Università di Padova   Facoltà di Ingegneria
Dipartimento di Ingegneria dell'Informazione

Scuola di Dottorato in Ingegneria dell'Informazione
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On the Design of Incentive Mechanisms in Wireless Networks: a Game Theoretic Approach

Dottorando

Luca Canzian

Supervisore:
Chiar.mo Prof. Michele Zorzi

Direttore della Scuola:
Chiar.mo Prof. Matteo Bertocco

Coordinatore di indirizzo:
Chiar.mo Prof. Carlo Ferrari

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To my parents, Walter and Olivana.

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List of Acronyms

3GPP	3rd Generation Partnership Project
ACK	ACKnowledgement
ARQ	Automatic Repeat reQuest
BIC	Bayesian Information Criterion
BN	Bayesian Network
BNE	Bayesian Nash Equilibrium
BPSK	Binary Phase Shift Keying
CDMA	Code Division Multiple Access
CSMA/CA	Carrier Sense Multiple Access with Collision Avoidance
DAG	Directed Acyclic Graph
FDD	Frequency Division Duplexing
FDMA	Frequency Division Multiple Access
GT	Game Theory
IEEE	Institute of Electrical and Electronics Engineers
IMT	International Mobile Telecommunications
LTE	Long Term Evolution
MAC	Medium Access Control
MIMO	Multiple Input Multiple Output

NE Nash Equilibrium

OFDMA Orthogonal Frequency Division Multiple Access

OLSR Optimized Link State Routing

PP Performance Parameters

QAM Quadrature Amplitude Modulation

QoS Quality of Service

QPSK Quadrature Phase Shift Keying

RRA Radio Resource Allocator

SC Single Carrier

SE Stackelberg Equilibrium

SINR Signal to Interference plus Noise Ratio

SNR Signal to Noise Ratio

TCP Transmission Control Protocol

TDD Time Division Duplexing

TP Topological Parameters

UMTS Universal Mobile Telecommunications System

UTRAN Universal Terrestrial Radio Access Network

Abstract

In wireless communication networks, many protocols (e.g., IEEE 802.11 a/b/g Medium Access Control (MAC) protocols) have been designed assuming that users are compliant with the protocol rules. Unfortunately, a self-interested and strategic user might manipulate the protocol to obtain a personal advantage at the expense of the other users. This would lead to socially inefficient outcomes.

In this thesis we address the problem of designing protocols that are able to avoid or limit the inefficiencies occurring when the users act selfishly and strategically. To do so, we exploit the tools offered by Game Theory (GT), the branch of mathematics that models and analyzes the interaction between strategic decision makers.

The dissertation covers aspects related to wireless communications at different levels. We start analyzing the downlink radio resource allocation issue of a cellular network based on Orthogonal Frequency Division Multiple Access (OFDMA). We propose a suboptimal game theoretic algorithm able to preserve the modularity of the system and to trade–off between sum–rate throughput and fairness among the users of the network.

Successively, we address the problem of promoting cooperation in wireless relay networks. To give the incentive for the users of a network to relay the packets sent by other users, we consider a dynamic scheduling in which cooperative users are rewarded with more channel access opportunities.

Infrastructure sharing is another form of cooperation that might be exploit to meet the increasing rate demands and quality of service requirements in wireless networks. We analyze a scenario where two wireless multi–hop networks are willing to share some of their nodes – acting as relays – in order to gain benefits in terms of lower packet delivery delay and reduced loss probability. Bayesian Network analysis is exploited to compute the correlation between local parameters and overall performance, whereas the selection of the nodes to share is made by means of a game theoretic approach.

Afterwards, our analysis focuses on channel access policies in wireless ad–hoc networks. We design schemes based on pricing and intervention to give incentives for the users to access the channel efficiently.

Finally, we consider another important issue that arises when the users are strategic and selfish: when asked to report relevant information, the users might lie, if it is in their individual interest to do so. For a class of environments that includes many resource allocation problems in communication networks, we provide tools to design an efficient system, in which the users have the incentive to report truthfully and to follow the instructions, despite the fact that they are self–interested. We then apply our framework and results to design a flow control management system.

Sommario

Nelle reti di comunicazione wireless, molti protocolli (ad esempio, i protocolli di accesso al mezzo IEEE 802.11 a/b/g) sono stati progettati assumendo che gli utenti rispettino le regole. Purtroppo un utente, guidato da interessi personali, potrebbe manipolare il protocollo per ottenere un beneficio a discapito degli altri utenti. Di conseguenza, la rete wireless sarebbe sfruttata in maniera inefficiente da un punto di vista sociale.

Questa tesi si occupa della progettazione di protocolli in grado di prevenire le inefficienze dovute al comportamento egoistico e strategico degli utenti. Per raggiungere questo scopo, vengono sfruttati gli strumenti offerti dalla teoria dei giochi, la scienza matematica che modella e analizza l'interazione tra soggetti che possono prendere delle decisioni in maniera autonoma.

La tesi copre aspetti legati alla gestione delle comunicazioni wireless a differenti livelli. Si inizia analizzando l'allocazione delle risorse radio, in fase di downlink, di una rete cellulare basata sulla tecnologia di accesso al mezzo di multiplazione a divisione di frequenza ortogonale (OFDMA). Viene proposto un algoritmo sub–ottimo basato sulla teoria dei giochi che permette di preservare la modularità del sistema ed é in grado di trovare un compromesso tra la massimizzazione del throughput totale e un livello equo delle prestazioni degli utenti.

Successivamente, si analizza il problema di incentivare la cooperazione nelle reti wireless in cui gli utenti agiscono opportunisticamente da relay. Per incentivare gli utenti della rete a inoltrare i pacchetti spediti da altri utenti viene adottato uno scheduling dinamico, in cui gli utenti cooperativi sono premiati aumentando le loro opportunità di accesso al mezzo.

La condivisione dell'infrastruttura é un'altra forma di cooperazione che potrebbe essere sfruttata per soddisfare la crescente esigenza di rate e qualità di servizio nelle reti wireless. A tal fine, si considera uno scenario in cui due reti wireless multi–hop sono disposte a condividere alcuni nodi, che agiscono da relay per entrambe le reti. Un'analisi basata sulle reti Bayesiane permette di stimare le prestazioni globali da alcuni parametri locali, mentre un'analisi basata sulla teoria dei giochi permette di selezionare in modo opportuno i nodi da condividere.

In seguito, la nostra analisi si concentra sulle politiche di accesso al mezzo in reti wireless ad–hoc. Viene progettato un protocollo basato sugli schemi di pricing e intervention per incentivare gli utenti ad utilizzare il canale wireless efficientemente.

Infine, si considera un altro importante problema che sorge nel momento in cui gli utenti sono egoisti e strategici: quando viene richiesto di riportare delle informazioni rilevanti, gli utenti potrebbero mentire, se ció fosse nel loro interesse. Partendo da uno scenario generico, comprendente molte problematiche associate all'allocazione di risorse nelle reti di comunicazione, vengono forniti degli strumenti per progettare un sistema efficiente, in cui gli utenti sono incentivati a comunicare le informazioni veritiere e seguire le istruzioni del protocollo. Tali strumenti e risultati vengono applicati per progettare un sistema di controllo della congestione in una rete di comunicazione.

Chapter 1

Introduction

Mobile communications have grown exponentially over the last two decades, and will continue to grow: Cisco projected a 18-fold increase of global mobile data traffic between the end of 2011 and 2016 and over 10 billion mobile-connected devices in 2016 [1]. While this exceptional pace of growth is exciting, it also presents a whole new set of challenges. To meet the increasingly high rate demands and quality of service requirements, future wireless networks must be reliable, able to interoperate and to manage dynamically and efficiently a large set of devices. As a consequence, wireless communication networks are migrating towards more distributed approaches, shifting network intelligence from the core network towards the edges of the network. This transformation is supported by the increase of mobile terminals computation capabilities and leads to more scalable, flexible and reliable networks, decreasing the information exchange and removing the single point of failure of completely centralized approaches.

Distributed algorithms, in which each device of the wireless network is capable of independently adapting its operation based on the current environment, have been studied extensively. Most of these works assume that devices comply with the rules of the algorithm. However, the distribution of the decision making process leads to a new fundamental issue: what happens if the algorithm used by a device is manipulated to pursue a personal benefit? In centralized approaches, such deviations from a prescribed protocol are not authorized and can be detected, because every action is dictated by a central entity. In decentralized approaches, each device has some degree of freedom in setting parameters or changing the mode of operation. By exploiting such leeway, a device might be programmed, by the manufacturer or by the final user, to accomplish a certain objective, at the cost of overall network performance.[1] As a consequence, there is the necessity to design systems able to cope with *selfish*

[1] In [2] the 802.11 MAC protocol of a commercial Broadcom chipset is replaced with a state machine execution engine

users.

To reach this goal wireless engineers need novel analytical approaches to study modern wireless networks, which exploit the tools offered by *game theory*. Game theory is a branch of applied mathematics that models the interaction among *decision makers*, each of them pursuing a personal objective, defines the solution concepts of such interaction and, based on them, provides analytical tools to predict the outcomes. The ability to model independent decision makers, whose actions potentially affect all other decision makers, renders game theory particularly attractive to analyze networking issues. The earlier applications of game theory to wireless networks problems were limited to the analysis of the impact of user selfishness on the performance of existing distributed algorithm. Only recently have they been used in a constructive way: to *design distributed protocols*.

In this dissertation we present some contributions on the design of efficient game-theoretic schemes in wireless networks. This research field can be divided into two main branches. In the first one, devices connected to a wireless network are assumed to pursue the objectives *assigned* by the protocol. Game theory is used to predict the outcomes for different sets of objectives and to design the objectives that allow to achieve the most efficient outcomes. Notice that, in this case, devices are *compliant* with the protocol rules, in that they accept passively the designed objectives, which may differ from the objectives of the users that operate the devices. Such an approach may help to design distributed algorithms, demonstrating and predicting the convergence of such algorithms, but does not answer the initial question, i.e., what happens if the algorithm used by a device is manipulated to pursue a personal benefit?

In the second branch, the devices connected to a wireless network are assumed to follow *personal objectives*, which are aligned with the objectives of the users that operate the devices. In this case game theory is used to design algorithms that are able to achieve efficient outcomes, despite the fact that devices seek to optimize their personal objectives. Such an approach allows to design protocols that provide the incentive to follow the rules: it will be in the self-interest of each user not to manipulate the algorithm.

Except for Chapter 3, in which we follow the first approach, in this thesis we follow the second approach, i.e., we assume that devices are autonomous decision makers that pursue their own interest, and we design *incentive schemes* to drive the outcome of the system toward an efficient point, covering aspects related to wireless communications at different levels.

which allows to program and use the desired MAC protocol. Such a capability of modifying protocols results in our concerns for self-interested users in future wireless networks.

1.1 Game Theory in Wireless Networks

The application of game theory to the modeling and analysis of wireless communication networks has received considerable attention in recent years, and has led to numerous tutorials [3–5] and books [6, 7] outlining game-theoretic concepts and their usage in wireless networks. Wireless communication networks are full of scenarios that can be modeled as *games*, examples are

- resource allocation [8–12]: sharing of the networks resources, such as channels, bandwidths and time slots
- power control [13–16]: adjustment of the transmission power
- relay network [17–22]: opportunistic packet forwarding
- flow/congestion control [23–29]: adjustment of the rate to the available bandwidth of the network
- network routing [30–33]: selection of paths with certain desirable properties

All these scenarios have in common the following features: (1) there is a set of users, (2) each user takes some actions based on a certain objective, and (3) the achievement of the objective depends on the actions taken by *every users*. As an example, in a flow control scenario each user connected to a network may decide to modulate its transmission rate to achieve a desired trade-off between its experienced throughput and delay. However, the delay depends on the total congestion of the network, which in turn depends on the transmission rates adopted by every user. Thus, the best action for a user depends on the actions adopted by the others, and it is not trivial to foresee the outcome of this interaction: game-theoretic tools must be exploited to do it.

The earlier applications of game theory to wireless networks problems were limited to the computation of the outcome of the interaction among selfish users adopting the existing schemes. This analysis provides insights on how robust the considered scheme is in presence of selfish users. Unfortunately, the operation of the network by selfish users usually leads to substantial inefficiencies, because the considered scheme has not been designed with this issue in mind. For example, [9–12] shows that the IEEE 802.11, the slotted Aloha and the CSMA/CA MAC protocols can lead to inefficient outcomes, if not to a network collapse. [31] demonstrates that the total latency of the routes chosen by selfish network users is at most $\frac{4}{3}$ times the minimum possible total latency if the latency cost of each edge is a linear function of its congestion, but for general cost function the total latency can be arbitrarily larger than the minimum possible. [34] shows that most congestion control schemes used, such as TCP, encourage a behavior that leads to congestion.

As a consequence, game theory was later applied by wireless engineers to design schemes able to cope with users who behave selfishly. In these schemes incentives for the users to adopt efficient actions are provided For example, *pricing scheme* that charge the users for their resource usage are used by [35–39] to design efficient slotted-Aloha like random access protocols, and by [30, 40, 41] to design efficient flow control management systems. *Intervention schemes*, in which a device provides the incentive for the users to adopt efficient actions by threating punishments, are applied to situations of medium access control [12, 42] and power control [16]. In [43, 44] efficient outcomes in power control problems are obtained introducing hierarchy in the scheme, allowing some users to move before others, and this is further advanced in [45] by considering a repeated interaction in which cooperation among users is obtained by punishing deviating users in subsequent stages.

1.2 Organization and Contributions of the Thesis

The rest of this thesis work is organized as follows:

Chapter 2: we introduce some important concepts, notations and tools of game theory that are extensively used in the dissertation. This chapter provides a useful background information for the remaining part of the thesis, in particular for the reader who is not familiar with game theory.

Chapter 3: we propose a novel approach, based on game theory, for radio resource allocation in the downlink of cellular networks using OFDMA. The reference technology is the LTE of the 3GPP UTRAN. The main contribution is to identify a model for the allocation objectives, and how to approach them in a tunable manner. The resource management issue is framed in the context of *spectrum sharing*, where multiple entities agree on utilizing the radio access channel simultaneously. A trade-off between sum-rate throughput and fairness among the users is identified and addressed through game theory, i.e., moving the operation of the system towards a stable Pareto efficient point. Such a methodology can be implemented with low complexity while ensuring the modularity of the overall system. Numerical results are also shown, to exemplify the validity of the proposed approach.

Chapter 4: we apply game theory to constructively derive practical network management policies for wireless relay networks. We focus on the problem of medium sharing and opportunistic packet forwarding in wireless relay networks, and we show how, by properly modeling the agents involved in such a scenario, and enabling simple but effective incentives towards cooperation for the users, we obtain a resource allocation scheme which is meaningful from both perspectives of game theory and network engineering. Such a result is achieved by introducing throughput redistribution as a way to transfer utilities, which enables cooperation among the users.

Chapter 5: we analyze a scenario where two wireless ad hoc networks are willing to share some of their nodes, acting as relays, in order to gain benefits in terms of lower packet delivery delay and reduced loss probability. Bayesian Network analysis is exploited to compute the correlation between local parameters and overall performance, whereas the selection of the nodes to share is made by means of a game theoretic approach. Our results are then validated through use of a system level simulator, which shows that an accurate selection of the shared nodes can significantly increase the performance gain with respect to a random selection scheme.

Chapter 6: we consider a number of users who compete to gain access to a channel, a slotted-Aloha like random access protocol and two incentive schemes: *pricing* and *intervention*. We provide some criteria for the designer of the protocol to choose one scheme between them and to design the best policy for the selected scheme, depending on the system parameters. Our results show that intervention can achieve the maximum efficiency in the *perfect monitoring* scenario. In the *imperfect monitoring* scenario, instead, the performance of the system depends on the information held by the different entities and, in some cases, there exists a threshold for the number of users such that, for a number of users lower than the threshold, intervention outperforms pricing, whereas, for a number of users higher than the threshold pricing outperforms intervention.

Chapter 7: we study the interaction between a designer and a group of strategic and self-interested users who possess information the designer does not have. Because the users are strategic and self-interested, they will act to their own advantage, which will often be different from the interest of the designer, even if the designer is benevolent and seeks to maximize (some measure of) social welfare. In the settings we consider, the designer and the users can communicate (perhaps with noise), the designer can observe the actions of the users (perhaps with error) and the designer can commit to (plans of) actions – *interventions* – of its own. The designer's problem is to construct and implement a *mechanism* that provides incentives for the users to communicate and act in such a way as to further the interest of the designer – *despite* the fact that they are strategic and self-interested and possess private information. To address the designer's problem we propose a general and flexible framework that applies to many scenarios. In an important class of environments, we find conditions under which the designer can obtain its benchmark optimum – the utility that could be obtained if it had all information and could command the actions of the users – and conditions under which it cannot. More broadly we are able to characterize the solution to the designer's problem, even when it does not yield the benchmark optimum. Because the optimal mechanism may be difficult to construct and implement, we also propose a simpler and more readily-implemented mechanism that, while falling short of the optimum, still yields the designer a "good" result. We then apply our framework and results to design a flow control management system, in both the complete and the incomplete information scenarios. Illustrative results show that the considered schemes can considerably improve the efficiency of the network.

