Working Paper Series, N. 11, November 2007

Value-at-Risk prediction by higher moment dynamics

Matteo Grigoletto
Department of Statistical Sciences
University of Padua
Italy

Francesco Lisi
Department of Statistical Sciences
University of Padua
Italy

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Keywords: VaR prediction, GARCH models, Skewness, Time-varying skewness, Time-varying kurtosis, Financial returns.

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Department of Statistical Sciences
Via Cesare Battisti, 241
35121 Padova
Italy

tel: +39 049 8274168
fax: +39 049 8274170
http://www.stat.unipd.it

Corresponding author:
Matteo Grigoletto
tel: +39 049 827 4169
matteo.grigoletto@unipd.it
http://www.stat.unipd.it/~mgri

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Matteo Grigoletto
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1 Introduction

It is well known that the Basel Committee recommends the Value-at-Risk (VaR) as a standard measure of exposure within a given portfolio (Basel Committee on Banking Supervision, 1995, 1996). Financial institutions are left free to develop their own internal model for VaR computation, but the Committee defines penalties for institutions adopting models that do not satisfy some requirements. The models for VaR computation should then be very carefully analyzed, and those implying a reduction of capital requirements can be preferred only if the declared coverage levels are respected.

The latter point is particularly delicate, since no particular backtesting procedures are today indicated by the regulatory authorities, and also because of the number of technical issues related to this operation (see e.g. Campbell, 2005).

In our view, even if often overlooked, the dynamics of conditional skewness and kurtosis should be accounted for together with the standardly used time-varying volatility, since their consideration would imply a better performance in VaR computation.

In particular, while the presence of excess kurtosis in the marginal or conditional return distributions is treated as a stylized fact in the literature, skewness has been quite neglected and relatively little work has been done to detect it.

Several economic theories have been offered as explanations of the the mechanism generating the asymmetry, including leverage effects (Black, 1976, Christie, 1982), the volatility feedback mechanism (Campbell and Hentschel, 1992), stochastic bubble models (Blanchard and Watson, 1982) and investor heterogeneity (Hong and Stein, 2003).

However, the objective of the present work is not to describe the causes of skewness, but rather to investigate – through empirical analyses – the effects of conditional skewness and kurtosis modeling in risk management and, in particular, on the value at risk computation.

Within this context, we consider nine time series of stock index returns for which the significance of the conditional asymmetry and kurtosis was studied in Grigoletto and Lisi (2006). For these series we analyze – in terms of VaR – the performances of two Gaussian models (a Gaussian GARCH and the Riskmetrics model), of a non-Gaussian symmetric model (a GARCH with Student's t innovations), and of a non-Gaussian asymmetric model (the GARCHDSK, introduced by Grigoletto and Lisi, 2006). The GARCHDSK model, in particular, allows to describe skewness and kurtosis of the conditional return distributions, both when they are assumed constant and when they are time-varying.

In Grigoletto and Lisi (2006) the analyses were limited to an in-sample context for shorter time series. Furthermore, the focus was on studying performances connected to Market Risk Capital Requirements (MRCR) as a function of past VaRs and of their violations, as defined by the guidelines of the Basel Accord on Banking Supervision. Here, on the other hand, both in-sample analyses and out-of-sample predictions are considered, and particular attention is dedicated to the testing of predictive VaR accuracy. Furthermore, in order to highlight the effects of asymmetric modeling of return time series, the Value-at-Risk is computed both for long and short positions, as suggested by Giot and Laurent (2003). The analysis is based on the study of sequences of VaR violations, for which the hypotheses of correct coverage rates and of absence of autocorrelation are assessed.

The paper is organized as follows. Section 2 introduces the GARCHDSK model. Empirical evidences, statistical significance and economic relevance of skewness are investigated in Section 3. Concluding remarks are presented in Section 4.

2 Time-varying variance, skewness and kurtosis models

Marginal and conditional skewness can be studied by means of tests or models. The more general test for studying skewness is probably that due to Bai and Ng (2005).

Conditional skewness can also be assessed by using suitable models for asymmetric behavior. In this study, we propose to analyze the presence of conditional skewness employing a GARCH-type model with innovations having a Pearson's Type IV (henceforth Pearson$_{IV}$) distribution. This model represents a generalization of the standard GARCH model because it can account for asymmetry and kurtosis in the conditional distribution. Conditional skewness and kurtosis can be time-varying, thus allowing to study possible dynamics in higher-order moments. In the following, the acronym GARCHDSK (GARCH with dynamic skewness and kurtosis) will be used to denote this model.

Time varying skewness and kurtosis were first introduced by Hansen (1994), who extended the ARCH framework by proposing the adoption of a conditional generalized Student's t distribution, and modeling its parameters as functions of the lagged errors. Approaches in which dynamics are imposed on shape parameters, thus inducing time-varying skewness and kurtosis, have also been adopted, among others, by Jondeau and Rockinger (2003) and Yan (2005). In other cases, higher order moments are modeled directly. For example, Harvey and Siddique (1999) introduce a GARCH-type expression for the conditional skewness, while Brooks et al. (2005) use a similar representation for the kurtosis. León et al. (2005) employ a GARCH specification for both conditional skewness and kurtosis.

In the spirit of Hansen (1994), here dynamics on skewness and kurtosis are introduced by modeling shape parameters, rather than directly skewness and kurtosis. As remarked by Yan (2005), this approach is less computationally intensive and allows skewness and kurtosis to explode, while the shape parameters remain stationary. This is particularly useful when modeling extremal events.

The approach suggested by Yan (2005) involves first estimating the dynamics of volatility. Then, conditionally on the results obtained, a model for skewness and kurtosis can be developed using the standardized residuals. However, this two-step procedure implies that the variability of the parameters ruling the dynamics of skewness and kurtosis is underestimated, being computed conditionally on the estimated volatility. For this reason, in the approach suggested here the parameters governing the dynamics of volatility, skewness and kurtosis are estimated together, in a single step.

