thus it is important to change methods and techniques of learning and teaching. This is more and more true, because otherwise we will have just people able to do standard computations or solve standard problems, but not people able to face new problems.

The impact of the new technologies has brought on many changes in activities, contents, strategies, and attitudes in education. With respect to the mathematics the new environment requires to choose an approach based on descriptive, intuitive, experimental and inductive aspects. There are multiple initiatives for

P.M.A. Sloot et al. (Eds.): ICCS 2003, LNCS 2657, pp. 754–763, 2003.

using the computer in learning process. It is sometimes used as a complementary tool for educational training, or sometimes as a learning instrument on its own. MoMAMath can be used both as complementary tool and as a learning instrument on its own for people who have not the chance to attend a classical course. We have experimented MoMAMath with engineering first year students, as complementary tool besides classical lectures.

2 General Description

The main aims of MoMAMath are first of all to support mathematics learning, then to give a qualitative improvement to the didactical properties of the traditional textbook.

The potential users are students of the scientific faculties of the universities, but more in general we can say any Internet user who is interested in learning basic mathematical concepts.

In the following we will describe the general characteristics of MoMAMath:

- Access trough a common web browser: MoMAMath is simply accessed by any web browser, so you need just a computer and an internet provider.
- Personal accounting by login e password: each user access to a personal working space by login and password and then s/he can choose topics, type of exercise, theory and so on.
- User-friendly interface: the access to the various sections can be done by clicking on some menu, the link to other exercises or theory pages is given by hot words or buttons. In the main page a help explains the use of MoMAMath and the syntax of the main functions.
- Hypermedia organization: the organization is hypermedial thanks to links that interconnect various sections. The hypermedial structure allows the autonomous organization of the connection links and of the logical consequence of the expositive text, so that the student can decide his personal learning path.
- Menu for the selection of the topics: menu are available for selecting topics and various types of exercises.
- Memory of the state of the exercise when the user logs out: when a user logs out because he wants to interrupt his/her exercise session, the system memorizes the state of the exercise so that, when the same user logs in again, s/he finds the exercises in the exact point in which s/he left it.

Let us see the main functionalities of MoMAMath.

First of all, you can make exercises: when, where, how many you want! In the next section we will precise what “how many” means.

Then you can require to do a simulation of exam, that is in a first phase you can learn some techniques to solve exercises choosing *a priori* what kind of exercise you are going to solve. In the exam session, you can require to the system to generate some exercises randomly chosen in the available set. So the student can check first his/her capability to recognize what kind of exercise s/he has to solve and then if s/he is able to solve that exercise.

Moreover an on-line teacher is present and the students can interact with him/her. At the moment, the interaction is asynchronous, that is just by email, but we plan to introduce some synchronous facilities such as forums.

Of course, the students have the possibility of consulting theory pages and help on-line.

3 Innovations and Advantages

In this section we want to point out the main innovations of MoMAMath.

First of all, our software allows to perform exercise sessions interactively on-line. It can generate infinity (and always different) exercises : this is an important feature because MoMAMath has not a database, containing exercises and solutions, but they are randomly generated *on the fly* by the software any time that a student requires one. Algorithms have been implemented in Mathematica in order to generate suitable exercises, which are consistent also from didactical point of view.

We have chosen the *divide and conquer* strategy, splitting each exercise into one or more elementary steps; that is the student is guided to the solution facing easier sub-problems. At each step a hint is given and an interaction is required, so that students have to give an answer to the current sub-problem. A suitable theoretical link is also available. If the student is wrong in his answer, the system for the first time impose to re-insert the answer in order to stimulate the students to try again, then if the student made mistake again, the system gives the chance of viewing the correct result if the student wants.

As you can image, the evaluation of the correctness of the given answer is done. We want to underline that it is not just to say "correct" or "uncorrect". The system in fact is able to recognize errors of a (most probably) theoretical character (e.g. logical inconsistencies) and computational errors.