Chapter 8: concludes the thesis with some remarks.

Chapter 2

Game Theory Preliminaries

This chapter introduces some important concepts, notations and tools of game theory. This is not meant to be a comprehensive and in-depth guide of game theory, for which we refer the interested reader to standard books such as [46–50], rather we lay the mathematical groundwork for the subsequent sections. The reader who is already familiar with game theory may want to skip this chapter.

2.1 Basic Concepts

Game theory is a branch of applied mathematics that attempts to capture rational behaviors in strategic situations – called *games* – in which an individual's success in making choices depends on the choices of others. This interdependence causes each individual – called *player* – to consider the other player's possible decisions – or *strategies* – in formulating his own strategy. Traditional applications of game theory assume that players are *self-interested* and *strategic*, meaning that they pursue a personal objective and they are aware of all consequences of their actions, and seek to find *equilibria* in these games: a sets of strategies in which players are unlikely to change their behavior. Many equilibrium concepts have been developed in an attempt to capture this idea. These equilibrium concepts, although they often overlap or coincide, are motivated differently depending on the considered scenario and on the game formulation. In the following sections we describe the game formulations, and the corresponding equilibrium concepts, that are of interest for this dissertation.

2.2 Static Games with Complete Information

In the most straightforward game formulation, each player selects a single *action* from a set of feasible actions, and each player evaluates the resulting outcome through a *utility function* quantifying the goodness coming from the adopted actions. If the players play the actions simultaneously (alternatively, one can think that they play the actions in different instants, but without knowledge of the actions played by the others), the game is said to be *static*. If the action sets and the utilities of all players are common knowledge among the players, the game is said to be with *complete information*.

Formally, a static game with complete information Γ can be represented by the tuple

$$\Gamma = (N, A, \{U_i\}_{i=1}^{n})$$

in which $N = \{1, \ldots, n\}$ is the set of players, labeled from 1 to n, $A = A_1 \times \ldots \times A_n$ is the set of action profiles, A_i is the set of actions player i can take, and U_i is player i's utility function – or utility for short. We write (a_i, a_{-i}) for the action profile in which player i chooses action $a_i \in A_i$ and other players choose the action profile $a_{-i} \in A_{-i} = A_1 \times \ldots \times A_{i-1} \times A_{i+1} \ldots \times A_n$; this is a common notation to specify a characteristic associated to all players except for player i, we use similar notations throughout the thesis. The utility $U_i : A \to \Re$ depends on the actions of *all* players, thus each player seeking to maximize his own utility has to consider the other player's possible actions in selecting his own action.

We say that an action a_i is weakly dominated by a_i' (equivalently, a_i' weakly dominates a_i) if player i's utility playing a_i' is greater than or equal to player i's utility playing a_i, for any actions of the other players, i.e.,

$$U_i(a_i', a_{-i}) \geq U_i(a_i, a_{-i}) \quad , \quad \forall\, a_{-i} \in A_{-i}$$

If the inequality is strict, then we say that a_i is strictly dominated by a_i' (equivalently, a_i' strictly dominates a_i). If an action weakly (strictly) dominates every other action, we say that it is a weakly (strictly) dominant action. It is quite obvious that a selfish and strategic player i would never adopt an action a_i which is strictly dominated by an action a_i', because action a_i' alway guarantees him a higher utility. Thus, from a practical point of view, action a_i can be eliminated from the set A_i. This procedure can be iterated and the same player i or other players can eliminate other strictly dominated actions[1]. This procedure is called *iterated elimination of strictly dominated strategies*, and

[1] Notice that, in doing so, a player $j \neq i$ may discover that the action $a_j \in A_j$ is strictly dominated by another action only after having eliminated action $a_i \in A_i$. This implicitly extends the notion of common knowledge: not only do players know the action sets and the utilities of the others, but they also know that all players are self-interested and strategic, and all players know that all players know, etc.

can be useful to obtain a "smaller" game or, in the rare cases in which only a single action profile is left, to compute the most likely outcome of a game.

2.2.1 Nash equilibrium

We define the *best response function* h_i^{BR} (though correspondence would be a more suitable name) of a player i as the set of player i's actions that maximize player i's utility for a given action profile of the other players, i.e., $h_i^{BR}(a_{-i}) = \operatorname{argmax}_{a_i} U_i(a_i, a_{-i})$. We say that the action a_i is a best response to the actions profile a_{-i} if $a_i \in h_i^{BR}(a_{-i})$.

Now we have the instruments to define one of the most important and best known concept of game theory: the *Nash Equilibrium* (NE). A NE is an action profile that corresponds to the mutual best response: for each player i, the action selected is a best response to the actions of all others. Equivalently, a NE is an action profile where no individual player can benefit from unilateral deviation, and for this reason it is said to be *self–enforcing* or *strategically stable*. Formally, a^{NE} is a NE if

$$U_i(a_i^{NE}, a_{-i}^{NE}) \geq U_i(a_i, a_{-i}^{NE}) \quad , \quad \forall\, i \in N,\ \forall\, a_i \in A_i$$

The action profiles corresponding to the Nash equilibria are a consistent prediction of the outcome of the game, in the sense that if all players predict that a NE will occur then no player has any incentive to choose a different action. For this reason a NE is commonly regarded as a *solution concept* of a game.

A Nash equilibrium may not exist, unless particular classes of games are considered[2], and there can be multiple Nash equilibria in a game, resulting in the issue on how players coordinate to a particular Nash equilibrium.

Another issue related to the NE (and to all equilibrium concepts we will define), which is of particular importance for this dissertation, is its *efficiency*: usually a NE does not correspond to an efficient outcome for a game. *Pareto optimality* is often used as a reference point for the efficiency of an outcome. An action profile is Pareto optimal if there is no other action profile that makes every player at least as well off while making at least one player better off. Formally, $a = (a_1, \ldots, a_n)$ is

[2]In some contexts players are allowed to *randomize* their actions, i.e., each player i adopts an action $a_i \in A_i$ following the strategy $s_i \in \Delta(A_i)$ which represents a probability distribution over the set A_i (if A_i has cardinality $|A_i|$, $\Delta(A_i)$ denotes the $|A_i| - 1$ unit simplex). These types of strategies are commonly called *mixed strategies*. In this case, each player is assumed to select a strategy that maximizes the expectation of his utility over the random action profile. For this particular situation the Nash theorem [49], which is an application of the Kakutani fixed-point theorem [51] to the best response functions, guarantees the existence of a NE.

Pareto optimal if there exists no other action profile $a' = (a'_1, \ldots, a'_n)$ such that $U_i(a') \geq U_i(a) \forall\, i \in N$ and $U_j(a') > U_j(a)$ for some $j \in N$. In an attempt to quantify the inefficiency of a game, the concept of *Price of Anarchy* has been introduced. After defining an efficiency measure for the players utilities (natural candidates are sum of utilities, or minimum utility, or some other measure of fairness among the utilities), the price of anarchy is defined as the ratio between the "worst" equilibrium and the "best" action profile – worst and best with respect to the efficiency measure considered. Notice that if the efficiency measure is increasing in each player utility, then the best action profile must be Pareto optimal.

2.3 Bayesian Games

There are many familiar situations in which some of the players are not certain about the characteristics of some of the other players. Bayesian games are designed for this purpose, to model static games with *incomplete information*. In Bayesian games each player is assumed to maximize his expected utility with respect to the unknown parameters. This implicitly assumes that each player has a *prior belief* about the characteristics of the other players.

Formally, a generic player i is characterized by an element of a set T_i of *types*; a player's type encodes all relevant information about the player, which will include the player's utility function and the influence the player's type has on other players and on the designer. We write $T = T_1 \times \ldots \times T_n$ for the set of possible type profiles. Players know their own type; players and the designer know the distribution of player types π (a probability distribution on T).[3] If player i is of type t_i then $\pi(\cdot \mid t_i)$ is the conditional distribution of types of other players. We allow for the possibility that types are correlated, which might be the case, for instance, if players have private information about the current state of the world and not only about themselves.

Finally, we can formalize a Bayesian game Γ by the tuple

$$\Gamma = (N, A, T, \pi, \{U_i\}_{i=1}^n)$$

in which N, A and U_i are the player set, action profile set and player i's utility respectively. Player i's utility $U_i : A \times T_i \rightarrow \Re$ depends on the actions of all players and on player i's type.

We define a *strategy* for player i as a function $g_i : T_i \rightarrow A_i$ that specifies which action to take, conditional on the type of player i. We may think of the type as given to the player at the beginning of the game, and the strategy tells which action he will adopt after being assigned a type. In general,

[3] We usually think of the distribution π as common knowledge but this is not entirely necessary.

a strategy for a player encodes all the strategic aspects of a game, while an action represents only a particular move (these two concepts coincide only for a static game with complete information).

In a Bayesian game each player is assumed to maximize his expected (with respect to the types of the other players) utility EU_i, which is a function of the strategy of all players and on player i's type. Given a strategy profile $g = (g_1, \ldots, g_n)$ and a type t_i, EU_i is given by

$$EU_i(g, t_i) = \sum_{t_{-i} \in T_{-i}} \pi(t_{-i} \mid t_i) U_i(g(t), t_i)$$

where $g(t) = (g_1(t), \ldots, g_n(t))$.

An important solution concept for a Bayesian game is the *Bayesian Nash equilibrium* (BNE), i.e., the Nash equilibrium applied to the expected utilities. A BNE is the strategy profile where no individual player can benefit (in terms of expected utility) from unilateral deviation. Formally, g^{BNE} is a BNE if

$$EU_i(g_i^{BNE}, g_{-i}^{BNE}, t_i) \geq EU_i(g_i, g_{-i}^{BNE}, t_i) \quad , \quad \forall\, i \in N,\ \forall\, t_i \in T_i,\ \forall\, g_i : T_i \to A_i$$

2.4 Stackelberg Games

A natural extension of static games are *dynamic* games, in which players are allowed to take actions sequentially. Before analyzing the dynamic games in general, we consider a simple dynamic game: the *Stackelberg game*.

A Stackelberg game is a 2-player game in which players move alternatively: first player 1 – the *leader* – then player 2 – the *follower*. As usual, both players are characterized by the action sets A_1 and A_2 and the utilities $U_1 : A \to \Re$ and $U_2 : A \to \Re$, $A = A_1 \times A_2$. We assume that this information is common knowledge among the players (i.e., complete information scenario), and we assume that player 2 can observe the move of player 1 before selecting his own action (this property is known as *perfect information*, we will formally define it in the next section). The strategy s_1 for player 1 coincides with the action he adopts, while the strategy s_2 for players 2 describes which action to adopt conditional on the action adopted by player 1, $s_2 : A_1 \to A_2$.

A Stackelberg game can conveniently be represented by a tree, as in Fig. 2.1, where the nodes represent the players allowed to move in that stage of the game and the links represent the actions the players can adopt. Following a particular path, i.e., given the actions adopted by the players, we end up in a particular leaf of the three, represented by a pair of numbers, which specify the utilities obtained by players 1 and 2 respectively.

A possible solution concept of a Stackelberg game is represented by a NE. However, Nash Equilibria can also include unlikely outcomes. For instance, a NE of the Stackelberg game represented in Fig. 2.1 is $s_1 = a_1$, $s_2(a_1) = a_2$, and $s_2(a'_1) = a_2$. Following this strategy player 1 and 2 obtain a utility respectively of 1 and 2. It is easy to see that this is a NE because no player could gain from deviating unilaterally: if player 1 changes his strategy (keeping fixed player 2's strategy) he would obtain a utility of 0; if player 2 changes his strategy (keeping fixed player 1's strategy) he would obtain a utility of 2 if $s_2(a_1) = a_2$ (i.e., he only changes $s_2(a'_1)$), or 0 if $s_2(a_1) = a'_2$. However, this equilibrium is based on the *threat* that player 2 adopts action a_2 if player 1 adopts action a'_1. In situations in which player 2 cannot commit to a particular strategy, this *threat is not credible*, and this NE is unlikely to happen. In fact, player 1 can foresee that the strategy player 2 will probably adopt is $s_2(a_1) = a_2$ and $s_2(a'_1) = a'_2$, and consequently select the action a'_1 which is his best strategy given the predicted strategy for player 2. The strategy profile obtained in this way, which is still a NE, is called *Stackelberg Equilibrium* (SE).

A SE is a refinement of a NE in Stackelberg games, and is obtained by means of *backward induction*: first the SE strategy of player 2, s_2^{SE}, is computed maximizing U_2 for each action of player 1, then the SE strategy of player 1, s_1^{SE}, is obtained maximizing U_1 given s_2^{SE}. Since this procedure requires a double maximization, the existence (but not the uniqueness) of the SE is guaranteed. Stackelberg games are commonly extended to situations in which a player moves first and the others move later. For the analysis of this type of games we refer the reader to the next section.

2.5 Dynamic Games

A dynamic game involves players moving sequentially. This means that we describe games taking place through *stages*. Dynamic games can conveniently be represented by trees, similarly to Fig. 2.1. We consider only dynamic games with complete information. If, in each stage of the game, the acting player knows the history of the game, we say that the game is with *perfect information*. If information is *imperfect* it means that some moves are simultaneous. For this reason we focus only on perfect information games, possibly allowing for simultaneous actions in some of the stages.

In a dynamic game, a player's strategy specifies the action to take in each stage, for each history of play through previous stages. We can regard any stage of a dynamic game as a static game, chosen among a number of possible alternatives (one per each game history!). However, the acting players must take into account how their actions in that stage influence the evolution of the game. After t

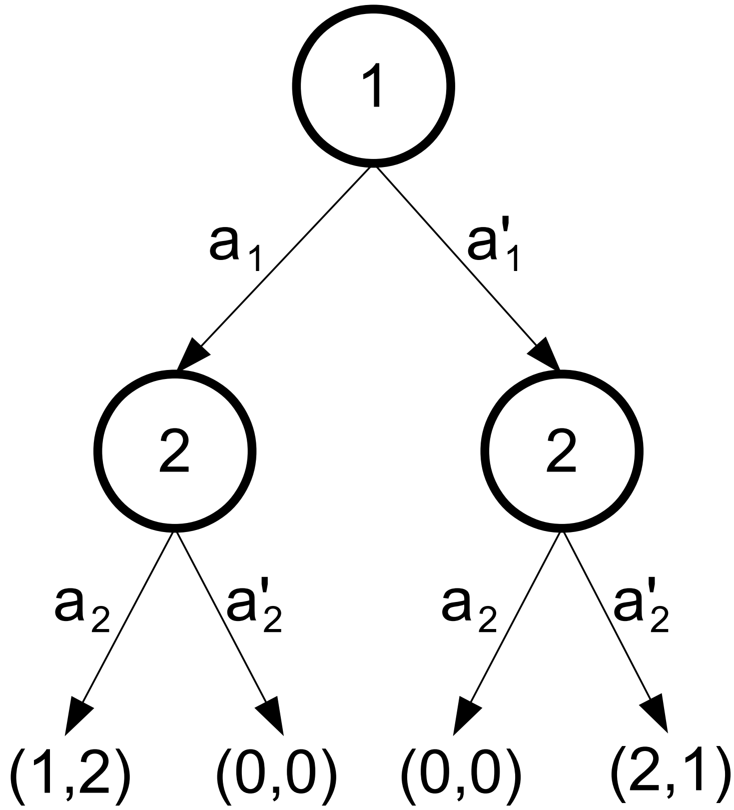


Figure 2.1. *Representation of a Stackelberg game*

stages, a *subgame* takes place from stage $t+1$ onwards. The whole game can be considered as a subgame of itself.

We say that a Nash equilibrium of a dynamic game is *subgame-perfect* if the players' strategies constitute a Nash equilibrium in every subgame. Subgame-Perfect Equilibrium (SPE) is a refinement of the NE that takes into account the credibility of the threats. It can be seen as an extension of the SE to more complex dynamic games (the two equilibrium concepts coincide in Stackelberg games). The backward induction procedure can be applied to compute a SPE: first a NE of the last stage is computed, then given this a NE of the second-last stage is computed, etc. Since at each step a NE computation is required, a SPE may not exists. However, if at each stage of the game only one player is allowed to play, the NE computation simplifies in a maximization (like in the Stackelberg game considered in the previous section), and the existence (but not the uniqueness) of the SPE is guaranteed.