Concerning the choice of the conditional distribution, in the present paper we follow Premaratne and Bera (2001) in the use of a Pearson$_{IV}$ distribution. This distribution is flexible, in the sense that it implies a wide range of feasible skewness-kurtosis couples. For example, the range associated with the Gram-Charlier density studied in Jondeau and Rockinger (2001) and adopted by León et al. (2005) is relatively rather limited (Yan, 2005). The Pearson$_{IV}$ is also found to approximate the generalized Student's t distribution on a large area of the skewness-kurtosis plane, but is computationally less demanding (see Premaratne and Bera, 2001, and the computational techniques discussed in Heinrich, 2004).

The GARCH-type model we will use to assess skewness has the following structure:

$$y_t = \mu_t + \varepsilon_t \ , \qquad t = 1, \ldots, n \ , \tag{1}$$

where $\mu_t = E(y_t|I_{t-1})$, and ε_t is such that $\varepsilon_t \mid I_{t-1} \sim \text{Pearson}_{IV}\,(\lambda_t, a_t, \nu_t, r_t)$. Hence, the conditional density is defined by

$$f(\varepsilon_t \mid I_{t-1}) = C_t \left[1 + \left(\frac{\varepsilon_t - \lambda_t}{a_t}\right)^2\right]^{-(r_t+2)/2} \exp\left[-\nu_t \arctan\left(\frac{\varepsilon_t - \lambda_t}{a_t}\right)\right] \ . \tag{2}$$

Jointly, parameters λ_t, a_t, ν_t and r_t control the conditional mean, variance, skewness and kurtosis. The parameter C_t is a normalizing constant depending on a_t, ν_t and

r_t. The distribution is symmetric for $\nu_t = 0$, positively skewed for $\nu_t < 0$ and negatively skewed for $\nu_t > 0$. For fixed ν_t, increasing r_t decreases the kurtosis. The Pearson$_{IV}$ distribution is essentially a skewed version of the Student's t and for $\nu_t = 0$, $r_t = g_t - 1$ and $a_t = \sqrt{g_t}$ reduces to a Student's t with g_t degrees of freedom. The normal distribution is a limit case where $\nu_t = 0$ and $r_t \to \infty$.

Setting $\lambda_t = a_t \, \nu_t / r_t$ in order to have a zero mean error term, for the conditional distribution of ε_t we have

$$E(\varepsilon_t | I_{t-1}) = 0 , \tag{3}$$

$$\sigma_t^2 = \mathrm{Var}(\varepsilon_t | I_{t-1}) = \frac{a_t^2 \left(r_t^2 + \nu_t^2\right)}{r_t^2 \, (r_t - 1)} , \tag{4}$$

$$S_t = S(\varepsilon_t | I_{t-1}) = \frac{-4 \, \nu_t}{r_t - 2} \sqrt{\frac{r_t - 1}{r_t^2 + \nu_t^2}} , \tag{5}$$

$$K_t = K(\varepsilon_t | I_{t-1}) = \frac{3 \, (r_t - 1) \left[(r_t + 6) \, (r_t^2 + \nu_t^2) - 8 r_t^2 \right]}{(r_t - 2) \, (r_t - 3) \, (r_t^2 + \nu_t^2)} , \tag{6}$$

where S_t and K_t are the conditional skewness and kurtosis coefficients, given by the standardized third and fourth moments.

In this framework, the conditional variance σ_t^2 depends jointly on a_t, ν_t and r_t, whereas conditional skewness and kurtosis depend only on ν_t and r_t. In particular, if $\nu_t = 0$ then $S_t = 0$ and this is why ν_t can be interpreted as the "skewness parameter". When $\nu_t = \nu$ and $r_t = r$, $\forall t$, conditional skewness and kurtosis are constant.

For a complete model specification a critical point is how to describe the dynamics of σ_t^2, S_t and K_t. Our proposal is to define it through the evolution of the parameters a_t, ν_t and r_t which, in turn, is induced by the following autoregressive GARCH-type structure:

$$a_t^2 = \omega_a + \alpha_a \, \bar{a}_{t-1}^2 + \beta_a \, a_{t-1}^2 , \tag{7}$$

$$\nu_t = \omega_v + \alpha_\nu \, \bar{\nu}_{t-1} + \beta_\nu \, \nu_{t-1} , \tag{8}$$

$$r_t = \omega_r + \alpha_r \, \bar{r}_{t-1} + \beta_r \, r_{t-1} , \tag{9}$$

with $\bar{a}_t$, $\bar{\nu}_t$ and $\bar{r}_t$ being moment-based estimators of a_t, ν_t and r_t (see Stuart and Ord, 1994, and Heinrich, 2004) defined by

$$\bar{a}_t = \frac{\sqrt{\bar{\sigma}_t^2 \left[16 \, (\bar{r}_t - 1) - \bar{S}_t^2 \, (\bar{r}_t - 2)^2 \right]}}{4} , \tag{10}$$

$$\bar{\nu}_t = - \frac{\bar{r}_t \, (\bar{r}_t - 2) \, \sqrt{\bar{S}_t}}{\sqrt{16 \, (\bar{r}_t - 1) - \bar{S}_t^2 \, (\bar{r}_t - 2)^2}} , \tag{11}$$

$$\bar{r}_t = \frac{6 \, (\bar{K}_t - \bar{S}_t^2 - 1)}{2 \, \bar{K}_t - 3 \, \bar{S}_t^2 - 6} . \tag{12}$$

By $\bar{\sigma}_t^2$, $\bar{S}_t$ and $\bar{K}_t$ we have denoted suitable estimates of the variance, skewness and kurtosis coefficients. In particular, the estimates defined in (10), (11) and (12) are "local", in the sense that only the m more recent values of the series are used in the computation of $\bar{\sigma}_t^2$, $\bar{S}_t$ and $\bar{K}_t$. In the following, the choice of m will be based on goodness-of-fit criteria.

Since a_t, ν_t and r_t depend only on past information, conditional variance, skewness and kurtosis at time t can be computed at time $t-1$.