Correspondingly, a different warning message is generated, suggesting the most likely nature of the error and suitable means of correcting it (within the MoMAMath environment). This feature proved particularly useful in saving time during the error correction phase, since students did not have to unusefully repeat the whole theoretical background in case of mere computational errors and, conversely, receiving a timely warning when they needed to get a better understanding of the underlying theory. Therefore, the previous functionality can be regarded - at least from the experimental points of view - as a kind of support, increasing the success rate for a final examination on the subject.

Moreover, MoMaMath offers a "simulation of exam". At any time of the his/her learning path, the student can decide to test its level of knowledge on one or more topics. S/He chooses the topics and how many exercises for each topics s/he wants to solve and the system generates an exam on the fly, choosing randomly the exercises in the selected topics of various difficulties. The student is required to solve by hand the problems checking directly the final answer. If s/he fails in some exercise, he is sent back to the step by step procedure.

Further advantages are the personalization of the instruction and diffusion of the instruction. Each user is able to personalize his/her instruction, choosing topics to study and time to dedicate. It could be possible to create personalized learning path based on the number and on the type of the carried out exercises stored by the system. The teacher can individuate the points of great difficulty in itself thanks to the possibility of generation of statistics of the performed exercises.

4 Development Environment

In this section we briefly describe the development environment.

For the implementation of MoMAMath we have used Mathematica for the management of the exercises; WebMathematica for the delivery of the exercises and uPortal for the management of the users.

The architecture is split into two servers: one that executes the uPortal application and one that executes WebMathematica and Mathematica. The user from the client side accesses to the system using a common web browser (at the moment Internet Explorer and Netscape are supported). The html pages which are delivered are generated by uPortal that manages the accounting (that is it communicates with a database in which all the users are registered). When a user requires a new page (for example clicking on a button or a hot word), such request is received by uPortal that interprets it and dialogues with WebMathematica calling the suitable page for that exercise. The MSP page contains the calls to the suitable Mathematica functions for that exercise and the needed instructions to construct the html page with the results of the current step of the exercise. Such instructions are exactly the functions of the package.

5 MoMAMath Portal

In the following figure 2 a representation of the areas that make the portal is given. The header area contains the logo, the accounting information and controls, and useful e-mail addresses. The user may contact the site-administrator for questions about the portal and its use, s/he may also send feedback with comments, suggestions and remarks about the MoMAMath application, and at least s/he may ask for a teacher support for problems related to the exercises.

The topics area is used to collect the exercises on the same subject. For example in MoMAMath for the course of “Matematica I” the topics are: Limits, Derivatives, Functions, and in addition the Exam Session.

List of Exercise Typologies area is a menu containing the list of the available exercises about the chosen topic. Selecting one item from the list it loads in the working area the last exercise of that kind that the user has studied and has left incomplete.

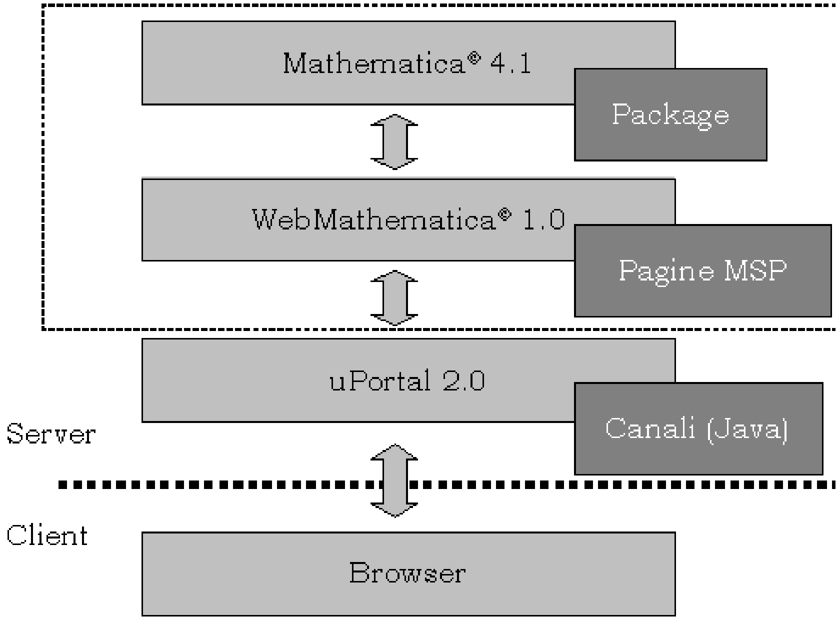

Fig. 1. MoMAMath architecture

The working area is plainly the area reserved to the execution of the exercise, and it contains also the number of exercises available to the user, since the number of exercises in MoMAMath per Account is limited.