2.6 Repeated Games

An interesting and well-understood class of dynamic games is that of *repeated games*. In a repeated game Γ^R the players play the *same stage game* Γ repeatedly, and the player's overall utility is a weighted average of the utilities in each stage. Repeated play introduces in general new equilibrium outcomes with respect to the stage game, because players can condition their play on the information they have received in the past. In this way a player has to take into account the effect of his current action on the other players' future behavior.

There exist two distinct versions of repeated games: finitely repeated games and infinitely repeated games. In *finitely repeated games* the stage game is played for a fixed number of times. The arithmetic mean of the utilities in each stage is usually adopted to quantify the overall utility of a player at the end of the game. If the stage game has a unique NE it is easy to check, using the backward induction, that the finitely repeated game has a unique SPE: to play the NE of the stage game in every stage. In fact, in the last stage players will play the unique NE of the stage game. In the second-last stage, given that in the last stage players will for sure play the unique NE, players will play again the unique NE of the stage game. And so on. More interesting is the situation if the single stage game has multiple Nash equilibria. In this case, for example, players can "agree" to play the "best" NE in the last stage if in the second-last stage they have adopted an efficient action profile (which might not be a NE of the stage game), to play the "worst" NE otherwise.

In *infinitely repeated games* the stage game is played infinitely. To quantify the utility U_i^R of a player i at the end of the game, the average utility $U_i^R = (1-\delta)\lim_{T\to+\infty}\sum_{t=1}^{T}\delta^{t-1}U_i^{(t)}$ is usually adopted, where $U_i^{(t)}$ is the utility obtained by player i at stage t and $\delta \in (0,1)$ is the *discount factor*. The discount factor is introduced mainly for mathematical reasons, but it can be useful to capture situations in which an imminent reward is better than a future reward, or in which each player can exit from the game with a certain probability. For infinitely repeated games there exists an important theorem, Friedman's theorem (also known as folk theorem), which states what the players can obtain with SPE strategies. We define a *feasible utility* as any convex combination of the utility obtainable in the single stage game, and let $(U_1^{NE}, \dots, U_n^{NE})$ be the utility obtainable with a NE of the single stage game. Let $(U_1, \dots, U_n)$ be a feasible utility such that $U_i > U_i^{NE}$, $\forall\, i \in N$. Friedman's theorem states that, if δ is close enough to 1, the infinitely repeated game has a SPE in which players obtain utilities $EU = (EU_1, \dots, EU_n) = (U_1, \dots, U_n)$. The intuition behind it is the adoption of a dynamic strategy in which the players adopt by default, for a certain number of stages, a certain action profile, and then change to another action profile, and so on. In this way any convex combination of the utility of the stage game can be obtained. As soon as a generic player i deviates from this "agreement", the other players *punish* it by adopting the NE action profile of the stage game for a certain number of stages. The condition $U_i > U_i^{NE}$ guarantees that the final utility foreseen for player i is higher than the utility player i would obtain during the punishment stages. The duration of the punishment can be set so that the gain obtained during the deviating stage does not compensate the loss incurred during the subsequent stages. Notice that a smaller δ makes the punishment less effective to deter deviations, from which the condition that δ must be close enough to 1.

2.7 Coalitional Games

Cooperative game theory is a branch of game theory that provides analytical tools to study the behavior of self-interested and strategic players when they try to find an "agreement" to cooperate.

The main area of cooperative games is represented by coalitional games [52], defined as a pair (N, v), where $N = 1, ..., n$ is a discrete set of players and v is a function that quantifies the *value* of a coalition in a game. Each coalition $S \subseteq N$ behaves as a single player, competing against other coalitions in order to obtain a higher value of v. A coalitional game may have the following properties:

Property 1. (Characteristic form) *The value of a coalition S depends only on who are the members of that coalition, regardless of other coalitions*

Property 2. (Transferable utility) *The value of a coalition is a real number, representing the total utility achieved by the coalition, and it can be arbitrarily divided among its members*

For coalitional games satisfying properties 1 and 2, the value $v : 2^N \to \Re$ is a function that assigns to each coalition S the total utility achieved by it. The utility value can be arbitrarily divided among the coalition members and the amount of utility that a player $i \in S$ receives, x_i, is the player's payoff. A payoff allocation is a vector $\mathbf{x} \in \Re^{|S|}$ (where $\mid S \mid$ is the cardinality of the set S) whose elements are the payoffs of players belonging to the coalition; in other words, it represents a redistribution of the total utility.

Another interesting property that a coalitional game may have is super-additivity, that for a game with properties 1 and 2 assumes the following form:

Property 3. (Super-additivity)
$v(S_1 \cup S_2) \geq v(S_1) + v(S_2) \;\; \forall S_1, S_2 \subset N$ *s.t.* $S_1 \cap S_2 = \emptyset$

The super-additivity property expresses in mathematical terms that formation of a larger coalition is always beneficial. Hence, for those games where it holds, the players are encouraged to stick together, forming the grand coalition N.

For a game having all properties listed before, the main aspects to analyze are:

- finding a redistribution of the total utility $v(N)$ such that the grand coalition is stable, i.e., no group of players has an incentive to leave the grand coalition
- finding fairness criteria for the redistribution of the total utility
- quantifying the gain that the grand coalition can obtain with respect to non cooperative behaviors

A payoff allocation is *group rational* if $\sum_{i=1}^{n} x_i = v(N)$ and it is *individually rational* if $x_i \geq v(\{i\})\ \forall i$, i.e., if every player does not obtain a lower utility by cooperating than by acting alone. A payoff allocation having both properties is said to be an *imputation*.

The concept of *core*, $\mathcal{C}$, is also very important. It is defined as the set of imputations that guarantee that the grand coalition is stable, i.e., all payoff allocations where no group of players $S \subset N$ have an incentive to refuse the proposed payoff allocation, leaving the grand coalition and forming coalition S instead. Mathematically,

$$\mathcal{C}=\left\{ \mathbf{x} \text{ s.t. } \sum_{i=1}^{n} x_i = v(N)\ ,\ \sum_{i \in S} x_i \geq v(S)\ \forall S \subset N \right\} \tag{2.1}$$

Indeed, the core may be empty, in which case the grand coalition is not stable. The existence of the core ought to be checked case by case, possibly exploiting some categories of games where the existence is guaranteed [47, Ch. 13].

Chapter 3

Exploiting Game Theory for Resource Allocation in LTE Systems

In this chapter[1] we propose a novel approach, based on game theory, for radio resource allocation in the downlink of cellular networks using OFDMA. The reference technology is the LTE of the 3GPP UTRAN. The main contribution is to identify a model for the allocation objectives, and how to approach them in a tunable manner. The resource management issue is framed in the context of *spectrum sharing*, where multiple entities agree on utilizing the radio access channel simultaneously. A trade-off between sum-rate throughput and fairness among the users is identified and addressed through game theory, i.e., moving the operation of the system towards a stable Pareto efficient point. Such a methodology can be implemented with low complexity while ensuring the modularity of the overall system. Numerical results are also shown, to exemplify the validity of the proposed approach.

3.1 Introduction

Cellular wireless systems have been able to improve their transmission rates, so as to reach "high speed" communication, thanks to the introduction of channel-aware radio resource allocation. This means that packet scheduling and the corresponding assignment of physical layer resources are dynamically performed according to the channel conditions and Quality of Service (QoS) experienced by the users.

[1]The material presented in this chapter has been published in:

[C1] L. Anchora, L. Badia, **L. Canzian**, and M. Zorzi, "A Characterization of Resource Allocation in LTE Systems Aimed at Game Theoretical Approaches," in *Proc. IEEE CAMAD*, Miami, FL, USA, Dec. 3-4, 2010

An important scenario where this principle finds application is represented by the Long Term Evolution (LTE) of Third Generation (3G) systems [53]. In this technology, the multiple access scheme in the downlink uses Orthogonal Frequency Division Multiple Access (OFDMA). Such a technology exploits multiple orthogonal subcarriers which can be used to take advantage of multi-user diversity [54]. However, given the key role played by the physical layer and the correlation of channel quality, the principles of a fair scheduling of multiple users are difficult to harmonize with the efficient resource allocation aiming at maximizing throughput.

In this chapter, along the lines of [55], we utilize a modular representation of the radio resource management procedure which is split between two functional entities, i.e., a credit-based scheduler and the actual resource allocator, operating at the transport layer and the medium access layer, respectively. The scheduler determines which packets, taken from different flows, are candidates to be served in the next allocation round. The resource allocator associates the packets with groups of OFDMA subcarriers, also accessed in a time division fashion, so that the resources to allocate are time/frequency resource blocks. In this choice, the resource allocator exploits a degree of freedom, represented by the number of packets selected by the scheduler (larger than the number of slots).

The resulting allocation can be regulated according to a trade-off between two contrasting objectives, i.e., that of throughput maximization, which is achieved by selecting the packets only according to a channel quality rationale, and fairness among the flows, which requires to pursue equity among the achieved rates. Indeed, this trade-off is reflected by the number of packets selected by the scheduler: when it is minimum, i.e., only the packets that fit the OFDMA frame are selected, all packets are mandatorily allocated, and the resource allocator has no choice. Here the allocation is only determined by the credit-based scheduler, which ensures fairness (the users with higher credits are allocated). Conversely, if the number of selected packets is high, the resource allocator can restrict the selection to the packets of the users with the best quality, entirely neglecting any fairness among flows. Therefore, to solve the trade-off we present an original approach based on game theory which tries to combine both objectives in an efficient yet easy to implement manner. The key idea is to treat the scheduler and the resource allocator as two players of a non-cooperative game. The resulting Nash equilibria are considered as possible solutions to the radio management problem, which exhibit a low computational cost, yet, under certain conditions, satisfactory performance. After discussing the proposed approach and its possible implementation, we also present some simple numerical evaluations for a two-person game which confirm the goodness of our approach and its ability to regulate the trade-off in a Pareto efficient allocation point.

Note that the scheduler and the resource allocator are part of a system operated by the same entity, that has a unique objective. We remark that in this chapter game theory is used to obtain a non-optimal but simple to implement algorithm, that preserves the modularity of the system. Conversely, in the following chapters we will exploit game theory to analyze the interaction between entities having different objectives.

The outline of the rest of the chapter is as follows. Section 3.1.1 outlines related approaches presented in the literature. Section 3.2 describes the properties of the LTE technology and discusses the layered characterization we gave to the resource allocation procedure. Section 3.3 introduces our proposal, whose rationale is based on game theory, which is used to determine a trade-off between throughput efficiency and fairness among the users. Supporting numerical results are shown in Section 3.4 and we conclude in Section 3.5.

3.1.1 Related work

Adaptive multi-user multi-carrier allocation schemes based on instantaneous channel state information in OFDMA systems allow significant performance improvements in terms of allocation efficiency. This happens thanks to the exploitation of the *multiuser diversity* principle, where subcarriers are preferably assigned to users experiencing favorable subchannel conditions and higher order modulation can be used to transport more bits per OFDMA symbol.

In chapter we focus on the resource allocation optimization problem in OFDMA downlink systems with perfect channel state information at the base station. In the literature there is no unique formulation for this type of problem. The most common formulation is the weighted sum rate maximization subject to some transmit-power constraints. For any fixed subchannel assignment, the optimal solution is achieved by multilevel waterfilling [56] for the continuous rate case (channel capacity is considered) and greedy or bisection allocation algorithms [57] for the integer-bit constellation case (bit rate constrained to real modulation schemes). When equal weights are considered, the optimal subchannel assignment is simply obtained by giving each subchannel to the user with the best gain to noise ratio [56]. This is called the max-sum-capacity rule, which results in the most efficient use of the resources in terms of throughput but can lead to unfairness and instability, especially for non-symmetrical channel conditions and non-uniform traffic patterns [54]. However, in the general case, finding the optimal subchannel assignment is a combinatorial problem whose complexity increases exponentially with the number of subcarriers. To find an efficient suboptimal algorithm, [56] considers a convex relaxation method, allowing time sharing in each subchannel. In this way the problem becomes convex and can be solved in polynomial time using interior-point methods. A further reduction in computational complexity is achieved considering a constant power for the used subchannels. In [58] a solution of the problem is efficiently computed using Lagrange dual decomposition and considering that the duality gap is zero when the number of subcarriers tends to infinity. Previously described works consider continuous rate adaptation. An additional constraint is added in [53] taking into account that real communication systems rely on integer-bit constellations. Moreover, since LTE is considered, the modulation and coding scheme for a given user has been considered fixed during a scheduling period. Also in this case the problem is combinatorial and a sub-optimal algorithm has been designed to reduce the computational complexity.

Another way of tackling the problem is power minimization subject to rate constraints for each user. In [58], similar to the weighted sum rate maximization, the Lagrange dual decomposition method has been proposed. In [59] an integer-bit constellation is considered and the power has been assumed to be a convex and increasing function of the bit rate (most popular coding and modulation schemes satisfy this condition). Due to the combinatorial nature of the problem, a convex relaxation has been used to obtain a sub-optimal solution.

Another approach is proposed in [60] where a fairness constraint is taken into account: the smallest capacity among all users is maximized, subject to a total transmit-power constraint. Variable bit rate traffic is considered, but the formulation can be slightly modified to consider constant bit rate traffic. This objective function can lead to inefficiencies if some users experience deeply faded subchannels. In [54], in order to support delay-sensitive applications, an approach that maximizes the total utility with respect to mean queue delays is proposed. Also in these last works, suboptimal solutions are computed due to the combinatorial nature of the problem. Finally, we cite the proportional fair scheduling [54], that aims at maximizing the logarithm of the average data rates to trade off spectrum efficiency and fairness among users.

To sum up, it is difficult to formulate the desired optimization goal and constraints for the multi-user multi-carrier allocation problem, in particular when mixed traffic with different QoS requirements is considered. Also, the set selection nature of the sub-carrier allocation leads to a combinatorial problem that requires an exhaustive search, with exponentially increasing complexity. Simplified approaches must be considered to design real time algorithms exploiting instantaneous subchannel information. This motivates us to consider an approach that does not claim optimality with respect to a subjective utility function, but rather is computationally lightweight and able to find a good trade-off between aggregate performance (in terms of throughput/spectrum efficiency) and fairness among flows.

3.2 Overview of LTE and System Model

LTE is a set of improvements to the Universal Mobile Telecommunications System (UMTS) introduced in the 3rd Generation Partnership Project (3GPP) Release 8 [61]. It represents efficient packet-based radio access networks allowing high throughput, low latency and low operating costs. Small enhancements have been introduced on LTE specifications in Release 9 [62]. The next step for LTE evolution is LTE Advanced which is currently being standardized in Release 10 [63], the major candidate technology for the so-called International Mobile Telecommunications (IMT)-Advanced.

Rel-8 LTE adopts OFDMA in the downlink for its robustness against multipath interference and to allow a high spectral efficiency exploiting time and frequency dependent scheduling and Multiple Input Multiple Output (MIMO) techniques. In the uplink, in order to maintain user orthogonality in the frequency domain, a Single Carrier Frequency Division Multiple Access (SC-FDMA) is adopted. Rel-8 LTE supports both Frequency Division Duplexing (FDD) and Time Division Duplexing (TDD) and uses multiple transmission bandwidths (i.e., 1.4, 3, 5, 10, 15 and 20 MHz) and multiple modulation schemes (i.e., QPSK, 16QAM and 64QAM) allowing peak rates of 300 Mb/s in downlink and 75 Mb/s in uplink.

We consider now the scheduling degree of freedom for the downlink of Rel-8 LTE. The basic unit of resource is the resource block, which is made of 12 adjacent subcarriers (15 kHz of subcarrier spacing) and has a duration of 0.5 ms (one slot), which correspond to 6 or 7 OFDM symbols depending on the cyclic prefix length chosen (4.7 μs or 16.7 μs). The scheduling block is the smallest resource unit that the scheduler can assign. It is made of two consecutive resource blocks, and therefore has a duration of 1 ms (one subframe). During the duration of a scheduling period, which is equal to the duration of a scheduling block (i.e., 1 ms), the modulation and coding scheme must be fixed for each user in the non MIMO configuration. For the MIMO configuration, a maximum of two different modulation and coding schemes can be used for data belonging to two different transport blocks [53].