The introduction of the constraints $\alpha_\nu = \alpha_r = \beta_\nu = \beta_r = 0$ allows to estimate models with constant skewness and kurtosis. However, note that for a dynamic behavior of both conditional skewness and kurtosis, it is sufficient that at least one of these parameters is different from zero.

Modeling a_t, ν_t and r_t rather than directly variance, skewness and kurtosis turns out to be easier because the latter quantities need to satisfy nonlinear constraints which are difficult to impose at each point in time, while the constraints concerning a_t, ν_t and r_t can be implemented straightforwardly.

The issue of what constraints are necessary and sufficient to ensure the stationarity of the model requires further study. However, by simulations, we found that the following conditions, besides guaranteeing the positivity of the variance and kurtosis parameters, are sufficient for a non-explosive behavior: $\omega_a > 0$, $\omega_r > 3$, $\alpha_i, \beta_i \geq 0$, $\alpha_i + \beta_i < 1$, for $i = a, \nu, r$. In particular, the constraint $\omega_r > 3$ is needed to ensure existence of the kurtosis.

Estimates for the ω_i, α_i and β_i $(i = a, v, r)$ parameters are obtained by maximizing the log-likelihood function

$$\sum_{t=1}^{n} \left\{ \log C_t - \frac{r_t + 2}{2} \log \left[1 + \left(\frac{\hat{\varepsilon}_t - \lambda_t}{a_t} \right)^2 \right] - \nu_t \arctan \left(\frac{\hat{\varepsilon}_t - \lambda_t}{a_t} \right) \right\} , \qquad (13)$$

where $\hat{\varepsilon}_t = y_t - \hat{\mu}_t$. The estimate $\hat{\mu}_t$ is computed in a first step of the procedure, by estimating a suitable ARMA model, which in the present context represents a very weak correlation structure or even reduces to a constant. Since parameters a_t, ν_t and r_t are functions of ω_i, α_i and β_i $(i = a, \nu, r)$, expression (13) can be maximized with respect to these latter. In principle, maximum likelihood can also be used to estimate the parameter m in the definition of $\bar{a}_t$, $\bar{\nu}_t$ and $\bar{r}_t$. However, this would imply a large computational burden. Hence, the choice of m will be based on goodness-of-fit considerations (see the next section).

3 Empirical results

3.1 In-sample results

We now apply the previous methodology to the daily returns, adjusted for split and dividends, of 9 major international stock indexes, namely the indexes CAC40, DAX, FTSE100, MIB30, SMI, Dow Jones, Nasdaq, S&P500, Nikkey225.

The time series refer to different periods but all end on March 10, 2007. However, in order to be able to make subsequent out-of-sample predictions, only the subseries ending on December 13, 2005 are considered for model building. These subseries are composed by a number of observations between 1547 and 4023 (Table 1).

Most of the series present some abnormal values that do not seem to belong to the standard dynamics, and were thus replaced with the mean of the data. All the analyses were conducted on these outlier-adjusted time series.

Sample skewness and kurtosis coefficients are given in Table 1: all indexes have negative skewness and severe excess kurtosis. Only the Nikkey225 index has positive, but very small, skewness. These results are consistent with other findings in the literature (e.g. Cont, 2001, Belaire-Franch and Peiró, 2003, Kim and White, 2004, and Peiró, 2004).

Series	n	$\hat{S}_u$	$\hat{K}_u$	$BN05_u$	$\hat{S}_c$	$\hat{K}_c$	$BN05_c$
CAC40	3977	-0.103	5.798	0.393	-0.366	5.336	0.083
DAX	3792	-0.117	6.204	0.326	-0.124	3.974	0.130
Dow Jones	4023	-0.223	7.548	0.227	-0.348	4.731	0.002
FTSE100	5483	-0.264	6.362	0.046	-0.208	3.941	0.008
MIB30	1547	-0.189	6.608	0.410	-0.419	4.237	0.003
Nasdaq	4023	-0.174	7.638	0.278	-0.412	4.316	0.000
Nikkey225	3926	0.038	5.098	0.691	-0.053	4.578	0.616
SMI	3797	-0.200	6.861	0.177	-0.280	3.915	0.000
S&P500	4023	-0.103	6.767	0.504	-0.345	4.759	0.002

Table 1: Symmetry tests for index returns. Columns $\hat{S}_u$ and $\hat{K}_u$ give the empirical skewness and kurtosis coefficients for the observed series; columns $\hat{S}_c$ and $\hat{K}_c$ give the same indices for the standardized residuals of a TGARCH(1,1) model; columns $BN05_u$ and $BN05_c$ give the $p-$values for the Bai and Ng (2005) test for unconditional and conditional asymmetry, respectively.

As a starting point, we looked for unconditional skewness by applying the Bai and Ng (2005) test and, as a benchmark, the asymptotic standard test. The $p-$values for the null hypothesis of symmetry, reported in Table 1, show that the Bai and Ng test accepts the symmetry, at the 5% level of significance, in 8 cases on 9, with a $p-$value of 0.0464 for the FTSE100.

These analyses indicate that no clear evidence of unconditional asymmetry was found in the considered time series.

As a second step, conditional skewness of the series is investigated by applying the Bai and Ng test to the residuals of a GARCH(1,1) model accounting for asymmetric effects (TGARCH). For the moment we will assume skewness to be constant.

Table 1 shows that results about the statistical significance of the conditional skewness are quite different from those on the marginal distributions and show more evidence of asymmetry. In particular, at the 5% significance level, the null hypothesis of symmetry is rejected in 6 cases on 9 by the $BN05_c$ test. The Nikkey225 index is the only one for which the test clearly agrees on accepting conditional symmetry.

The presence of significant conditional skewness was further investigated by estimating GARCHDSK models, as defined in Section 3, assuming constant conditional skewness and kurtosis. This amounts to imposing $\alpha_\nu = \beta_\nu = \alpha_r = \beta_r = 0$, thus obtaining a subset of models that we will indicate with GARCHSK. The Kolmogorov-Smirnov goodness-of-fit test described below led us to the choice of $m = 10$ in the definition of $\bar{a}_t$, $\bar{\nu}_t$ and $\bar{r}_t$.