6 Examples

Let us show now some examples to illustrate the system. The first chosen example is a single step exercise but it clarifies what we mean for "elementary step" and what kind of check the system is able to do.

Consider the following exercise regarding vector spaces:

Determine the dimension and a basis of the vector space V represented by the linear system:

$$\begin{cases} -x_1 + 4x_2 + x_3 - 3x_4 = 0 \\ -3x_1 + 2x_2 - 3x_3 + 2x_4 = 0 \end{cases}$$

The system suggests the steps to do:

Reduce the coefficients matrix of the given system to echelon form and determine a basis for the space V.

Moreover some instructions related to the syntax are given:

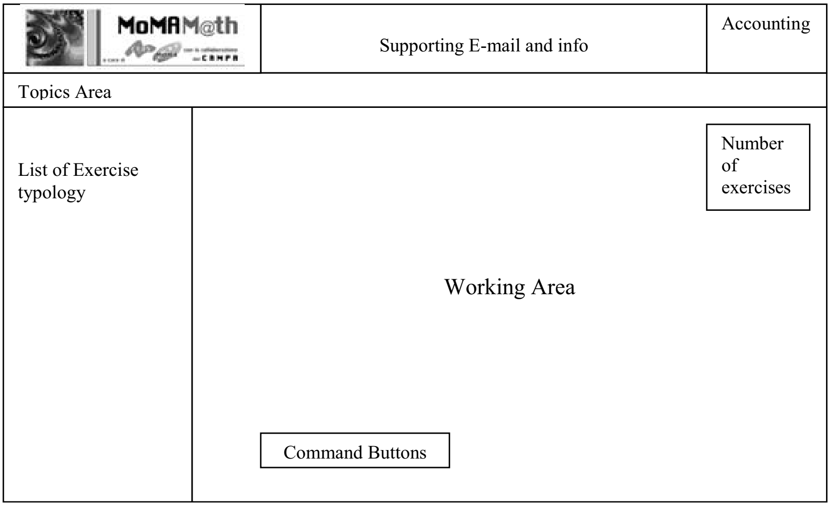

Fig. 2. MoMAMath portal

Note that each vector has to be insert between curly braces using commas to separate the components. If the basis contains a number of vectors less than the number of boxes, just leave blank.

The student is expected to insert the dimension and a basis of V. Suppose s/he answers $dimV = 3$ and gives the basis $\{\{1,-1,0\},\{2,-2,1\},\{-1,2,0\}\}$. The system check the given solution and give the following feedback:

The dimension of the space V is wrong. Note that the dimension of the space V the number of the variables minus the number of the linear independent rows of the coefficients matrix. Compute again the dimension and fill the blank boxes.

Finally pay attention to the length of the vectors you have inserted!!

In this case the system has revealed two errors. First, the dimension of the space is wrong: this can be due to a computational error (for example the rank of the matrix) or to a theoretical gap and the system recalls how to compute the dimension. Then, the system gives a warning because the inserted vectors cannot be in the space because of their wrong length.

At this stage the student has to try again to solve the exercise. Suppose that the new answer is $dimV = 2$ and the basis $\{\{1,-2\},\{3,-5\},\{0,-5\}\}$. The system check the given solution and give the following feedback:

The dimension of the space V is correct, but the given answer is incoherent because the dimension of the space V you have inserted is not equal to the dimension of the list of the vectors in the basis. Note that the dimension of a vector space is exactly the number of the vectors of any basis of the space. It is better you have another look at the theory.

Moreover note that not all the vectors you gave generate the space V, because they are not solutions of the given system. Compute again the basis and fill the blank boxes.