LTE Advanced is a further evolution of LTE Release 8 and 9 which is supposed to meet the requirements for IMT-Advanced and enhance them to future operator and user needs. It shall support a wider transmission bandwidth using both contiguous and non-contiguous carrier aggregation, achieving flexible spectrum usage while maintaining backward compatibility with Rel-8. Moreover, it shall enhance multi-antenna and Coordinated Multi-Point transmission/reception techniques. These improvements are expected to allow peak rates of 1 Gb/s in downlink and 500 Mb/s in uplink.

Different radio resource management strategies are required for organizing and bringing together multiple users and letting them receive data in an LTE system (note: we are considering the downlink, which is the only direction using OFDMA multiplexing). In particular, multiple flows directed to the users are to be coordinated, so that a number of packets are selected for possible transmission from each flow. In the following, this operation will be referred to as *scheduling*. However, according to the above discussion, actual transmission requires to match the selected packets to a given resource block in a channel-aware fashion. Thus, it is necessary to eventually select which resources to utilize for the selected packets. Such an operation will be referred to as *resource allocation*.

The design of policies for resource management is intentionally left open in the standards to allow developers to implement their own strategy of choice. However, in the following we adopt a two-fold model where scheduling and resource allocation are managed by two different modules: a scheduler, operating at the transport layer (thereby possibly distinguishing among different kinds of traffic) and a resource allocator, which actually implements the Medium Access Control (MAC) sublayer. The scheduler determines which packets must be passed to the allocator and their order according to an internal scheduling policy. The allocator selects for transmission a subset of them with the aim of maximizing the advantages of multiuser diversity. In this case only a loose cross-layer is introduced, guaranteeing a certain modularity between scheduler and Radio Resource Allocator (RRA).

In particular, we call L the number of resource blocks that the resource allocator is entitled to assign. This is subject to a constraint $L \leq L_{\max}$, where $L_{\max}$ is a maximum value which corresponds to assigning every resource block. For simplicity, we consider that, to limit the interference caused to the neighboring cells, L is set to a fixed value which is less than or equal to $L_{\max}$. The value assigned to L is communicated to the scheduler by the resource allocator. Actually, this represents a form of cross-layer interaction among the modules, which is intentionally kept to a minimum level, thereby promoting modularity and tunability of the approach.

Upon knowing L, the scheduler determines a number D of packets to send to the resource allocator, where in general $D \geq L$. The exact choice of D influences the entire allocation. As a matter of fact, if $D = L$, the resource allocator has no degree of freedom as to which packets to allocate (while, obviously, it must allocate the packets to the best channels as perceived by the users). By increasing D, the resource allocator can achieve a higher throughput by selecting only L packets out of D, according to a channel-aware policy, although at the price of a possibly decreased fairness.

3.3 Proposed Game Theoretic Approach

The choice of D determines a trade-off between the possible objectives of throughput and fairness. We now present a game-theoretic approach to set D; we remark that the main point of our discussion does not lie in optimizing the performance of the resulting algorithm, which is left for future research. Rather, our proposed methodology enables a dynamic setup of D without any need for a preliminary evaluation, e.g., where D is set to some arbitrary value, of the possible equilibria of the system, nor it is required to re-compute the system equilibria if the network and channel conditions change. Instead, the choice of D is directly derived from the definitions of the contrasting utilities between which a trade-off is sought (specifically, throughput and fairness). Together with the separation of the resource management process into two functional entities (scheduler and RRA), this is key to achieve a computationally efficient online allocation strategy.

In our formulation, the scheduler (player 1) and the RRA (player 2) are represented as players of a game whose aim is the decision of the value for D. Both players make a proposal s_j, with $j = 1, 2$, respectively. The idea is that, if proposals s_1 and s_2 coincide, D is selected as their common value. However, the choice of s_1 and s_2 is also done according to the utility of the proposer, i.e., the fairness for the scheduler and the throughput for the RRA, respectively.

In the following, we introduce some assumptions for the sake of simplicity in the exposition. We consider a network scenario with only two users (i.e., two flows); this is not to be confused with the two "virtual" players of the game, i.e., the scheduler and the resource allocator. Besides, this assumption is just made for ease of implementation in the simulator, but can be relaxed quite naturally to scenarios with $n > 2$ users. We model the system as a static game with complete information, as follows:

- the players are the scheduler and the RRA.
- their action spaces are the set of values of D that can be proposed, i.e. $S_1 = S_2 = \{L, L + 1, ..., 2L\}$.
- both utilities are 0 if the proposals s_1 and s_2 do not coincide, i.e., there is no agreement on the value of D.
- when $s_1 = s_2$, the utilities are assigned to fairness $F(s_1, s_2)$ for the scheduler, measured using Jain's index [64] (see Eq. 4.13 for a formal definition of the Jain's index), and the throughput $T(s_1, s_2)$ for the RRA.

The last point is arbitrary, as other definitions can be used; the important requirement is that $F(s, s)$ and $T(s, s)$ are decreasing and increasing in s, respectively. The game is represented in Fig. 3.1 through a matrix whose cells contain pairs of real numbers (therefore called a bi-matrix), representing the utilities obtained by the scheduler and the RRA respectively, for a given action profile

Resource Allocator

Scheduler		L	$L{+}1$	....	$2L$
	L	$1, T_{\min}$	$0, 0$	$0, 0$	$0, 0$
	$L+1$	$0, 0$	...	$0, 0$	$0, 0$
	...	$0, 0$	$0, 0$	...	$0, 0$
	$2L$	$0, 0$	$0, 0$	$0, 0$	$\frac{1}{2}, T_{\max}$

Figure 3.1. *Bi-matrix representation of the game*

– different actions of the scheduler are represented by different rows and different actions of the RRA are represented by different columns. The fairness is a decreasing function of D: its maximum value is 1 while the minimum is $1/2$ (i.e., $1/n$ where n is the number of flows). On the other hand, the throughput is an increasing function of D varying in the range $[T_{\min}, T_{\max}]$, where $T_{\min}$ is achieved when no degree of freedom is given to the allocator, while $T_{\max}$ is obtained when the RRA has enough freedom to allocate only the best L resources. Both maximum throughput and minimum fairness are reached for $D = 2L$, under the assumption that there are always at least L packets available for selection by the scheduler from each queue. All the strategies along the diagonal are Pareto efficient Nash equilibria.

To determine a trade-off point, we propose an algorithm which tries to automatically estimate an efficient value of D for each frame. The value is chosen considering the past proposals, thus we change the model into a a *repeated game with perfect information*. The aim is to reach an acceptable level for both utilities after a number of repetitions. Note that this proposed algorithm is just an example and can be replaced by other analogous procedures.

1) Both scheduler and RRA randomly pick a value for D.

2) If the choices coincide, D is set and the game ends, otherwise a bargaining phase goes on until a common point is chosen. Every time the players disagree, both get zero utility.

3) The goal of each round of the loop is moving towards the diagonal of the bi-matrix step-by-step. Each player decides whether or not to change its previous proposal based on its level of satisfaction (i.e., the ratio between the value actually achieved and the maximum achievable). The higher the satisfaction, the higher the probability that a player changes its choice with a value more convenient for the other. If S_D and RRA_D are the proposals for D made by the scheduler and the allocator, respectively, and S_s and RRA_s the respective levels of satisfaction when the game is played, we

select the changes as follows.

— If $S\_D > RRA\_D$, we are in the lower triangle of the matrix. We can move towards the diagonal by going up (decrement of S_D), or right (increment of RRA_D), or in both directions. For both players, these options lead to higher values in their own utility function to the detriment of the other's, thus the willingness to change should be a decreasing function of the respective satisfaction level. Thus, we select

$$Prob\{S\_D\ up\} = 1 - S\_s \tag{3.1}$$

$$Prob\{RRA\_D\ right\} = 1 - RRA\_s \tag{3.2}$$

— If $S\_D < RRA\_D$, we are in the upper triangle of the matrix. The diagonal can be reached by going down (S_D increment), or left (RRA_D decrement), or in both directions. The situation is now reversed, as a deviation in its own action implies a reduction in the utility of each player in favor of the other's. Therefore, the probability of moving must be an increasing function of the respective satisfaction, which is obtained for example by choosing

$$Prob\{S\_D\ down\} = S\_s \tag{3.3}$$

$$Prob\{RRA\_D\ left\} = RRA\_s \tag{3.4}$$

In this manner, we define an algorithm whose goal is to lead the choice of D towards an intermediate value which offers both good throughput and satisfactory fairness.

3.4 Numerical Results

We ran evaluations within a simple LTE simulator to verify the ability of the proposed approach to converge towards a trade-off among the utility functions of the two players. All the performance indices are characterized by a confidence interval of 95% with a maximum relative error of 5%.

We developed and used a simple asynchronous event-driven simulator, written in C++, which reproduces a base station transmitting to two different mobile users. The base station contains a packet scheduler with two queues (one for each user) and an RRA module. The scheduler is credit-based and tries to guarantee fairness by selecting packets from the queues according to their residual credit. Flows are assumed to have always backlogged traffic. The RRA manages the resource allocation according to a greedy criterion: slots and packets are matched in order to maximize the total throughput given the channel condition of each user, which are assumed to be independent of each other.

Parameter	**value**
number of flows	2
packet size	500 bytes
$Pr\{GOOD \rightarrow GOOD\}$	0.9
$Pr\{BAD \rightarrow BAD\}$	0.8
number of subcarriers	16
time slots per frame	24
frame duration	1 ms
transmission power per slot	1 mW

Table 3.1. *Main system parameters*

The radio channel model represents each frequency subchannel by means of a two-state Markov channel (Gilbert-Elliot model) whose state is updated after each time slot to take into account channel correlation over time. The number of subcarrier groups is 16 while the time slots for each frame are 24, for a total of 384 resource blocks. A different average noise power is associated with each of the two states of the chain, thus different values of capacity can be reached (according to the Shannon formula). For simplicity, when the Gilbert-Elliot channel is in the good state, interference and noise power are treated as a random variable with uniform distribution between 1 and 2 mW; similarly, in case of bad channel, the interference plus noise power is uniformly distributed between 1 and 200 mW. The transmission power per slot is fixed to 1 mW. The main system parameters are summarized in Table 3.1.

In Fig. 3.2 and Fig. 3.3 the fairness and the normalized throughput as a function of time are shown for several values of D when $L = 300$ packets. They confirm what was expected from our analysis: the fairness is a decreasing function of D while the throughput increases. When $D = L$, we have that the fairness is always 1, the maximum value according to Jain's index. On the other hand, the normalized throughput has its minimum value because the resource allocator has no freedom in the choice of the packets to transmit and the user diversity cannot be exploited.

When D is increased, the two performance indices considered have contrasting behaviors, as already expressed in the previous section: the fairness undergoes a decrease while the throughput starts increasing. The introduction of a certain freedom in the allocation choice shows its effects and

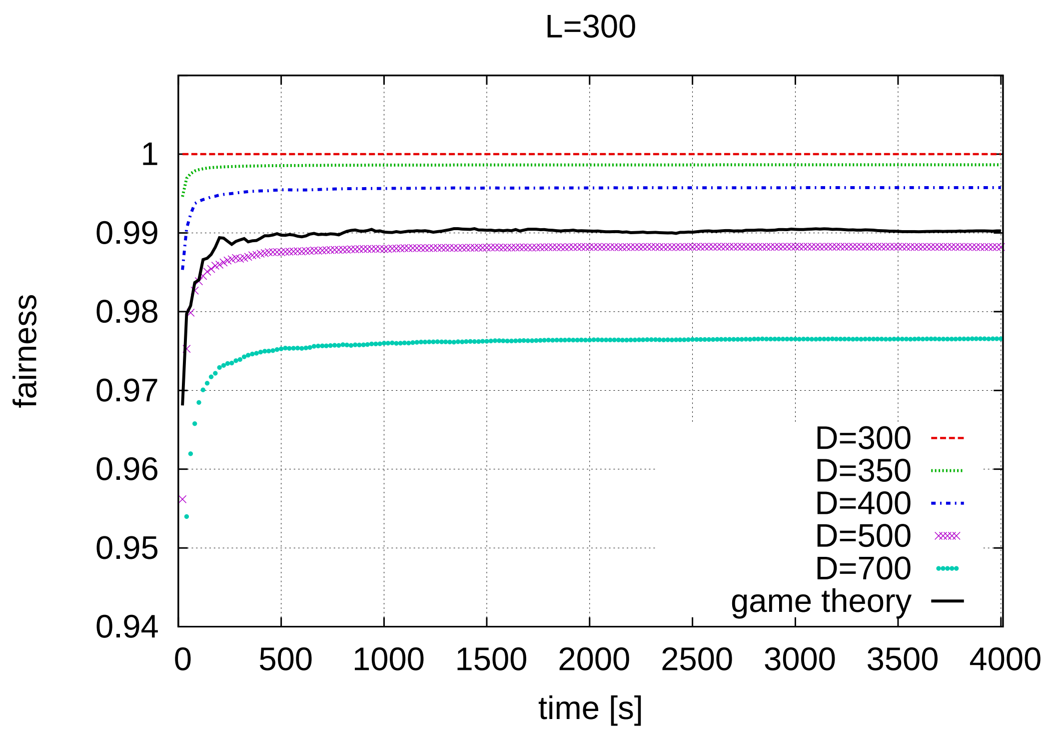


Figure 3.2. *Fairness over time for different values of D.*

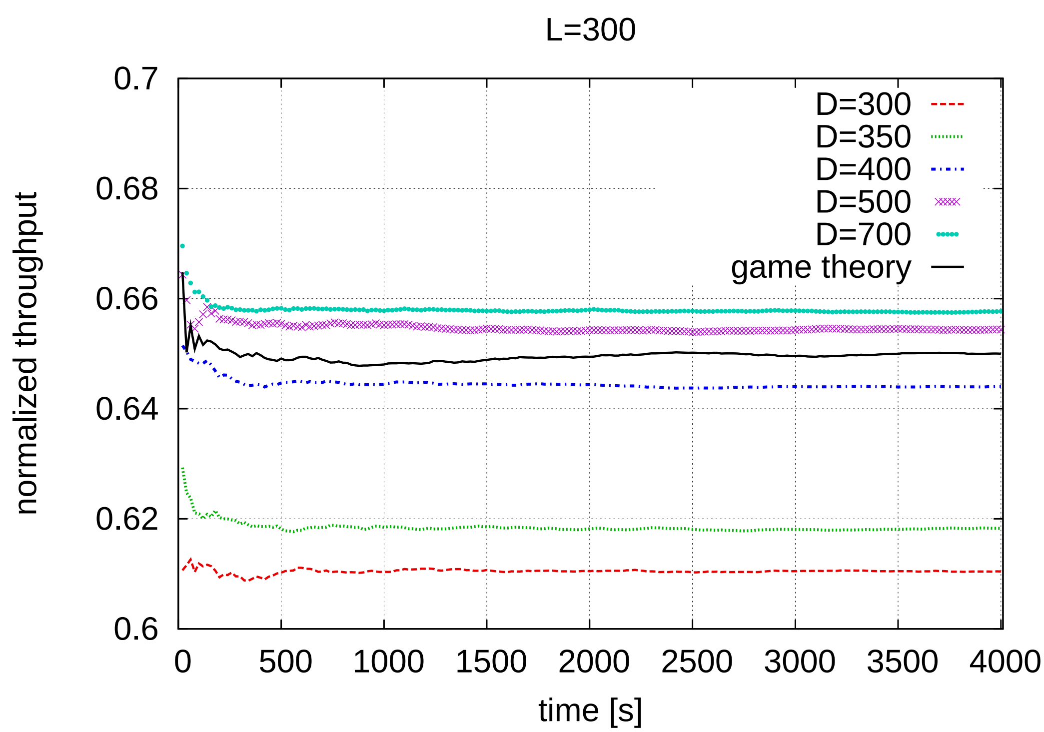


Figure 3.3. *Throughput over time for different values of D.*

the trade-off among the utility of the two players becomes evident. Figure 3.4 clearly shows this situation: the points along the curve are the Pareto solutions of the game, one for each value of D, and there is no possibility to reach a better solution for one player without worsening the other's one.

All figures report the outcome of the game theoretic algorithm. Both in Fig. 3.2 and in Fig. 3.3, the automatic choice of D leads to an intermediate value of both performance indices. This means that each player slightly reduces its own utility for the sake of a better joint solution. In Fig. 3.4 it is shown that this new operating point is localized close to the Pareto boundary. Moreover, the proposed algorithm is quite simple and the convergence to a common value of D is extremely fast, thus it is suitable for an online implementation. Indeed, in Figs. 3.2–3.3 the warm-up period is quite short, about 300 ms.

For completeness, we ran other tests by varying L in the range $[100, 350]$. In all these cases we obtained that the fairness increased with the value of D while the throughput decreased. The operation point reached by the proposed algorithm always approximately lies on the Pareto boundary.