The maximum likelihood parameter estimates, with their asymptotic standard deviations and t-statistics, are given in Table 2. The results indicate that conditional skewness is statistically significant for all series except Nikkey225. Table 3 lists the

Parameter	Estimate	Std. err.	t-stat.	Parameter	Estimate	Std. err.	t-stat.
CAC40				Nasdaq			
ω_a	0.214	0.073	2.92	ω_a	0.147	0.039	3.73
α_a	0.075	0.009	7.60	α_a	0.085	0.006	12.84
β_a	0.914	0.011	83.12	β_a	0.905	0.007	135.1
ω_ν	1.801	0.678	2.65	ω_ν	3.389	0.872	3.88
ω_r	11.46	2.109	5.43	ω_r	11.09	1.888	5.87
DAX				Nikkey225			
ω_a	0.191	0.056	3.37	ω_a	0.195	0.057	3.44
α_a	0.101	0.010	9.66	α_a	0.081	0.009	8.58
β_a	0.888	0.010	83.01	β_a	0.906	0.010	87.12
ω_ν	1.386	0.479	2.89	ω_ν	0.307	0.252	1.21
ω_r	9.602	1.587	6.05	ω_r	6.859	0.906	7.56
Dow Jones				SMI			
ω_a	0.052	0.013	3.78	ω_a	0.163	0.049	3.32
α_a	0.051	0.005	9.52	α_a	0.097	0.012	8.02
β_a	0.938	0.005	170.6	β_a	0.886	0.013	64.71
ω_ν	0.867	0.299	2.89	ω_ν	2.212	0.587	3.76
ω_r	7.042	0.909	7.74	ω_r	9.365	1.502	6.23
FTSE100				S&P500			
ω_a	0.141	0.046	3.03	ω_a	0.083	0.024	3.47
α_a	0.074	0.009	8.39	α_a	0.098	0.011	9.31
β_a	0.915	0.010	88.85	β_a	0.891	0.011	80.28
ω_ν	3.138	0.913	3.44	ω_ν	0.891	0.315	2.82
ω_r	14.39	2.512	5.73	ω_r	7.168	1.016	7.05
MIB30							
ω_a	0.112	0.045	2.49				
α_a	0.088	0.013	6.73				
β_a	0.899	0.013	68.67				
ω_ν	1.782	0.729	2.44				
ω_r	7.829	1.934	4.05				

Table 2: ML estimates of GARCHSK model parameters (constant conditional symmetry and kurtosis), asymptotic standard errors and t-statistics.

conditional skewness an kurtosis implied by models estimated in Table 2 and shows that all indexes are negatively skewed with the Nikkey225 having the smallest coefficient. Again, the absolute conditional skewness entailed by the models is generally greater than the marginal one.

The model introduced in Section 2 also allows to investigate the presence of dynamic, rather than constant, conditional skewness.

Table 4 lists the estimated parameters and shows that for 7 indexes the parameter α_ν is significant implying that both skewness and kurtosis are time varying. For all these models, the Ljung-Box test at lag 15 on standardized squared residuals accepts the hypothesis of no residual correlation.

In order to check goodness of fit, we applied the Kolmogorov-Smirnov test to assess the uniformity of the values $\hat{F}(\hat{\varepsilon}_t|I_{t-1})$, $t = 1, \ldots, n$, where $F(\cdot|I_{t-1})$ denotes the c.d.f. corresponding to the density defined in (2), and $\hat{F}(\cdot|I_{t-1})$ is obtained by

	GARCHSK		GARCHDSK	
Series	S	K	S_{av}	K_{av}
CAC40	-0.212	3.78	-0.288	3.71
DAX	-0.219	3.99	-0.264	3.99
Dow Jones	-0.239	4.59	-0.321	4.50
FTSE100	-0.253	3.62	-0.269	3.60
MIB30	-0.390	4.52	-0.354	3.40
Nasdaq	-0.407	4.02	-0.408	4.02
Nikkey225	-0.089	4.56	0.00	4.55
SMI	-0.365	4.17	-0.404	4.02
S&P500	-0.237	4.54	-0.329	4.23

Table 3: Conditional skewness and kurtosis implied by the GARCHSK and GARCHDSK models (in the latter case the average conditional skewness and kurtosis are given).

substituting the ML parameter estimates in the c.d.f. definition. Table 5 lists the test p-values for each series. The p-values for other models, described in the next section, are also shown. At the standard 5% significance level, the null is accepted for all models, suggesting that the GARCHDSK models are appropriate.

In summary, the results on the nine analyzed time series indicate that there are no strong evidences of unconditional skewness, which seems to be more the exception than the rule. On the contrary, conditional skewness appears to be more widespread. In particular, there are clear indications of dynamic skewness that, if modeled, may allow for a more realistic description of the evolution of financial quantities of interest.

3.2 Value-at-Risk prediction and model validation

In the previous section skewness was analyzed mainly from a statistical viewpoint, by looking at its statistical significance. In this section we mean to study the economic and financial importance of skewness by analyzing its role in risk modeling and examining the out-of-sample performance of GARCHDSK models in this context.

With this purpose, for the nine stock indexes the time-varying Values-at-Risk $\{\mathrm{VaR}_t\}$ were computed, using GARCHDSK and some alternative models, in order to compare them. Market Value-at-Risk is a crucial component of most risk analyses and management systems in financial and insurance industries. It measures how the market value of an asset, or of a portfolio of assets, of value P is likely to decrease over a certain time period under normal market conditions. It is typically used by security houses or investment banks to measure the market risk of their asset portfolios, but is actually a very general concept that has broad application. It is well known that VaR is a high quantile of the profit/loss distribution which defines a bound such that the loss over a fixed holding period is less than this bound with probability $1 - \alpha$.