Finally pay attention to the length of the vectors you have inserted!!

From now on, the student may choose to re-try to solve by himself the exercise or to see the solution.

At a first look, the previous exercise may seem too difficult to require to be solved in one step! Solving homogenous linear system and reducing a matrix to echelon form are considered elementary steps and no hints or guide are given. This is because such processes are developed step by step in previous exercises as we show in the following.

Compute the dimension and a basis of the nullspace of the homogeneous linear system:

$$\begin{cases} -2x_1 + 8x_2 + 6x_3 - 4x_4 = 0 \\ 4x_1 + 2x_2 - 2x_3 + 24x_4 = 0 \\ x_1 + 5x_2 + 2x_3 + 10x_4 = 0 \end{cases}$$

The steps in which the exercise is split are:

– *compute the rank of the matrix A and give the dimension of the nullspace S.*

– *how many solutions have the system?*

The following step depends on the number of the solutions given by the student:

– case 0: the system checks the answer and gives feedback;

– case 1: the system asks the student to give the component of the solution;

– case ∞: the system requires: *Give the indices of the columns corresponding to the independent variables. Consider as dependent variables those corresponding to the pivot columns. Insert the echelon form of the coefficients matrix.*

In the last case, a further step is available to have a basis of the nullspace:

Compute the solution of the system corresponding to the following value of the independent variables: $x_3 = 1, x_4 = 0$ and $x_3 = 0, x_4 = 1$.

Finally we point out that each exercise is algorithmized in a unique block so a finite sequence of "elementary steps" is presented and the student has to follow. What we mean by "elementary step" is a step actually simple or already explained in details in previous exercises. These decomposition actually is not formalized in the application, but it could be easily represented by means of an ontology.

By using an ontology, indeed, it could automatized the process of solving the elementary single steps, letting the user to open the linked exercise instantiated with the data of the studied example. This may be seen as an up-down procedure to solve the exercise that the student may use in dependence of the difficulties s/he faces.

The study of a function, being an exercise with a more complex structure, is the only kind of exercise that is exploded in a different form. As it is shown in the following figure 3 the student can select from the combo box which phase s/he is interested in. Indeed, the study of a function is composed of a well-known sequence of phases, each one made by more elementary steps. For example the student may choose to study the asymptotes of the function and s/he is required for these steps:

1– limits to the finite extremes of the domain
2– equations of vertical asymptotes
3– limits to the $+\infty$ and $-\infty$
4– equations of horizontal asymptotes
5– equations of oblique asymptotes

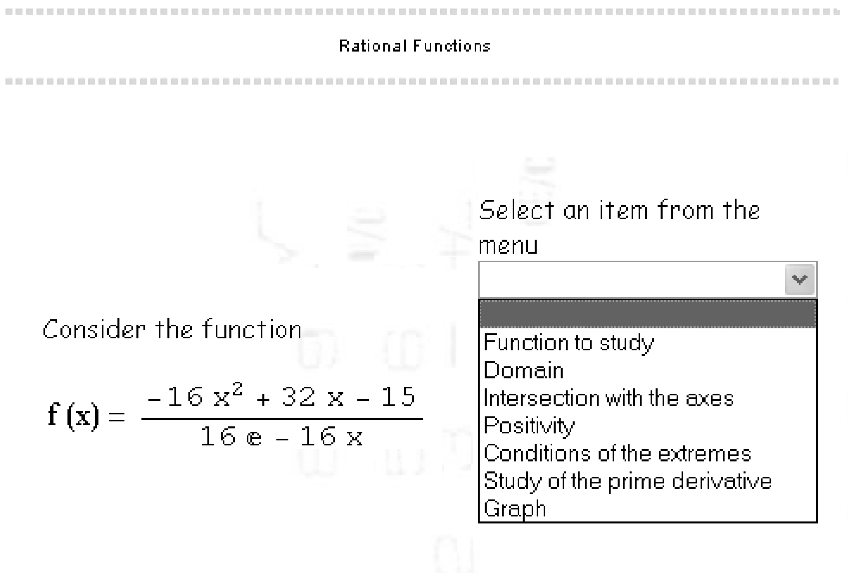

Fig. 3. Study of a function

7 Students' Feedback, Difficulties, and Future Work

MoMAMath has been experimented at the Faculty of Engineering of the University of Salerno. It has been used as complementary tool for the first year mathematics students. The students attended classical lectures, then they were habilitate to use the software for their own extra exercise sessions.