3.5 Conclusions

In this chapter we have presented a novel design approach for resource management in OFDMA/TDMA cellular networks such as LTE. A cross-layer approach has been explored, where scheduler and radio resource allocator exchange a limited amount of information to provide both an adequate level of fairness among flows and a high throughput. A game theoretic model of the system has been proposed and a feasible algorithm for the dynamic setting of a system parameter has been evaluated. The results obtained through simulation show that the proposed solution is able to trade-off fairness requirements and throughput.

Possible future works include the extension to a multicellular network, where several base stations coexist and share resources trying to minimize mutual interference through a proper resource allocation. Moreover, we plan to implement the proposed approach in a more detailed network simulator.

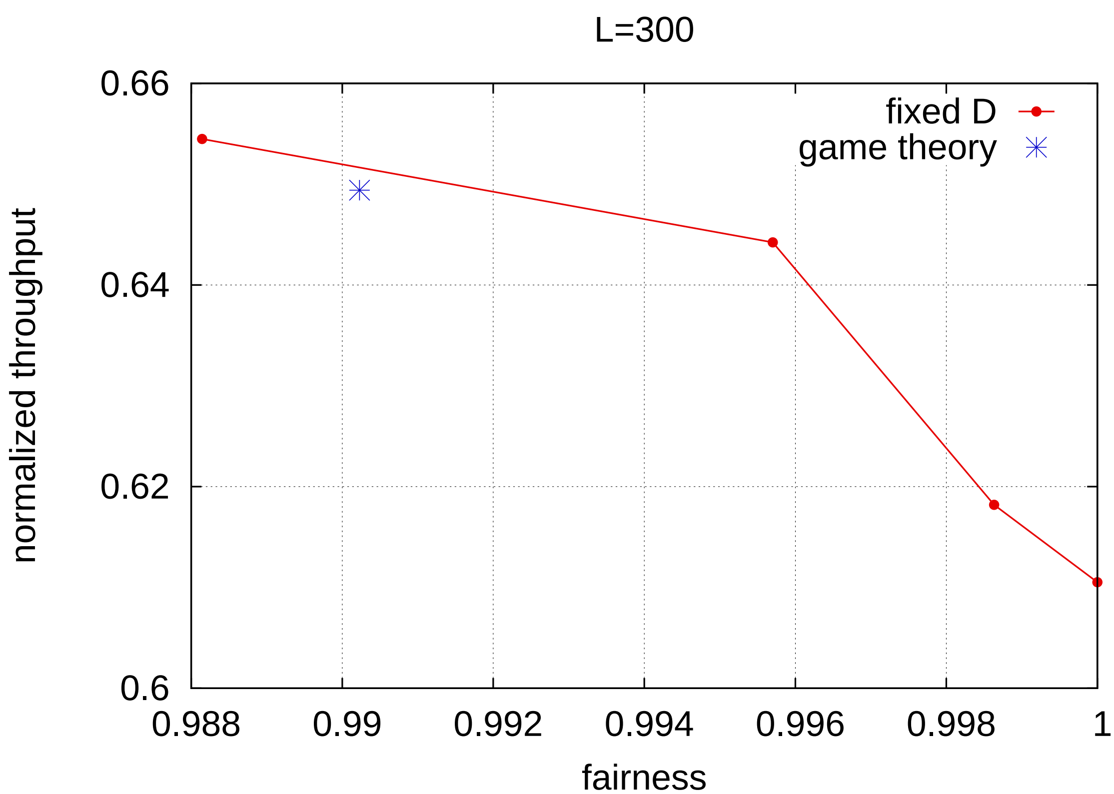


Figure 3.4. *Pareto boundary and operating point of the algorithm.*

Chapter 4

Promoting Cooperation in Wireless Relay Networks

In this chapter[1] we apply game theory to constructively derive practical network management policies for wireless relay networks. We focus on the problem of medium sharing and opportunistic packet forwarding in wireless relay networks, and we show how, by properly modeling the agents involved in such a scenario, and enabling simple but effective incentives towards cooperation for the users, we obtain a resource allocation scheme which is meaningful from both perspectives of game theory and network engineering. Such a result is achieved by introducing throughput redistribution as a way to transfer utilities, which enables cooperation among the users.

4.1 Introduction

Cooperation has emerged as a new networking concept that has a dramatic effect of improving the performance from the physical layer up to the networking layers, and it is considered as one of the most promising enabling technologies to meet the increasingly high rate demands and quality of service requirements in wireless networks. In this chapter we consider the simplest form of physi-

[1]The material presented in this chapter has been published in:

[C4] **L. Canzian**, L. Badia, and M. Zorzi, "Relaying in Wireless Networks Modeled through Cooperative Game Theory," in *Proc. IEEE CAMAD*, Kyoto, Japan, Jun. 10-11, 2011

[J1] **L. Canzian**, L. Badia, and M. Zorzi, "Promoting Cooperation in Wireless Relay Networks through Stackelberg Dynamic Scheduling," *to appear in IEEE Trans. Commun.*

cal layer cooperation: an opportunistic relay channel, in which each user connected to the network forwards the packets of the other users.

We investigate cooperative relaying not only improving the social welfare of the network, but also increasing the individual benefit of each single user, that is assumed to act selfishly and strategically. The motivation behind this approach is that relaying is possible only if incentives are given to each user to overcome the disadvantage of consuming energy to forward the packets of the other users. We first prove the potential gain of cooperation through a cross-layer scheme involving joint routing and medium access, which is analyzed by means of renewal process theory [65]. However, such a globally efficient allocation may not match the allocation equilibrium in a game theoretic sense. To overcome this difficulty, we first consider a simple 2–users case and model users' interaction as a *coalitional* game, introducing throughput redistribution as a way to transfer utilities. This will enable cooperation among the users. Unfortunately, it is very difficult to generalize such an approach to larger networks, both because it is computationally expensive to characterize the core for a number of users higher than 3 [52], and because it requires the definition of a proper negotiation protocol to establish the cooperation roles, an overhead which may considerably limit the cooperation gain in large networks. Thus, as a main contribution of this chapter, we propose another incentive scheme which follows the approach of the coalitional game, redistributing the throughput among users through a dynamic scheduling rule. This scheme involves a coordinator, that triggers cooperative behaviors increasing the access opportunities of users acting as relays. This kind of approach is framed as a Stackelberg game involving the coordinator as the leader and the users, whose strategic decision involves whether to act collaboratively, as followers. It can also be considered as an intervention scheme [66] (which will be described accurately in Chapters 6 and 7): the coordinator represents the intervention device and the dynamic scheduling rule represents the intervention rule. However, differently from most of the intervention schemes in the literature in which the intervention action represents a punishment for non compliant users, here the scheduling action represents an award for cooperative users.

The rest of this chapter is organized as follows. We describe the scenario under investigation and the key assumptions in Section 4.2. Then, Section 4.3 formalizes the analysis of cooperative versus non cooperative schemes by means of renewal process theory. Section 4.4 introduces the throughput redistribution concept and studies the coalitional game in the 2–users scenario. Section 4.5 represents the main contribution of this chapter: the dynamic scheduling scheme to provide network incentives towards cooperation is defined. Numerical results are provided in Section 4.6. We discuss possible relaxations of some hypotheses in Section 4.7, and Section 4.8 concludes the chapter with some remarks.

4.1.1 Related work

Relay networks have been widely studied in information theory [67]. In particular, the relay channel represents one of the most common scenarios studied. Several theoretical results about the capacity of this basic network have been available in the literature since long [68], and others keep being proposed even very recently [69–72]. These studies assume that each user is willing expend energy in forwarding packets for other users, without having nothing in return. However, relaying is possible in practice only if incentives are given to the individual users to overcome the disadvantages of their limited energy budgets. In this spirit, [17] promotes a fair packet forwarding mechanism balancing the relaying opportunities that each node gives to and receives from other nodes. Similarly, [18] introduces a virtual currency and mechanism for charging/rewarding service usage/provision. Both papers assume the application of a tamper-resistant module in each node to store the forwarding balance or the virtual currency credit. The virtual currency concept is also used in [19], while in [20] cooperation is reached by using a reputation mechanism. A distributed and scalable acceptance algorithm was proposed in [21], in order for the nodes of an ad hoc network to decide whether to accept or reject a relaying request. Finally, [22] considers an incentive mechanism where the nodes flexibly give transmission bandwidth in exchange for forwarding data.

Differently from [17–20], that are based on the exchange on a network scale of abstract notions of worth (e.g., currency and reputation), our opportunistic relaying scheme represents a more tangible and immediate incentive mechanism. The repeated game formulation considered in [21] is efficient only if a user asking for a relay service can return the favor in future interactions. Our scheme can be applied in more general situations, even in strongly asymmetric scenarios where some users only ask for relay services and other users are only asked to act as relays. In fact, users acting as relays are immediately rewarded, independently of the future interactions with the other users. Our approach is closer in spirit to [22]. The main difference is that, instead of rewarding cooperative users in the frequency domain, giving them more bandwidth, we reward cooperative users in the time domain, increasing their access opportunities. Moreover, there are some different hypotheses that make the analysis of the two schemes very different, e.g., in this chapter we assume that the users can select their modulation scheme which in turn determines the packet reception probability, while [22] adopts a more abstract formulation based on channel capacity.

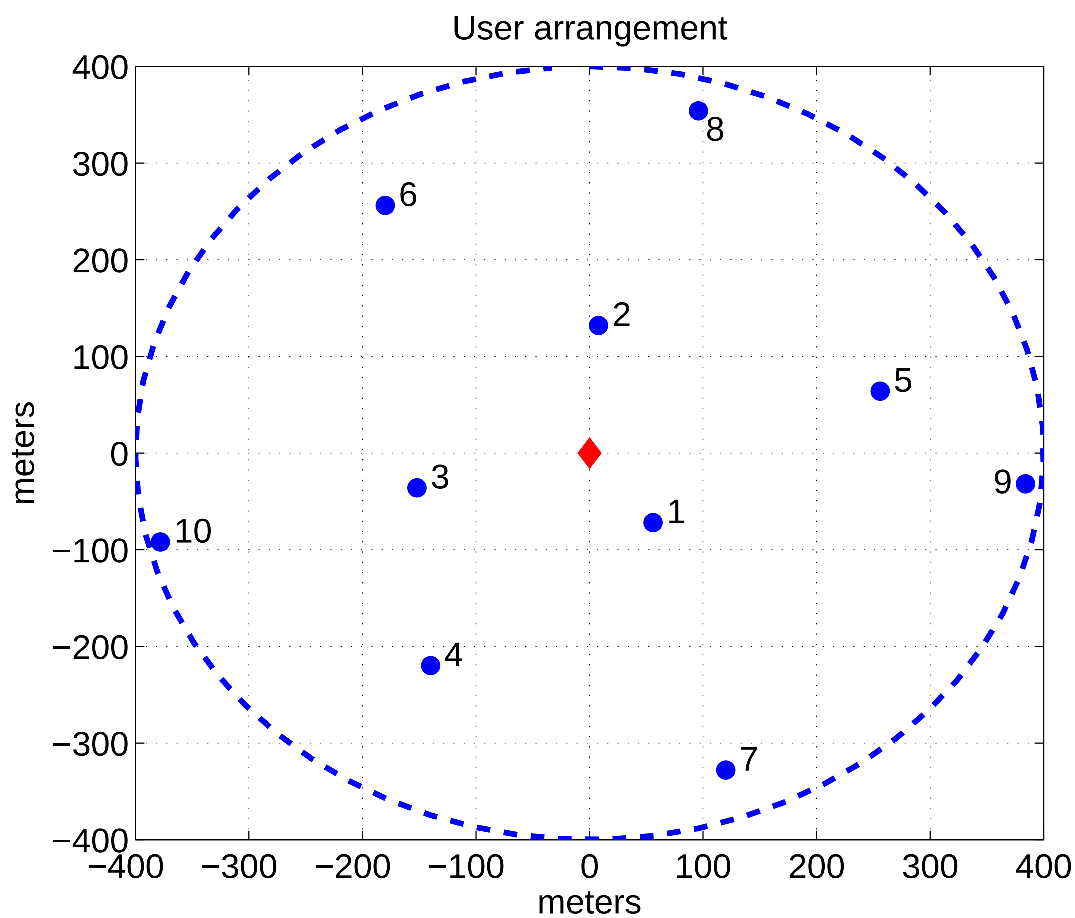


Figure 4.1. *The considered scenario: an access point surrounded by user nodes*

4.2 Problem Statement

Consider a scenario as reported in Fig. 4.1, where a set $N = \{1, 2, \ldots, n\}$ of n nodes, hereafter called *users*, are distributed around a further node called *node 0*. This may represent an access point of a wireless local area network, or a base station of a cellular network. We focus on the uplink between each user and node 0; yet, we assume that node 0 is not only the end destination, but also a resource manager, as explained later.

We denote the signal to noise ratio (SNR) between user i and node 0 as γ_i and the SNR between users i and j as γ_{ij}. Users are labeled in decreasing order of SNR to node 0, i.e., $\gamma_1 \geq \gamma_2 \geq \ldots \geq \gamma_n$. We consider time invariant channels and fixed transmission powers P_{pkt}, so that the γ_i and γ_{ij} terms are constant over time. We also assume perfect channel state knowledge.

A Time Division Multiple Access (TDMA) scheme is adopted, with a fixed slot duration T_{pkt}. Node 0 controls the time shares of the users by selecting, in each slot t, a specific user that is allowed to transmit. The probability that user i is selected in slot t is $P_i^{(t)}$. The selected user transmits a single packet over the entire slot, comprising a number of bits that depends on its modulation scheme M_i. M_i is chosen over a finite set $\mathcal{M}$ according to the channel quality and in turn determines the probabilities q_i and q_{ij} that the packet is correctly received by 0 and j. We denote with $E_{pkt} = P_{pkt}T_{pkt}$ the energy consumed by a user for a single packet transmission.

Automatic Repeat reQuest (ARQ) is used as the mechanism to achieve reliable communication [73]. If the packet transmitted by user i is not correctly received by node 0, the packet is retransmitted the next time user i is scheduled, until the packet is received or the maximum number of retransmissions is reached. For the sake of simplicity, we consider at most one retransmission per packet, although the extension to multiple retransmissions would be conceptually straightforward. Users are assumed to be backlogged, i.e., they always have packets to transmit. In the following, we will start by considering that retransmissions of a packet are only performed by the node that has originated that packet, i.e., the node that performed the first transmission attempt. We will refer to this situation as the *no cooperation* case and denote its corresponding quantities with a superscript $\mathcal{N}$. $P_i^{(t)}$ can be set as a constant/static value for all t, which makes the selection process independent and identically distributed (iid). The scheduling policy can be described by a vector $P = (P_1, P_2, \ldots, P_n)$, where $\sum_{i=1}^{n} P_i = 1$, so that $P_i^{(t)} = P_i$ for all t; for example, a fair sharing is represented by $P = (1/n, 1/n, \ldots, 1/n)$.

We will also consider two evolutions of this scheme, where retransmissions of faulty packets may not be carried out by the same node performing the first attempt. This is enabled by assuming that during the transmission phase of a generic node i the other nodes listen to the channel and store i's packet if they have correctly received it. Thus, they can retransmit it if needed. If more than one user can retransmit the packet, node 0 selects the one with the best channel.

In the first scheme, called *forced cooperation* (denoted by superscript $\mathcal{F}$), we assume that the users have no say in deciding whether or not to cooperate, but must follow node 0's directions when instructed to do so, hence the name. Since cooperation does not come from a free decision, there is no need for rewarding the collaborative users with a higher access probability. Thus, similarly to the no cooperation case, the access probabilities $P_i^{(t)}$ stay the same for every t. However, their physical meaning changes: they represent the event that the packet originated from i is transmitted during slot t; if it is the first transmission attempt, it will be performed by i, while this is not necessarily true for a retransmission.