We computed time-varying VaR_t by using the GARCHDSK models estimated in the previous subsection, the Riskmetrics approach with the usual smoothing parameter $\lambda = 0.94$ (see Alexander, 2001), a Gaussian GARCH(1,1) and a GARCH(1,1)

Parameter	Estimate	Std. err.	t-stat.	Parameter	Estimate	Std. err.	t-stat.
CAC40				Nasdaq			
ω_a	0.242	0.085	2.83	ω_a	0.147	0.039	3.73
α_a	0.062	0.007	7.97	α_a	0.084	0.007	12.85
β_a	0.925	0.009	103.9	β_a	0.905	0.007	135.07
ω_ν	4.069	1.611	2.52	ω_ν	3.389	0.872	3.88
α_ν	0.218	0.048	4.54	α_ν	–	–	–
ω_r	14.63	3.598	4.06	ω_r	11.090	1.888	5.87
DAX				Nikkey225			
ω_a	0.166	0.053	3.11	ω_a	0.195	0.057	3.44
α_a	0.087	0.009	9.22	α_a	0.081	0.009	8.58
β_a	0.903	0.009	93.83	β_a	0.906	0.010	87.12
ω_ν	2.151	0.705	3.04	ω_ν	0.306	0.252	1.21
α_ν	0.239	0.056	4.22	α_ν	–	–	–
ω_r	10.58	1.997	5.29	ω_r	6.859	0.906	7.56
Dow Jones				SMI			
ω_a	0.058	0.017	3.41	ω_a	0.176	0.057	3.08
α_a	0.054	0.006	8.80	α_a	0.085	0.011	7.73
β_a	0.935	0.007	138.4	β_a	0.897	0.012	74.7
ω_ν	1.658	0.551	3.00	ω_ν	3.918	1.215	3.221
α_ν	0.280	0.068	4.10	α_ν	0.224	0.052	4.33
ω_r	8.064	1.334	6.05	ω_r	11.77	2.485	4.73
FTSE100				S&P500			
ω_a	0.155	0.047	3.28	ω_a	0.079	0.027	2.85
α_a	0.063	0.006	9.21	α_a	0.076	0.008	9.17
β_a	0.923	0.008	113.5	β_a	0.913	0.008	106.6
ω_ν	4.656	1.20	3.85	ω_ν	2.296	1.192	1.92
α_ν	0.210	0.041	5.13	α_ν	0.275	0.061	4.47
ω_r	16.86	3.119	5.40	ω_r	9.431	2.533	3.72
MIB30							
ω_a	0.410	0.253	1.61				
α_a	0.072	0.010	6.85				
β_a	0.913	0.011	83.5				
ω_ν	23.59	19.38	1.21				
α_ν	0.153	0.071	2.14				
ω_r	35.87	21.99	1.63				

Table 4: ML estimates of GARCHDSK model parameters (time-varying conditional skewness and kurtosis), asymptotic standard errors and t-statistics.

with Student's t innovations.

An analysis of the one-day-ahead 1% VaR for these series and models can be found in Grigoletto and Lisi (2006). They compared nominal and observed VaR coverages and showed, by using the Kupiec LR test (Kupiec, 1995), that only for the GARCHDSK models they are not significantly different, at the 5% significance level.

However, since in real situations VaR models are used to deliver out-of-sample predictions, a deeper evaluation of out-of-sample performances of VaR is now conducted, for the same indexes and for the period going from December 13, 2005 to

	Riskmetrics	GARCH-N	GARCH-t	GARCHDSK
CAC40	0.427	0.044	0.450	0.090
DAX	0.183	0.003	0.339	0.182
Dow Jones	0.028	<0.001	0.364	0.243
FTSE100	0.033	0.049	0.022	0.104
MIB30	0.002	<0.001	0.006	0.415
Nasdaq	<0.001	<0.001	<0.001	0.295
Nikkey225	0.174	0.005	0.768	0.658
SMI	0.056	0.002	0.040	0.117
S&P500	0.011	<0.001	0.297	0.090

Table 5: p-values for the Kolmogorov-Smirnov goodness-of-fit test.

March 10, 2007. This period was not considered for model fitting and includes 317 working days.

Moreover, in order to evidence the effects of conditional asymmetry, we computed VaR for both long and short positions. We assume the portfolio value at time t is P_t and the profits and losses over h time units are represented by the log-returns of the portfolio, $r_{t,h} = \log(P_t/P_{t-h})$, with distribution F_h. Then, following Giot and Laurent (2003), the VaR for long position VaR_l is defined by

$$\mathrm{VaR}_l = -P_{t-h} \ \left[F_h^{-1}(\alpha)\right] , \tag{14}$$

This represents the point of view of traders who bought assets, and are mainly concerned with the possibility that prices decrease. On the contrary, portfolio managers who have short positions, i.e. who sell assets, lose money when prices increase. This is represented by VaR for short position, VaR_s, defined by

$$\mathrm{VaR}_s = P_{t-h} \ \left[F_h^{-1}(1-\alpha)\right] . \tag{15}$$

When the distribution of $r_{t,h}$ is not constant over time, VaR is also time-varying. In the following, we will assume to have a unitary position ($P = 1$) in a portfolio defined by each index.

Let us now turn to the problem of computing the return predictive distributions. In the present framework, it is very difficult to determine analytically the distribution of future observations and of their volatility, skewness and kurtosis, when an arbitrary prediction horizon ($h > 1$) is considered. We therefore resort to the bootstrap technique, extending the approach proposed by Pascual et al. (2006) in the context of GARCH models. Our goal is to estimate the distribution of y_t, a_t^2, ν_t and r_t, for $t = n+1, n+2, \ldots$, conditionally on the available observations $y_1, \ldots, y_n$. Once the parameters $(\omega_i, \alpha_i, \beta_i)$, $i = a, \nu, r$, have been estimated, it is possible to define bootstrap forecasts through the recursions