The students have found MoMAMath a good tool at least to learn standard solution techniques and to help them in recovering personal gaps since the step-by-step procedure is able to guide them in reaching the solution.

Anyway most of the students guess that MoMAMath is not able to substitute a human tutor: their feedback stress the importance of the interaction with the teacher! The difference between a software and a human teacher is that the latter is *intelligent*, that is people can reason, have the capability of understanding the actual difficulties even if they are tacit or not well-expressed. Students require more interactivity with the tutor, at least by the software support: at the moment they have just email to communicate with the tutor, but they miss synchronous and direct tools which allow them to clarify doubts exactly when they arise.

We have to mention technical difficulty too, related in particular to the mathematics: the mathematical notation on the web is an open problem, not so many browsers support MathML. Our solution at the moment consists in giving output in symbolic mathematical language by using images while the input requires textual Mathematica syntax. This implies that students become familiar with Mathematica syntax and this requires an extra effort. We are working to a solution which allows to give input using WebEquation.

Last, but not the least, difficulty consists in constructing such software is the didactical planning. We need different competencies and different people: the expert of the contents, the teacher with didactical and disciplinary experience, the expert of pedagogy, the computer scientist, the expert of communication and so on. It is difficult to have people from different areas together, and it is more and more difficult that they dialogue among them because of different languages they use.

Here are some ideas for future work:

– personalization of the hints w.r.t. to the cognitive user profile;
– more collaboration with the teacher and with other students: in particular introduction of synchronous communication tools;
– personalization of the environment for the student: in particular, the chance to have possibility of having personal notes linked with some particular exercise, save some exercises, personal links to other connected material in the web and so on;
– introduction of virtual scientific experiments that can suggest both the comprehension of complex phenomena and the learning of mathematical concepts and results.

Acknowledgement. MoMAMath is the result of a project in which are involved different instituions: the Department of Information Engineering and Applied Mathematics of the University of Salerno; the Centre of Excellence "Methods and Systems for Learning and Knowledge", which is approved and financed by the Italian Ministry of Scientific and Technological Research, singular in Italy on such theme; MOMA which is a society oriented to advanced services and to the diffusion of the innovation; the Centre of Research in Pure and Applied Mathematics which is a centre of research which has developed competences and experiences of high level with many European projects.

References

1. Albano, G., Cavallone, A., D'Apice, C., Salerno, S.: Mathematica and didactical innovation. Proceedings IMS'99 (International Mathematica Symposium), (1998) available at
http://south.rotol.ramk.fi/ keranen/IMS99/ims99papers/ims99papers.html
2. Albano, G., D'Apice, C., D'Auria, B., Manzo, R.: A new approach to teaching/learning mathematics. Proceedings of EAEEIE2000Ulm (Annual Conference on Innovations in Education for Electrical and Information Engineering) (2000) 1–5
3. Albano, G., D'Apice, C., D'Auria, B., Salerno, S.: Statistical interpretation of mechanical models by using simulation. Proceedings ICSEE 2000 (International Conference on Simulation and Multimedia in Engineering Education) (2000) 215–220
4. Galliani, L.: Open Distance Learning: innovazioni pedagogiche e didattiche. in Matteuzzi M., Banzato M. (1999)
5. Ligorio, M.B., Caravita, S.: Technology For Communication and Metacognition. International Symposium on Mathematics/Science Education and Technology (1994)
6. Miner, R., Topping, P. (Design Science, Inc): Standards-based Math on the Web. (2001) available at http://www.dessci.com/webmath/status/status_Jan_01.stm
7. Miner, R., Topping, P. (Design Science, Inc): Authoring Tools. (2002) available at http://www.dessci.com/webmath/status/status_Jan_02.stm