Finally, we will consider a further cooperative case, called *voluntary cooperation* (denoted by superscript $\mathcal{V}$), where the users freely decide whether or not they want to cooperate in the retrans-

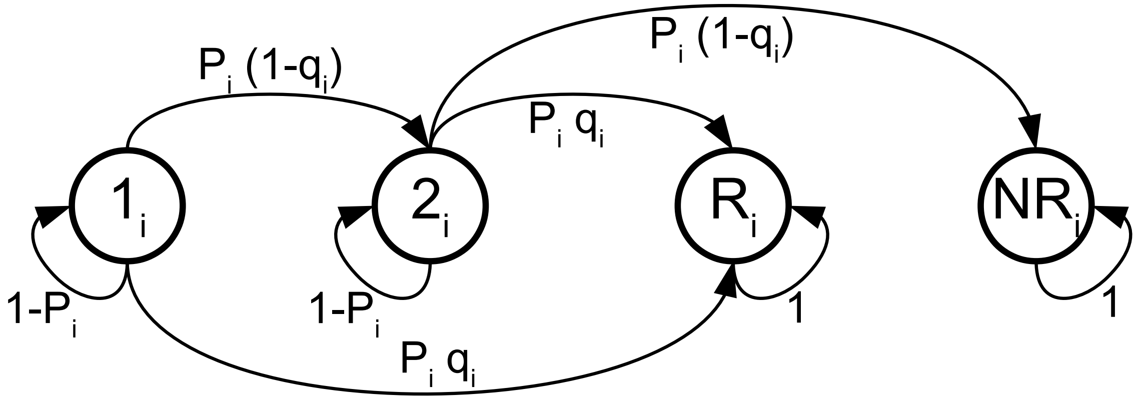


Figure 4.2. *Non-cooperative transmission process of a packet of user i*

mission process of other users. In this case, node 0 rewards them with a higher access probability, decreasing by the same amount the access probability of the users being helped. Thus, $P_i^{(t)}$ changes over time. Suppose node i cooperates with node j in slot t, retransmitting a packet originated from node j. We define K_{ij} as the number of scheduling instants, after slot t, where the scheduling policy is changed, and $\Delta P_{ij}^{(s)} > 0$ as the variation of the scheduling policy, with respect to the reference policy $P = (P_1, P_2, \dots, P_n)$, in slot s, i.e.,

$$P_j^{(s)} = P_j - \Delta P_{ij}^{(s)} \quad ; \quad P_i^{(s)} = P_i + \Delta P_{ij}^{(s)} \quad ; \quad s = t+1, \dots, t+K_{ij} \tag{4.1}$$

To compare the three cases, we define the bit rate of user i in slot t as

$$BR_i^{(t)} = \begin{cases} \dfrac{N_i}{T_{pkt}} & i\text{'s packet correctly received by } 0 \text{ in slot } t \\ 0 & \text{otherwise} \end{cases}$$

where N_i is the number of bits in user i's packet, which depends on the chosen modulation scheme M_i. Finally, we define the asymptotic bit rate of user i as

$$BR_i = \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} BR_i^{(t)}$$

4.3 Renewal Theory Analysis

In the no cooperation scheme, the transmission process of a generic packet originated from user i can be represented by the Markov Chain of Fig. 4.2. The successful reception probabilities q_i and q_{ij} depend on the modulation scheme M_i and the SNR values γ_i and γ_{ij}. In the following we will omit all these dependencies in favor of a clearer notation.

The initial state of the Markov Chain is 1_i, which means that the next time user i is scheduled it will transmit the packet for the first time. Analogously, state 2_i implies that by scheduling user i the packet will be transmitted for the second time. State 2_i is entered if the first attempt failed. The term P_i that influences the transition probabilities results from the scheduling process. The absorbing states R_i and NR_i represent the events that user i's packet is eventually received or not, respectively, by node 0. When either of the absorbing states is entered, the transmission process of another packet of node i is considered, restarting again from state 1_i.

The time intervals of the packet transmission processes are positive, independent, identically distributed random variables. These variables define a renewal process which can be studied exploiting renewal theory results [65]. The asymptotic metrics of the network can be obtained studying the (statistical) average behavior of the Markov process. In particular, the asymptotic throughput of each user is equal to the average number of received bits divided by the average time to be absorbed in the Markov chain associated to that user.

We denote with $P_{R_i}^{\mathcal{N}}$ the probability to be absorbed in state R_i and with $v_i^{\mathcal{N}}$ the average number of time slots to be absorbed starting from state 1_i. Therefore,

$$P_{R_i}^{\mathcal{N}} = q_i + (1-q_i)\, q_i = q_i\,(2-q_i)$$
$$v_i^{\mathcal{N}} = \frac{1}{P_i} + \frac{1}{P_i}\,(1-q_i) = \frac{2-q_i}{P_i}$$

Thus, i's asymptotic bit rate for the no cooperation case is

$$BR_i^{\mathcal{N}} = \frac{P_{R_i}^{\mathcal{N}} N_i}{v_i^{\mathcal{N}} T_{pkt}} = P_i q_i \frac{N_i}{T_{pkt}} \tag{4.2}$$

The best modulation scheme for user i is simply obtained maximizing its throughput

$$M_i^{\mathcal{N}} = \underset{M_i \in \mathcal{M}}{\arg\max}\, q_i N_i \tag{4.3}$$

Recall that both N_i and q_i depend on M_i. Finally, the asymptotic bit rate of the network for the no cooperation scenario is

$$BR^{\mathcal{N}} = \sum_{i=1}^{n} BR_i^{\mathcal{N}} = \frac{1}{T_{pkt}} \sum_{i=1}^{n} P_i q_i N_i$$

where the modulation scheme for each user is selected according to (4.3).

In the forced cooperation scheme, the packet transmission process of user i follows the Markov Chain in Fig. 4.3. Differently from the no cooperation case, the retransmission of i's packet is performed by the best user k among those that have received the packet during i's first attempt, $k < i$,

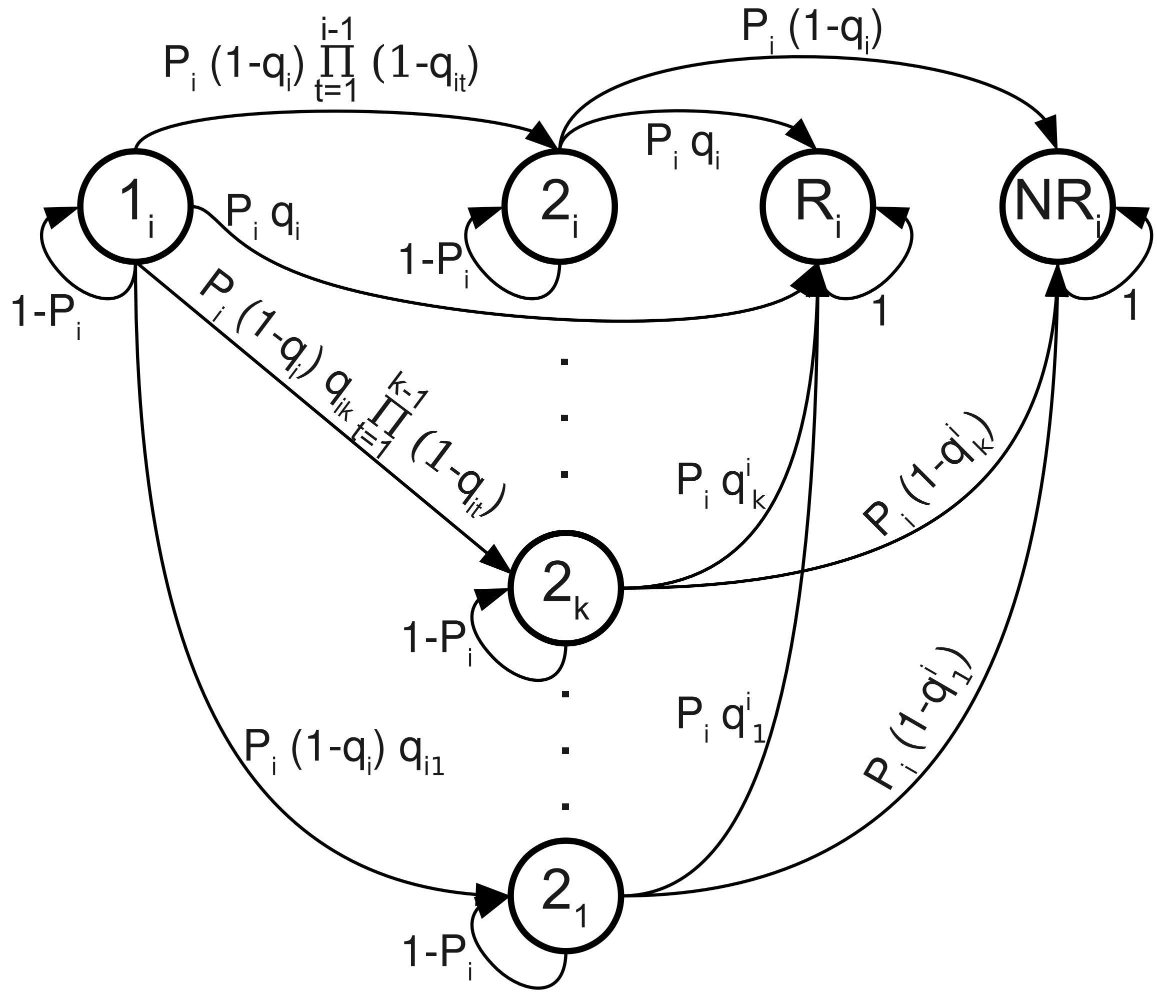


Figure 4.3. *Transmission process of a packet of user i in the forced cooperation scheme*

otherwise the retransmission is performed by i itself. In the retransmission, k will use the same modulation order used by i, M_i. In fact, although the optimal modulation M_k for k may be higher, i's packet dimension cannot be increased.[2] We define q_k^i as the correct reception probability of a packet transmitted by k using the same modulation scheme of i. Since $k < i$, we have $q_k^i \geq q_i$.

The probability $P_{R_i}^F$ to be absorbed in R_i and the mean number of steps $v_i^{\mathcal{F}}$ to absorption are

$$P_{R_i}^{\mathcal{F}} = q_i + (1-q_i) \sum_{k=1}^{i} q_{ik} \prod_{j=1}^{k-1} (1-q_{ij})\, q_k^i = q_i\,(2-q_i) + \sum_{k=1}^{i-1} (1-q_i)\, q_{ik} \prod_{j=1}^{k-1} (1-q_{ij})\,(q_k^i - q_i)$$

$$v_i^{\mathcal{F}} = \frac{2-q_i}{P_i}$$

where we took $\prod_{j=1}^{0} (1-q_{ij}) = 1$ and $q_{ii} = 1$. In particular, $(1-q_i)\, q_{ik} \prod_{j=1}^{k-1} (1-q_{ij})\,(q_k^i - q_i)$

[2]Actually, node k can even improve its amount of transmitted data by stuffing i's packet with its own data up to $N_k - N_i$ bits. We neglect this further advantage which, however, would be immediate to include.

is the probability that 0 has not correctly received i's packet in the first attempt while k has received it but no user better than k has received it, multiplied by the difference between the probabilities that the packet is correctly retransmitted by k and i. It represents the contribution of k to the probability that i's packet is eventually received by 0.

Considering for the moment that user i is adopting the same modulation scheme $M_i^{\mathcal{N}}$ as in the no cooperation case, we have obtained $P_{R_i}^{\mathcal{F}} \geq P_{R_i}^{\mathcal{N}}$ and $v_i^{\mathcal{F}} = v_i^{\mathcal{N}}$. The latter is a consequence of considering a single retransmission (for the multiple retransmission case, $v_i^{\mathcal{F}} \leq v_i^{\mathcal{N}}$ in general).

Similar to (4.2), the asymptotic bit rate of user i in the cooperative scenario is

$$BR_i^{\mathcal{F}} = P_i \left[q_i + \frac{1-q_i}{2-q_i} \sum_{k=1}^{i-1} q_{ik} \prod_{j=1}^{k-1} (1-q_{ij}) \, (q_k^i - q_i) \right] \frac{N_i}{T_{pkt}}$$

and the best modulation scheme $M_i^{\mathcal{F}}$ for the cooperative case is

$$M_i^{\mathcal{F}} = \arg\max_{M_i \in \mathcal{M}} \left[q_i + \frac{1-q_i}{2-q_i} \sum_{k=1}^{i-1} q_{ik} \prod_{j=1}^{k-1} (1-q_{ij}) \, (q_k^i - q_i) \right] N_i \tag{4.4}$$

Finally, for the aggregate throughput we obtain

$$BR^{\mathcal{F}} = \frac{1}{T_{pkt}} \sum_{i=1}^{n} P_i \left[q_i + \frac{1-q_i}{2-q_i} \sum_{k=1}^{i-1} q_{ik} \prod_{j=1}^{k-1} (1-q_{ij}) \, (q_k^i - q_i) \right] N_i \tag{4.5}$$

where the modulation scheme for each user is selected according to (4.4). Comparing this result with the no cooperation case, if in both cases users are adopting the modulation schemes according to (4.3), we obtain $BR^{\mathcal{F}} \geq BR^{\mathcal{N}}$. This relation is further enforced if we calculate $BR^{\mathcal{F}}$ considering the best modulation schemes for the forced cooperation case, according to (4.4).

To study the *voluntary cooperation* scheme, we need to introduce a game theoretic framework modeling interactions among selfish users and their decision to cooperate / not to cooperate. In Section 4.4 we will study this interaction as a transferable utility coalitional game, in which the users can redistribute among them the total gain obtained through cooperation. We carry on this analysis considering a simple 2–user case in which users are interested in maximizing their throughput, and the redistribution of the throughput is physically possible by changing the access opportunities of the users – of course each user is free to decide if such an agreement is convenient for him or if it is better to leave the coalition and refuse to cooperate with the other user. Then, in Section 4.5, following the idea of the throughput redistribution, we give an active role to node 0, assuming that it can modify the access opportunities of each user following a dynamic scheduling rule which is a function of each user's decision to cooperate or not to cooperate with the other users. This kind of approach is framed as a Stackelberg game, in which node 0 is the leader and the users are the followers. In this case we consider a more realistic scenario in which n users are interested in maximizing their throughput and minimizing their energy consumption.

4.4 Coalitional Game and Throughput Redistribution

In this section we study the interaction of 2 users through a coalitional game, assuming that they can form a coalition in which they agree to cooperate with each other, and to redistribute the total throughput obtained in order to get a higher throughput compare to the no cooperation scenario. We assume that the coalitional game satisfies properties 1 and 2 of Section 2.7. Note that in the 2–user case the former property is automatically satisfied. However, the property still holds true even if the analysis is extended to a network with more than two users, since the TDMA approach guarantees that different coalitions do not interact: each coalition tries to obtain the maximum throughput by using the slots assigned exclusively to it. For what concerns property 2, the problem of the throughput redistribution is addressed at the end of this section.

The value $v(\cdot)$ of the coalitional game is the throughput obtained by each coalition. In a 2–user case, three coalitions are possible: the two coalitions formed by the single users, 1 and 2, and the coalition formed by both users, i.e., the grand coalition $N = \{1, 2\}$. The value of each coalition is:

$$v(\{1\}) = BR_1^{\mathcal{N}} \quad , \quad v(\{2\}) = BR_2^{\mathcal{N}} \quad , \quad v(N) = BR^{\mathcal{F}} \geq BR^{\mathcal{N}} = v(\{1\}) + v(\{2\})$$

Therefore the game satisfies also property 3 of Section 2.7.

Now we want to find a utility allocation that belongs to the core and is fair under certain parameters. Note that, for a super-additive two player game, the core is not empty and coincides with the set of imputations. In the considered game, the set of imputations is given by:

$$x_1 = BR_1^{\mathcal{N}} + w(BR_2^{\mathcal{F}} - BR_2^{\mathcal{N}}) \quad , \quad x_2 = BR_2^{\mathcal{N}} + (1-w)(BR_2^{\mathcal{F}} - BR_2^{\mathcal{N}}) \tag{4.6}$$

where the *cooperation weight* w belongs to the interval $[0, 1]$. It is immediate to see that $x_1 + x_2 = v(N)$, $x_1 \geq v(\{1\})$, and $x_2 \geq v(\{2\})$, $\forall w \in [0, 1]$.

The cooperation weight determines the throughput share that each user gets. If $w = 0$ (i.e., the throughput is not redistributed) we obtain $x_1 = v(\{1\})$, hence only user 2, whose channel quality to node 0 is worse, can directly benefit from being helped by user 1's cooperative relaying. If $w > 0$, also user 1 can benefit from the cooperation. The greater w, the greater the incentive for user 1 to cooperate. For $w = 1$, $x_2 = v(\{2\})$, hence only user 1 can benefit from the cooperation[3]. Thus, by setting the value of w we decide the right level of fairness of the subdivision.