$$\begin{aligned}
\hat{a}_t^{*2} &= \hat{\omega}_a + \hat{\alpha}_a \ \bar{a}_{t-1}^{*2} + \hat{\beta}_a \ \hat{a}_{t-1}^{*2} \ , \\
\hat{\nu}_t^* &= \hat{\omega}_\nu + \hat{\alpha}_\nu \ \bar{\nu}_{t-1}^* + \hat{\beta}_\nu \ \hat{\nu}_{t-1}^* \ , \\
\hat{r}_t^* &= \hat{\omega}_r + \hat{\alpha}_r \ \bar{r}_{t-1}^* + \hat{\beta}_r \ \hat{r}_{t-1}^* \ , \\
\hat{y}_t^* &\sim \mathrm{Pearson}_{IV}\left(\hat{\lambda}_t^* = (\hat{a}_t^* \ \hat{\nu}_t^*/\hat{r}_t^*), \hat{a}_t^*, \hat{\nu}_t^*, \hat{r}_t^*\right) \ ,
\end{aligned}$$

for $t = n+1, n+2, \ldots$, and where $\bar{a}_t^{*2}$, $\bar{\nu}_t^*$ and $\bar{r}_t^*$ have the same definitions as $\bar{a}_t^2$, $\bar{\nu}_t$ and $\bar{r}_t$, but concern

$$\tilde{y}_t^* = \begin{cases} y_t & \text{if } t \leq n \\ \hat{y}_t^* & \text{if } t > n \end{cases} .$$

The recursions are initialized by setting $\hat{a}_n^{*2}$, $\hat{\nu}_n^*$, and $\hat{r}_n^*$ as suitable functions of the observed series, so that the state of the process at the end of the observation period is taken into account. In particular, a_t^2 may be written as

$$a_t^2 = \frac{\omega_a}{1-\alpha_a-\beta_a} + \alpha_a \sum_{j=0}^{\infty} \beta_a^j \left(\bar{a}_{t-j-1}^2 - \frac{\omega_a}{1-\alpha_a-\beta_a} \right) ,$$

and analogous expressions hold for ν_t and r_t. On the grounds of this result we set

$$\hat{a}_n^{*2} = \frac{\hat{\omega}_a}{1-\hat{\alpha}_a-\hat{\beta}_a} + \hat{\alpha}_a \sum_{j=0}^{n-2} \hat{\beta}_a^j \left(\bar{a}_{n-j-1}^2 - \frac{\hat{\omega}_a}{1-\hat{\alpha}_a-\hat{\beta}_a} \right) ,$$

and similarly for $\hat{\nu}_n^*$ and $\hat{r}_n^*$. It should be noted that, since $\hat{a}_{n+1}^{*2}$, $\hat{\nu}_{n+1}^*$ and $\hat{r}_{n+1}^*$ are observable, no variability is involved in their prediction.

Once B bootstrap replications $(\hat{y}_t^*, \hat{a}_t^{*2}, \hat{\nu}_t^*, \hat{r}_t^*)_i$, $i = 1, \ldots, B$, have been obtained, the predictive distributions of y_t, a_t^2, ν_t and r_t can easily be computed for each time $t = n+1, \ldots$ of interest. This is most simply achieved by considering the empirical distribution of the B replications. Suitable transformations of $(\hat{a}_t^{*2}, \hat{\nu}_t^*, \hat{r}_t^*)$ immediately lead to the prediction of volatility, skewness and kurtosis.

The present approach can easily be extended to consider the uncertainty due to parameter estimation, although we will not pursue this goal here.

We will now consider the assessment of performance in VaR computation, for several models. Since the late 1990's a variety of tests have been proposed to measure the VaR accuracy and validate underlying models (e.g. Kupiec, 1995, and Christoffersen, 1998). The literature highlighted three main features that a good VaR model should have:

i) *unconditional coverage property*: for correctly specified models, the number of expected VaR violations must not be (statistically) different from the observed violations (i.e. nominal and observed coverages don't have to differ significantly);

ii) *independence property*: VaR violations, concerning the same coverage rates, but observed at different dates, must be independent;

iii) properties i) and ii) should hold even when several coverage rates are considered simultaneously.

Berkowitz and O'Brien (2002) showed that properties i) and ii) are satisfied when the process associated to VaR violations is a martingale difference or a weak white noise. Property iii) is a generalization of the two previous ones, since it requires that i) and ii) hold not only at a given coverage rate, but for the whole distribution.

Recently, Hurlin and Tokpavi (2007) proposed a multivariate portmanteau test based on the weak white noise property of the process of VaR violations. This test is able to jointly validate this hypothesis, for a finite set of coverage rates; e.g., properties i) and ii) can be assessed jointly for levels 1% and 5%, rather than for a single level.

In order to study the validity of the VaR models considered, some of the tests previously described have been applied to the processes of VaR violations for the nine time series under study. In particular, we considered

- the Kupiec test for assessing property i), at the 1% nominal level;
- the Christoffersen test to jointly study properties i) and ii), at the 1% nominal level;
- the Hurlin and Tokpavi test to jointly investigate properties i) and ii), considering the 1% and 5% levels at the same time; the validity of property iii) was therefore examined, to some extent.

The last test is also suitable when considering both long and short VaR at the same time. Results concerning the Kupiec, Christoffersen and Hurlin and Tokpavi tests are reported in Tables 6, 7 and 8-11, respectively.

In order to interpret the results, it should first be noted that the variability of the out-of-sample period, for the time series analyzed here, is always smaller than the variability observed in-sample; the ratio between the variances observed in the in- and out-of-sample periods ranges, for the different series, between 1.5 and 2.9. This explains why even the Gaussian model gives generally acceptable results. As a consequence we see that, for example, for the traditional 1-day-ahead 1% long VaR, both the Kupiec and the Christoffersen tests almost always accept, at the 5% significance level (the Riskmetrics for MIB30 being an exception), the null of correct coverage and independence from past violations (Tables 6 and 7).

However, if we consider the whole set of results for 1- and 5-day-ahead long and short VaR, only for the GARCHDSK model the null hypothesis is never rejected, whereas all other models show some cases where the null is not accepted at the 5% significance level.

To study in more depth the performance of the considered models, the Hurlin and Tokpavi test has also been applied in two different ways: first, the 1% and 5% levels on the same tail (i.e. concerning VaR_l or VaR_s) were jointly investigated (Tables 8 and 9); then, the test was applied considering the 1% or 5% levels on both tails at the same time (Tables 10 and 11).