So far we have supposed that the total throughput can be divided by users rather arbitrarily. From a practical point of view, the only thing that can be controlled is the allocation policy, P_1 and P_2. We suppose therefore that the allocation policy is changed from P_1 and P_2 to P_1' and P_2' in order to satisfy the subdivision proposed. Is the new allocation policy feasible? That is, is $P_1' + P_2' \leq 1$? It is easy to show that the new allocation policy is feasible. In fact, we have to increase the allocation probability of the cooperating user 1 and decreasing the allocation probability of 2, while keeping constant the total bit rate $v(N)$. Since 1 has a better channel, it results that the increase $P_1' - P_1$ is lower than the decrease $P_2 - P_2'$ in order to keep the total bit rate constant. Therefore:

$$P_1' - P_1 < P_2 - P_2' \Rightarrow P_1' + P_2' < P_1 + P_2 = 1$$

This means that the allocation is feasible and that there is a positive probability that some slots are not assigned to anybody, which would not be meaningful. Therefore, the quantity $P' = 1 - P_1' - P_2'$ can be divided among users, increasing for example both P_1' and P_2' by the same amount, or increasing them by a weighted amount of P', where we can use again the cooperation weight w. Finally, this means that both users have a further benefit in obtaining an even higher bit rate compared to the subdivision proposed.

4.5 Dynamic Scheduling Scheme

It is very difficult to generalize the approach of Section 4.4 to larger networks, both because it is computationally expensive to characterize the *core* for a number of users higher than 3 [52], and because it requires the definition of a proper negotiation protocol to establish the cooperation roles, an overhead which may considerably limit the cooperation gain in large networks. Thus, as a main contribution of this chapter, we propose in this section a dynamic scheduling scheme which follows the idea of redistributing the throughput among users, awarding cooperative users.

In the voluntary cooperation scheme we allow the user to freely choose whom to cooperate with, as well as its own modulation scheme. We model their interaction as a static game with complete info and, for the time being, we consider that the strategy[4] of user i consists only in choosing the set of users it cooperates with, which we denote as $a_i \subseteq N$ (i.e., the action set A_i is the power set of

[3] Actually, in this case user 2 can still benefit in that it saves energy, because some of its packets are retransmitted by user 1. We will introduce the energy consumption in the users' utilities in Section 4.5.

[4] In static games the user strategy coincides with the user action. In this chapter we keep using the word strategy.

N). Since each user can cooperate only with users having a worse channel, a_i is actually a subset of $\{i+1,\ldots,n\}$. The choice of the modulation scheme can be added in a later step as a superposition to the choice of a_i, and it does not represent a strong interaction factor among users. Also, denote with W_i the set of users that cooperate with i, i.e., $W_i = \{j \in N : i \in a_j\}$.

We represent the preference of each user i through a utility function $\Psi_i(B_i, E_i)$ which depends on the number of transmitted bits B_i and on the energy spent E_i per unit time. Actually, for the analysis of the game we use the *incremental utility* $\psi_i(\Delta B_i, \Delta E_i)$ representing the increase in Ψ_i with respect to the no cooperation case[5], i.e.,

$$\psi_i(\Delta B_i, \Delta E_i) = \Psi_i(B_i^{\mathcal{N}} + \Delta B_i, E_i^{\mathcal{N}} + \Delta E_i) - \Psi_i(B_i^{\mathcal{N}}, E_i^{\mathcal{N}})$$

By definition, $\psi_i(0,0) = 0$. Also, it is reasonable to assume that ψ_i is a continuous and increasing (respectively, decreasing) function of the variation of transmitted bits (respectively, energy consumption) per unit time, ΔB_i (respectively, ΔE_i).

Note that ΔB_i and ΔE_i can be split into the contributions due to the individual interactions with other users: $\Delta B_i = \sum_{j\in N\setminus\{i\}} \Delta B_{ij}$ and $\Delta E_i = \sum_{j\in N\setminus\{i\}} \Delta E_{ij}$, where ΔB_{ij} and ΔE_{ij} are the variations, per unit time, of transmitted bits and energy expenditure of i due to the interaction with j. Now, we assume that the incremental utility $\psi_i(\Delta B_i, \Delta E_i)$ can be additively split as a sum of local contributions $\psi_{ij}(\Delta B_{ij}, \Delta E_{ij})$, each due to the interaction between i and j, with ψ_{ij} having the same characteristics of ψ_i (continuity and monotonicity). Then we can write:

$$\psi_i(\Delta B_i, \Delta E_i) = \sum_{j\in N\setminus\{i\}} \psi_{ij}(\Delta B_{ij}, \Delta E_{ij}) = \sum_{j\in W_i} \psi_{ij}(\Delta B_{ij}, \Delta E_{ij}) + \sum_{j\in a_i} \psi_{ij}(\Delta B_{ij}, \Delta E_{ij}) \quad (4.7)$$

where we exploited the fact that if $j \notin W_i \cup a_i$, i.e., j has no interaction with i, then $\psi_{ij}(\Delta B_{ij}, \Delta E_{ij}) = 0$.[6] In (4.7), ψ_i is re-arranged in two sum terms. The former involves the users of set W_i offering their cooperation to i; therefore, in the corresponding terms, ΔB_{ij} and ΔE_{ij} are positive (as we will see in Section 4.5.1) and negative, respectively. This means that user i will always benefit from cooperation by another user j with a better channel; however, the strategic choice whether to cooperate or not is left to user j. The latter term includes instead the variation of ψ_i due to i offering cooperation to other nodes belonging to set a_i, which is where the decision of i comes into play.

The term ψ_{ij} can therefore be regarded as the specific utility of user i in a simple 2-player game between i and j, $i < j$, where the only user who can make a non-trivial decision is i. It will cooperate

[5]The game's outcomes are invariant to this choice. In fact, they depend only on the ranking of the preference of each user, which is preserved if a (user-dependent) constant is subtracted from the utility of each user.

[6]A linear $\psi_i(\cdot,\cdot)$ will satisfy (4.7). In particular, if $\psi_i(\cdot,\cdot)$ is linear then $\psi_i(\cdot,\cdot) = \psi_{ij}(\cdot,\cdot), \forall i,j$. Moreover, the converse is also true: if $\psi_i(\cdot,\cdot)$ satisfies (4.7) and $\psi_i(\cdot,\cdot) = \psi_{ij}(\cdot,\cdot), \forall i,j$, then $\psi_i(\cdot,\cdot)$ is a linear function.

with j if and only if $\psi_{ij} \geq 0$ (it is not restrictive to assume cooperation in the equality case). Note that i's strategy has no influence on the utilities of lower index users and, therefore, on their decision process. Hence, i's decision to cooperate or not with j, with $i < j$, can be made by maximizing just the partial utility ψ_{ij}. In this way, the original n-player game is decoupled into $\binom{n}{2}$ 2-player games whose outcomes can be easily predicted.

In particular, without any incentive mechanism, the option of relaying packets for another node would never be advantageous. In fact, in this case $\Delta B_{ij} = 0$ and $\Delta E_{ij} > 0$, hence, ψ_{ij} is negative. Thus, no node would ever relay a packet. This is why we also include node 0 that can provide incentives for cooperation, through a reshaping of the transmission probabilities. In this way, users can now get a positive utility when they act as relays, since they may have higher energy consumption but also higher throughput.

4.5.1 Stackelberg formulation

In light of the above discussion, we consider node 0 as an active player in the game, which, to promote cooperation in the network, can change the scheduling policies of users, with respect to the reference scheduling policy $P = (P_1, P_2, \ldots, P_n)$, according to (4.1). We want that, after this intervention by node 0, the users exploiting a collaborative relay still have a throughput improvement, i.e., if $j \in a_i$ then $\Delta B_{ji} \geq 0$; note that they always have an energy saving, i.e., $\Delta E_{ji} < 0$, since i performs a retransmission in j's stead. Moreover, as cooperation rewards are granted by node 0, the transmission probability of i can be increased according to (4.1) only if node 0 correctly received the packet retransmitted by i. In order to reach both objectives, we impose the following change in the allocation conditioned on the event that the packet retransmitted by i is correctly received by node 0

$$\sum_{s=1}^{K_{ij}} \Delta P_{ij}^{(t+s)} q_j^N N_j^N = w_{ij} \frac{q_i^j N_j - q_j^N N_j^N}{q_i^j} \tag{4.8}$$

where $w_{ij} \in [0,\, 1]$ is the *cooperation weight* of i with respect to j. The left hand side represents the average decrease of the number of bits transmitted by j during the following K_{ij} slots, given that $P_j^{(t+s)} = P_j - \Delta P_{ij}^{(t+s)}$, $s = 1, \ldots, K_{ij}$. Therefore, the average (non conditioned) decrease of the number of bits is obtained multiplying it by the probability that the packet retransmitted by i is correctly received by node 0, and we have imposed it equal to $w_{ij}\left(q_i^j N_j - q_j^N N_j^N\right)$. Since $w_{ij} \in [0,\, 1]$, the average increase in the number of bits transmitted by j during slot t, $q_i^j N_j - q_j^N N_j^N$, is higher than the average decrease of the number of bits transmitted by j during the subsequent K_{ij} slots, hence, $\Delta B_{ji} \geq 0$ as we wanted.

The cooperation weight w_{ij} is a tunable parameter describing how valuable it is to reward cooperation by i towards j. If w_{ij} is equal to 1, during the $K_{ij}+1$ time slots from t to $t+K_{ij}$ user j transmits an average number of bits equal to what it would have transmitted during the same interval in the no cooperation case. The lower w_{ij}, the higher the throughput of user j, but at the same time the lower the incentives given to user i, until $w_{ij}=0$, where no incentives are given to user i.

The cooperation weight $w_{ij}, \forall i,j : i<j$, represents the strategy of node 0, i.e., the strength of incentives given to cooperating users. We suppose that w_{ij} are fixed by node 0 at the beginning of the communication and are transmitted to all users. In this way, any user knows in advance the gain it obtains by cooperating with each other user and can select its best strategy. This type of interaction between node 0 and other users can be cast in the framework of the Stackelberg games, where node 0 plays first and the users act afterwards. The player moving first can predict the behavior of other players and optimize its own strategy.

We can rewrite (4.8) as

$$\sum_{s=1}^{K_{ij}} \Delta P_{ij}^{(t+s)} = \frac{w_{ij}}{q_i^j}\left(\frac{q_i^j N_j}{q_j^N N_j^N} - 1\right)$$

under the constraint $\Delta P_{ij}^{(t+s)} \leq P_j$, $s=1,\ldots,K_{ij}$.

There are infinitely many solutions $\left\{K_{ij}, \Delta P_{ij}^{(t+s)}, s=1,\ldots,K_{ij}\right\}$ that satisfy the above equation. However, cooperating users should be rewarded as early as possible, so as to enable faster convergence to the asymptotic throughput. Thus K_{ij} is set as the lowest integer such that

$$K_{ij}P_j \geq \frac{w_{ij}}{q_i^j}\left(\frac{q_i^j N_j}{q_j^N N_j^N} - 1\right)$$

which results in the following scheduling policy variation:

$$\begin{aligned} \Delta P_{ij}^{(s)} &= P_j \quad ; \quad s=t+1,\ldots,t+K_{ij}-1 \\ \Delta P_{ij}^{(t+K_{ij})} &= \frac{w_{ij}}{q_i^j}\left(\frac{q_i^j N_j}{q_j^N N_j^N} - 1\right) - (K_{ij}-1)P_j \end{aligned} \tag{4.9}$$

4.5.2 User strategies

Now, we study the interaction between users considering generic cooperation weights w_{ij} and introducing the selection of the modulation scheme M_i.

In the voluntary cooperation scheme, the packet transmission process of user i follows the Markov Chain in Fig. 4.4, which is conceptually similar to Fig. 4.3 with the difference that only users belonging to W_i cooperate with i and the scheduling is dynamic according to (4.1). The access probability of user i at the beginning of a slot depends on the users i has cooperated with and on the users that have relayed i's packets in the preceding slots. In order to derive the exact metrics associated to the voluntary cooperation scheme, the Markov chain of Fig. 4.4 should be expanded to take into account that i might cooperate with other users when it is not scheduled. The transition associated to the probability $1 - P_i^{(t)}$ should be divided into a number of transitions equal to the cardinality of a_i plus 1, representing the events that i is not scheduled and it does not act as a relay or it acts as a relay for one of the users belonging to a_i. These transitions would end in as many chains, all of them similar to the lower chain of Fig. 4.4, with the only difference that the access probabilities of user i are different. To obtain simple analytical expressions of the asymptotic metrics of the voluntary cooperation scheme, instead of exactly tracing the temporal variation of the scheduling probability we consider an approximate approach that takes into consideration just the average value $\overline{P}_i$ of the scheduling probability of a generic user i. This allows us to obtain the following results

$$\begin{aligned}
P_{R_i}^{\mathcal{V}} &= q_i\,(2-q_i) + \sum_{k\in W_i}(1-q_i)\,q_{ik}\left[\prod_{j\in W_i, j<k}(1-q_{ij})\right](q_k^i - q_i)\\
v_i^{\mathcal{V}} &= (2-q_i)\,/\overline{P}_i\\
BR_i^{\mathcal{V}} &= \overline{P}_i\left[q_i + \frac{1-q_i}{2-q_i}\sum_{k\in W_i} q_{ik}\prod_{j\in W_i, j<k}(1-q_{ij})\,(q_k^i - q_i)\right]\frac{N_i}{T_{pkt}}\\
BR^{\mathcal{V}} &= \sum_{i=1}^{n} BR_i^{\mathcal{V}} = \frac{1}{T_{pkt}}\sum_{i=1}^{n}\overline{P}_i\left[q_i + \frac{1-q_i}{2-q_i}\sum_{k\in W_i} q_{ik}\prod_{j\in W_i, j<k}(1-q_{ij})\,(q_k^i - q_i)\right]N_i
\end{aligned} \tag{4.10}$$

As per (4.1)

$$P_i^{(t)} = P_i + \sum_{j\in a_i}\Delta P_{ij}^{(t)} - \sum_{k\in W_i}\Delta P_{ki}^{(t)}$$

where $\Delta P_{ij}^{(t)}, \Delta P_{ki}^{(t)} \geq 0$ are according to (4.9). $\Delta P_{ij}^{(t)} > 0$ if and only if i cooperated with j during one of the preceding K_{ij} slots. $\Delta P_{ki}^{(t)} > 0$ if and only if k cooperated with i during one of the preceding K_{ki} slots. As per (4.9), ΔP_{ki} depends on q_k^i and N_i that in turn depend on the modulation scheme M_i. This must be taken into account when optimizing M_i. In particular, since the access opportunity of user i is decreased after being helped, the *net* average increase of i's transmitted bits due to the cooperation of user k is scaled by a factor $(1-w_{ik})$. We define

$$D_i = q_i + \frac{1-q_i}{2-q_i}\sum_{k\in W_i}(1-w_{ik})\,q_{ik}\left[\prod_{j\in W_i, j<k}(1-q_{ij})\right](q_k^i - q_i)$$