Results relative to the Hurlin and Tokpavi test are less systematic, also because the test can be applied only if at least one violation occurs. The symbol "–", used in the tables, means that there were no violations and thus the p-value could not be computed. When considering a single tail (Tables 8 and 9) for the GARCHDSK model this situation takes place only once. Apart from this, for the 1-day-ahead VaR (Table 8) the test rejects the null hypothesis only in the case of long VaR computed for the FTSE100 series. For 5-day-ahead VaRs (Table 9), instead, all models show problems. This is supposedly due to the fact that existing violations most of the time concern both the levels considered, thus defying the hypothesis of uncorrelation between "hits" at different levels.

When the test is applied in order to jointly account for the performance of long and short 1% 1-day-ahead VaR (Table 10), the null hypothesis is always accepted only for GARCHDSK. Again, the performance of all models is worse when a 5 day predictive horizon is taken into account (Table 11).

4 Conclusions

This paper has focused on the issue of empirical evidence concerning in- and out-of-sample Value-at-Risk computation for time series of financial returns. Nine series of daily stock index returns have been analyzed, with the objective to compare the performance of several widely used models with that of the GARCHDSK, a GARCH-type model which allows to take into account both skewness and kurtosis. A characteristic feature of this approach is that skewness and kurtosis are allowed to evolve dynamically. This is done by assuming Pearson's Type IV errors and defining suitable dynamics for the distribution parameters. The dynamic structure depends on moment-based estimators.

Our results indicate that for the considered series there are no strong evidences of unconditional asymmetry which, therefore, does not appear to be a common feature of financial returns.

Different conclusions are drawn with respect to conditional skewness, which was found to be significantly present in eight of the nine stock index returns analyzed. In particular, in seven of the eight cases, we found significant time-varying skewness and kurtosis. These findings are consistent with those of studies by, among others, Brooks et al. (2005), León et al. (2005) and Cappuccio et al. (2006).

To investigate the economic importance of a correct modeling of skewness, different models were compared with respect to the computation of the 1- and 5-day-ahead long and short 1% Value-at-Risk. The analyses were carried out taking into account also a predictive perspective, that is in an out-of-sample framework.

The out-of-sample period, including 317 working days, was not particularly turbulent and its variability was always lower than that of the in-sample period. This is why also standard models, which usually perform poorly in this context, yield adequate results.

Even in this situation, however, the VaR computation with the GARCHDSK model resulted to be best. This confirms that skewness is important not only from a statistical point of view, but also from a financial perspective, particularly in risk management.

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CAC40	1d long	1d short	5d long	5d short
GARCH-N	0.977	0.013	0.602	0.013
GARCH-t	0.977	0.013	0.977	0.013
Riskmetrics	0.304	0.519	<0.001	0.051
GARCHDSK	0.159	0.159	0.495	0.159
DAX	1d long	1d short	5d long	5d short
GARCH-N	0.304	0.169	0.304	0.169
GARCH-t	0.977	0.013	0.977	0.169
Riskmetrics	0.304	0.304	0.002	0.005
GARCHDSK	0.325	0.159	0.945	0.945
Dow Jones	1d long	1d short	5d long	5d short
GARCH-N	0.601	0.519	0.601	0.519
GARCH-t	0.519	0.169	0.977	0.519
Riskmetrics	0.133	0.304	0.005	0.017
GARCHDSK	0.977	0.977	0.977	0.169
FTSE100	1d long	1d short	5d long	5d short
GARCH-N	0.601	0.169	0.304	0.169
GARCH-t	0.977	0.013	0.601	0.169
Riskmetrics	0.133	0.304	0.017	0.017
GARCHDSK	0.963	0.508	0.963	0.508
MIB30	1d long	1d short	5d long	5d short
GARCH-N	0.304	0.169	0.304	0.170
GARCH-t	0.602	0.013	0.977	0.169
Riskmetrics	0.018	0.977	0.005	0.018
GARCHDSK	0.977	0.519	0.977	0.602
Nasdaq	1d long	1d short	5d long	5d short
GARCH-N	0.601	0.304	0.601	0.977
GARCH-t	0.601	0.169	0.601	0.977
Riskmetrics	0.304	0.304	0.002	0.002
GARCHDSK	0.977	0.133	0.977	0.304
Nikkey225	1d long	1d short	5d long	5d short
GARCH-N	0.601	0.169	0.304	0.013
GARCH-t	0.169	0.013	0.977	0.013
Riskmetrics	0.304	0.1699	<0.001	0.519
GARCHDSK	0.995	0.176	0.585	0.176
SMI	1d long	1d short	5d long	5d short
GARCH-N	0.977	0.013	0.601	0.013
GARCH-t	0.519	0.013	0.977	0.013
Riskmetrics	0.051	0.519	<0.001	0.017
GARCHDSK	0.512	0.166	0.512	0.610
S&P500	1d long	1d short	5d long	5d short
GARCH-N	0.601	0.601	0.304	0.602
GARCH-t	0.519	0.169	0.977	0.601
Riskmetrics	0.052	0.134	<0.001	0.002
GARCHDSK	0.977	0.304	0.977	0.978

Table 6: p-values for the Kupiec test for the 1- and 5-day-ahead long and short 1% VaR (1d long, 1d short, 5d long and 5d short, respectively).

CAC40	1d long	1d short	5d long	5d short
GARCH-N	0.960	0.046	0.816	0.046
GARCH-t	0.960	0.046	0.960	0.046
Riskmetrics	0.533	0.796	<0.001	0.046
GARCHDSK	0.369	0.369	0.777	0.369
DAX	1d long	1d short	5d long	5d short
GARCH-N	0.533	0.387	0.533	0.387
GARCH-t	0.960	0.046	0.960	0.387
Riskmetrics	0.533	0.099	0.004	0.387
GARCHDSK	0.559	0.369	0.959	0.959
Dow Jones	1d long	1d short	5d long	5d short
GARCH-N	0.816	0.796	0.816	0.796
GARCH-t	0.796	0.387	0.960	0.796
Riskmetrics	0.282	0.533	0.015	0.796
GARCHDSK	0.960	0.960	0.960	0.387
FTSE100	1d long	1d short	5d long	5d short
GARCH-N	0.816	0.387	0.099	0.387
GARCH-t	0.960	0.046	0.091	0.387
Riskmetrics	0.079	0.533	0.025	0.387
GARCHDSK	0.960	0.788	0.960	0.788
MIB30	1d long	1d short	5d long	5d short
GARCH-N	0.533	0.387	0.533	0.387
GARCH-t	0.816	0.046	0.960	0.387
Riskmetrics	0.047	0.960	0.015	0.387
GARCHDSK	0.960	0.796	0.960	0.816
Nasdaq	1d long	1d short	5d long	5d short
GARCH-N	0.816	0.533	0.816	0.960
GARCH-t	0.816	0.387	0.816	0.960
Riskmetrics	0.533	0.533	0.005	0.960
GARCHDSK	0.960	0.282	0.960	0.533
Nikkey225	1d long	1d short	5d long	5d short
GARCH-N	0.816	0.387	0.533	0.046
GARCH-t	0.387	0.046	0.960	0.046
Riskmetrics	0.533	0.387	0.001	0.046
GARCHDSK	0.970	0.397	0.805	0.397
SMI	1d long	1d short	5d long	5d short
GARCH-N	0.960	0.046	0.816	0.046
GARCH-t	0.796	0.046	0.960	0.046
Riskmetrics	0.049	0.796	<0.001	0.046
GARCHDSK	0.790	0.382	0.790	0.822
S&P500	1d long	1d short	5d long	5d short
GARCH-N	0.816	0.816	0.533	0.816
GARCH-t	0.796	0.387	0.960	0.816
Riskmetrics	0.125	0.282	<0.001	0.816
GARCHDSK	0.960	0.533	0.960	0.960

Table 7: p-values for the Christoffersen test for the 1- and 5-day-ahead long and short 1% VaR (1d long, 1d short, 5d long and 5d short, respectively).

Index		GARCH-N	GARCH-t	Risk.	GARCHDSK
CAC40	long	<0.001	<0.001	<0.001	0.225
	short	–	–	0.824	0.605
DAX	long	0.559	0.985	0.371	0.980
	short	0.495	–	0.389	–
Dow Jones	long	0.464	0.175	0.026	0.093
	short	1	1	1	0.987
FTSE100	long	0.059	<0.001	0.234	<0.001
	short	1	–	0.922	0.910
MIB30	long	0.766	0.102	0.342	0.997
	short	1	–	<0.001	0.903
Nasdaq	long	1	0.991	0.131	1
	short	0.999	1	0.994	0.908
Nikkey225	long	0.997	1	0.994	0.597
	short	0.172	–	0.591	0.079
SMI	long	<0.001	0.094	0.005	0.448
	short	–	–	0.969	0.249
S&P500	long	0.784	0.657	0.665	0.997
	short	1	1	0.987	0.999

Table 8: p-values for the Hurlin and Tokpavi test; the test concerns two levels on a single tail ($\alpha_1 = 0.01$ and $\alpha_2 = 0.05$, or $\alpha_1 = 0.95$ and $\alpha_2 = 0.99$) and a *1 day* predictive horizon.

Index		GARCH-N	GARCH-t	Risk.	GARCHDSK
CAC40	long	<0.001	<0.001	0.056	<0.001
	short	–	–	0.999	0.529
DAX	long	0.005	<0.001	0.010	0.001
	short	<0.001	<0.001	0.756	<0.001
Dow Jones	long	<0.001	0.006	<0.001	<0.001
	short	1	–	1	0.499
FTSE100	long	<0.001	<0.001	<0.001	<0.001
	short	0.187	1	0.472	0.765
MIB30	long	0.007	<0.001	0.002	<0.001
	short	0.999	–	0.999	0.999
Nasdaq	long	<0.001	0.010	0.131	<0.001
	short	0.956	1	1	0.987
Nikkey225	long	0.312	<0.001	0.644	0.001
	short	–	–	1	1
SMI	long	<0.001	<0.001	<0.001	<0.001
	short	–	–	0.934	1
S&P500	long	<0.001	<0.001	0.001	0.004
	short	0.979	1	<0.001	0.857

Table 9: p-values for the Hurlin and Tokpavi test; the test concerns a single tail ($\alpha_1 = 0.01$ and $\alpha_2 = 0.05$, or $\alpha_1 = 0.95$ and $\alpha_2 = 0.99$) and a *5 day* predictive horizon.

Index	GARCH-N	GARCH-t	Risk.	GARCHDSK
CAC40	–	–	0.003	0.998
DAX	0.997	–	0.862	0.999
Dow Jones	0.998	0.998	0.772	0.998
FTSE100	0.977	–	0.989	0.399
MIB30	0.999	–	0.008	0.998
Nasdaq	0.998	0.998	0.998	0.998
Nikkey225	0.998	–	0.998	0.421
SMI	–	–	0.230	0.998
S&P500	0.998	0.998	0.998	0.998

Table 10: p-values for the Hurlin and Tokpavi test; the test concerns both tails ($\alpha_1 = 0.01$ and $\alpha_2 = 0.99$) and a *1 day* predictive horizon.

Index	GARCH-N	GARCH-t	Risk.	GARCHDSK
CAC40	–	–	0.003	$<$0.001
DAX	0.999	0.998	0.862	0.998
Dow Jones	0.226	–	0.771	1
FTSE100	$<$0.001	$<$0.001	0.989	$<$0.001
MIB30	0.997	–	0.008	0.421
Nasdaq	0.803	$<$0.001	0.998	0.998
Nikkey225	–	–	0.998	–
SMI	–	–	0.230	0.998
S&P500	0.611	$<$0.001	0.998	0.998

Table 11: p-values for the Hurlin and Tokpavi test; the test concerns both tails ($\alpha_1 = 0.01$ and $\alpha_2 = 0.99$) and a *5 day* predictive horizon.

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Department of Statistical Sciences
University of Padua
Italy