Then, the optimal modulation scheme of user i can be computed as

$$M_i^{\mathcal{V}} = \arg\max_{M_i\in\mathcal{M}} D_i N_i \tag{4.11}$$

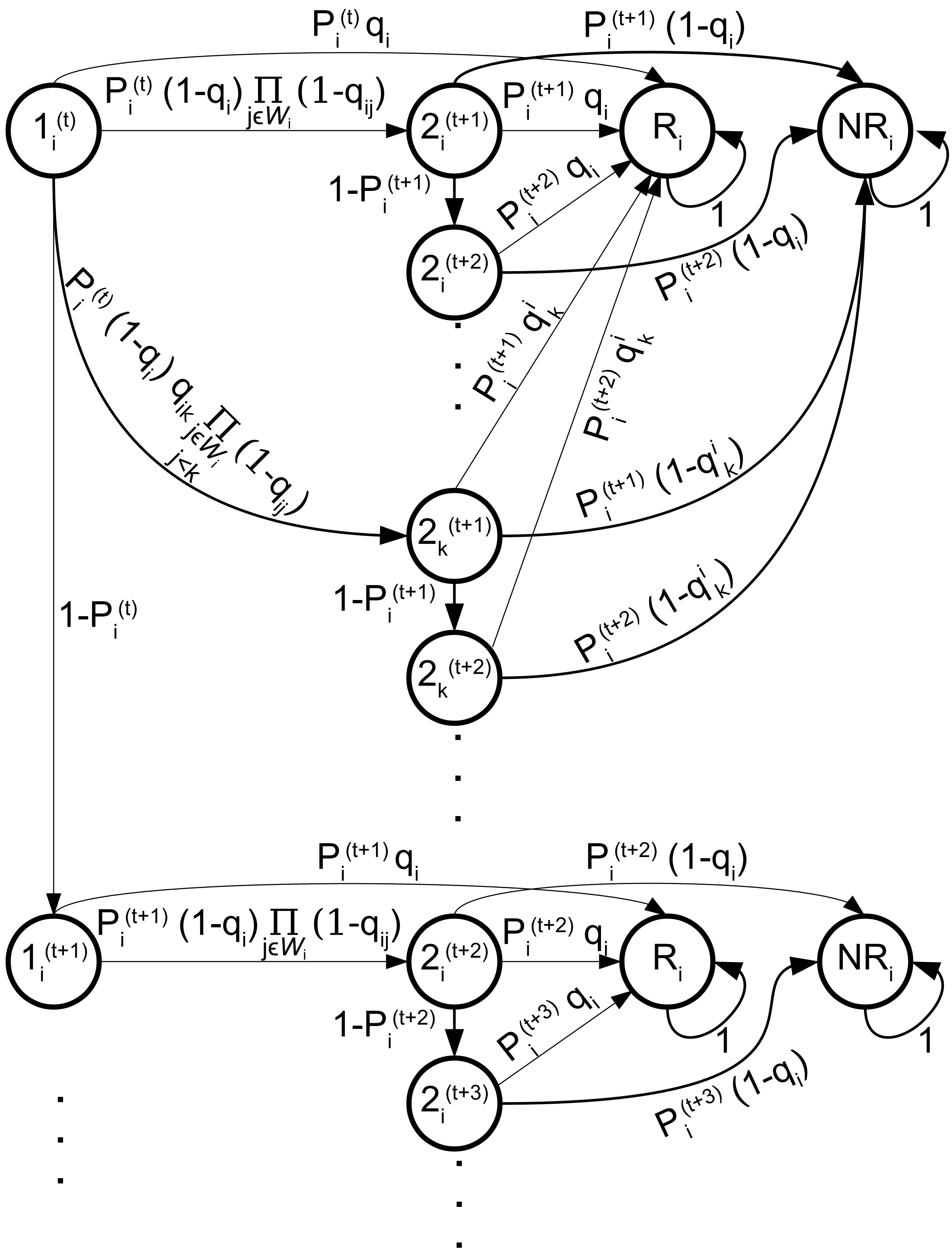


Figure 4.4. *Transmission process of a packet of user i in the voluntary cooperation scheme*

where both D_i and N_i depend on M_i. If i cooperates with j, the average variation $\Delta B_{ij} > 0$ and $\Delta E_{ij} > 0$ of i's transmitted bits and energy consumption per unit time are equal to

$$\Delta B_{ij}=q_i^j \sum_{s=1}^{K_{ij}} \Delta P_{ij}^{(t+s)} D_i \frac{N_i}{K_{ij}T_{pkt}} = w_{ij} \left(\frac{q_i^j N_j}{q_j^N N_j^N} - 1 \right) D_i \frac{N_i}{K_{ij}T_{pkt}}$$
$$\Delta E_{ij}=\left[1 + q_i^j \sum_{s=1}^{K_{ij}} \Delta P_{ij}^{(t+s)}\right] \frac{E_{pkt}}{(K_{ij}+1)\,T_{pkt}} = \left[1 + w_{ij}\left(\frac{q_i^j N_j}{q_j^N N_j^N} - 1\right)\right] \frac{E_{pkt}}{(K_{ij}+1)\,T_{pkt}} \tag{4.12}$$

where $M_i^{\mathcal{V}}$ is chosen according to (4.11). Thus, the evaluation of the partial utility $\psi_{ij}(\Delta B_{ij}, \Delta E_{ij})$ depends (through D_i) on W_i, i.e., the cooperation choices adopted towards i by users with lower indices.

Proposition 1. *Assuming that users cooperate in case their utility is flat with respect to this choice, the sub-game between users admits one and only one NE,* $a^{NE} = \{a_1^{NE}, \dots a_n^{NE}\}$.

Proof. The proof follows a constructive and iterative procedure. Let us consider user 1, which can not be helped by any other node: $W_1 = \emptyset$, $P_{R_1}^{W_1} = P_{R_1}^{\mathcal{N}}$ and $D_1 = P_{R_1}^{\mathcal{N}} / (2 - q_1)$. Since the probability error function q_1 varies with continuity, the set of allocation policies that optimizes (4.11) is a singleton, therefore user 1 can uniquely select its best modulation scheme $M_1^{\mathcal{V}}$. Then user 1 can compute the optimal set of users to cooperate with, i.e., its best strategy a_1^{NE}, depending on the modulation selected by each user. This can be done by calculating ΔB_{1j} and ΔE_{1j} according to (4.12) and evaluating $\psi_{1j}, \forall j \neq 1,\ \forall M_j \in \mathcal{M}$.

This procedure can be repeated for any other user. For a generic user i and for each modulation scheme M_i, if we know the strategies of users $1, 2, \dots, i{-}1$, we can uniquely calculate W_i, $M_i^{\mathcal{V}}$, $P_{R_1}^{W_i}$, ΔB_{ij}, and ΔE_{ij}, $\forall j > i,\ \forall M_j \in \mathcal{M}$; from these, we obtain ψ_{ij}, depending on the modulation selected by the users with worse channels. In the end, we obtain the best modulation scheme for all users and the unique NE strategy profile a^{NE}. □

Corollary 1. *The Nash Equilibrium is Pareto Efficient.*

Proof. The utility of user 1 is the highest possible since it is not affected by other users' strategies and it selects its own strategy to maximize its own utility. In the same way, the utility of user 2 is the highest possible given the strategy of user 1. Moreover, if we change the strategy of user 1 we make user 1 worse off, except for the case in which user 1's utility is flat in its choice to cooperate with user 2. However, in this case we have assumed that 1 chooses to cooperate with 2, hence, if 1 changes its strategy, the utility of 2 can not increase. This procedure can be repeated for any other user. □

4.5.3 Access point strategy

Theorem 1 states that the sub-game between the users has only one possible outcome. Moreover, the constructive proof provides an algorithm to calculate this outcome. The access point can predict, for each strategy $w = (w_{ij})_{ij} \in [0,1]^{\binom{n}{2}}$, the strategies of all users. Therefore, it can choose its best strategy $w^* = (w^*_{ij})_{ij}$ to drive the network performance toward a desired outcome.

Assume that the network performance is quantified by a utility function $u_0 : [0,1]^{\binom{n}{2}} \to \Re$, whose argument is the strategy selected by node 0. It can be thought as the composition of two functions f and g, i.e., $u_0 = g \circ f$, such that $f : [0,1]^{\binom{n}{2}} \to \Re^n$ gives the utility of the users as a function of 0's strategy and $g : \Re^n \to \Re$ gives the utility of 0 as a function of all users' utilities. It is reasonable to assume that g is a continuous function.

Take w^{th}_{ij} as the value such that $\psi_{ij}(\Delta B_{ij}, \Delta E_i) = 0$, which can be derived from (4.12). It is the minimum w_{ij} such that i cooperates with j. The only interesting case is when w^{th}_{ij} exists and $w^{th}_{ij} \in [0,1]$, otherwise it is not possible to trigger i's cooperation with respect to j without decreasing the throughput of j. Since ψ_{ij} are continuous, then f is continuous in $[0,1]$ except in w^{th}_{ij}. Indeed, user i changes its cooperation behavior towards j at w^{th}_{ij}. However, from a practical point of view, if $w_{ij} \in [w^{th}_{ij}, 1]$ the utility of both users i and j increases. In fact, user j achieves at least the same throughput, while decreasing its energy consumption, whereas the increase in throughput of i compensates the additional energy spent to cooperate with j. That is, promoting cooperation under this scheme is always beneficial for both users involved. For this reason, it is reasonable to assume that u_0 is upper semi-continuous.

Proposition 2. *If u_0 is upper semi-continuous then there exists at least one Stackelberg Equilibrium (SE). Moreover, all SEs are equivalent from a network performance point of view.*

Proof. The utility u_0 can be maximized since the sub-game NE exists and is unique. The strategy space of node 0 is closed and bounded, and $u_0(\cdot)$ is upper semi-continuous. An SE can be found by combining the best strategy w^* of node 0 and the NE strategy profile of the sub-game among the users when the strategy of node 0 is w^*. There may be more than one optimal w^*, but they all achieve the same maximum utility of node 0. □

Finally, for result comparison, we consider the following access point strategy

$$w^*_{ij} = \begin{cases} w^{th}_{ij} & \text{if } 0 \le w^{th}_{ij} \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

We chose this strategy to promote cooperation, i.e., increase network performance while keeping a high level of fairness (fairness metrics will be defined in the following section).

Note that $w_{ij}^{th} \notin [0,1]$ means that it is impossible, with the considered scheme, to provide an incentive for user i to cooperate with j. In this case the system functionality is independent of w_{ij}^*, and we have arbitrarily chosen $w_{ij}^* = 0$.

4.6 Performance Evaluation

Prior to comparing the 3 cooperation schemes, we introduce some performance metrics.

For any vector of n real numbers, $\mathbf{x} = (x_1, \dots, x_n)$, we define a fairness metric $J(\mathbf{x})$ over **x**, called Jain index [64], as

$$J(\mathbf{x}) = \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n \sum_{i=1}^{n} x_i^2} \tag{4.13}$$

We will evaluate this index for the vectors of throughput ($BR = (BR_1, \dots BR_n)$) and utility values ($\mathbf{\Psi} = (\Psi_1, \dots \Psi_n)$). We use superscripts $\mathcal{N}$, $\mathcal{F}$, and $\mathcal{V}$ to relate these metrics to the no cooperation, forced cooperation and voluntary cooperation schemes, respectively.

A scenario with n users uniformly placed within a 400 meters radius from an access point has been simulated in Matlab. We consider a time slot $T_{pkt} = 1\ ms$ and a symbol period of $T_{sym} = 1$ μs, that is, each packet is made of 1000 symbols. The number of bits per packet for a generic user depends on the number of bits per symbol, i.e., on the modulation scheme selected by that user. We consider $\mathcal{M} = \{BPSK,\ QPSK, 16\!-\!QAM,\ 64\!-\!QAM\}$, that correspond to the rates represented in Fig. 4.5.

Each user transmits with a fixed power of $P_{pkt} = 100$ mW. The time invariant channel attenuation coefficient is given by the superposition of two effects: a power law decay with exponent equal to 3 and a Rayleigh distributed coefficient. The signal to noise ratio obtained at a reference distance of 10 m considering a unit-power Rayleigh coefficient is 10. We consider the initial allocation policy $P = (1/n, 1/n, \dots, 1/n)$.

We take $\Psi_i(B_i, E_i) = B_i - c_i E_i$, i.e., $\psi_i(\Delta B_i, \Delta E_i) = \Delta B_i - c_i \Delta E_i$, which satisfies (4.7) with $\psi_{ij}(\Delta B_{ij}, \Delta E_{ij}) = \Delta B_{ij} - c_i \Delta E_{ij}$, $\forall i, j$, where $c_i > 0$ is a measure on how important the throughput is for user i with respect to its power expenditure. We consider $c_i = \dfrac{q_i N_i}{2E_{pkt}}$ where q_i and N_i are calculated with a modulation scheme according to (4.3), i.e., c_i is equal to half i's energy efficiency (rate divided by power consumption) in the non cooperative case. In this way, users having a low non cooperative rate are more inclined to cooperate with other users, consuming their energy

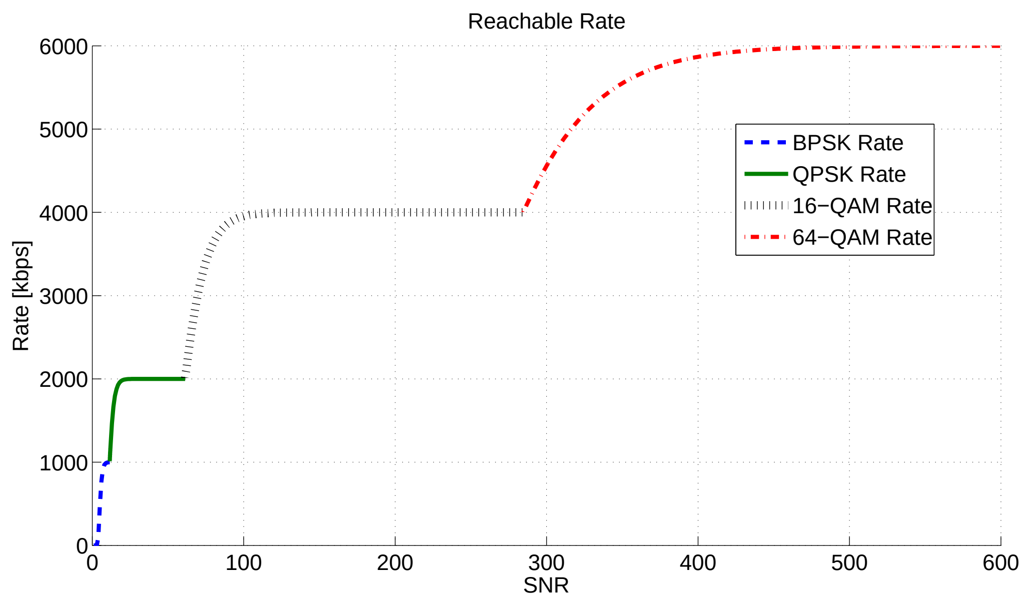


Figure 4.5. *Reachable rate varying the modulation depending on the SNR*

to obtain a higher throughput, with respect to users having already a high non cooperative rate. We obtain

$$w_{ij}^{th} = \frac{c_i E_{pkt}}{\left(\frac{q_i^j N_j}{q_j^N N_j^N} - 1 \right) (D_i N_i - c_i E_{pkt})}$$

We first present some results for a specific topology with $n = 10$, which is actually the one in Fig. 4.1. Fig. 4.6 shows the evolution of throughput over time for user 10 (the one with lowest SNR), for the 3 different schemes. The dashed lines represent the average throughput of no cooperation and forced cooperation schemes according to (4.2) and (4.5). The cumulative throughput asymptotically converges to these average values. This convergence is quite fast, as the curves are already stable after few iterations and become practically indistinguishable from the asymptotic value within 10 seconds.

Fig. 4.7 compares the asymptotic throughput reached by each user. Roughly speaking, this specific topology includes some users (with indices 1-3) that are able to reach a maximal throughput of 600 kb/s already under the no cooperation scheme, by using the highest modulation (64-QAM) without ever incurring in packet retransmission. Conversely, users 7-10 have very poor channel conditions (lower modulation scheme, and possibly frequent retransmissions), and users 4-6 are in an intermediate condition. Interestingly, in the forced cooperation scheme the users with the highest indices obtain the greatest benefit. They know that users 1, 2 and 3 are forced to act as relays. Thus, since they have a good channel towards at least one of these relays, they select the highest modulation

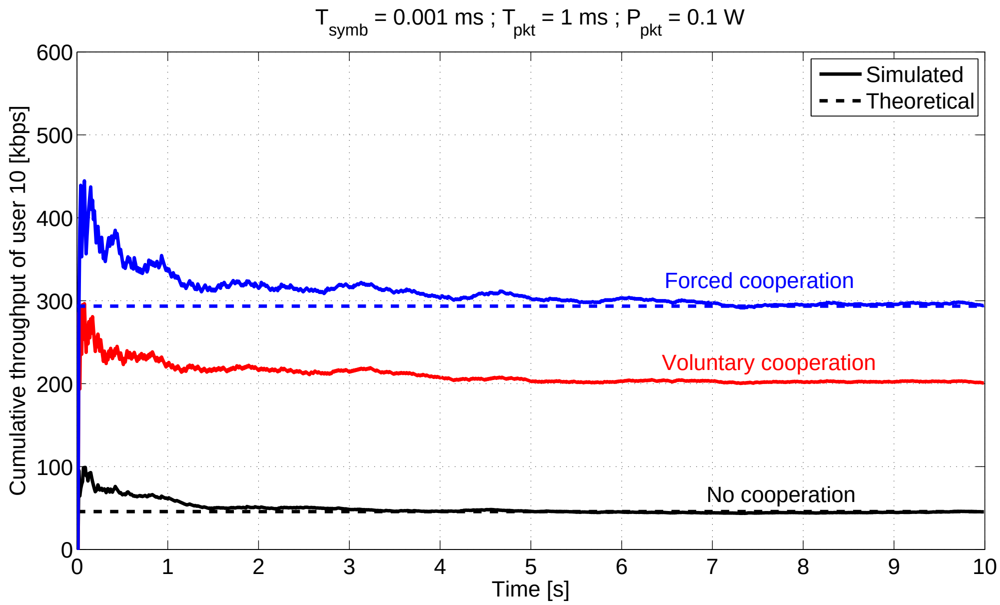


Figure 4.6. *Cumulative throughput of user* 10

and their packets are transmitted in two hops exploiting the relays, allowing them to reach a bit-rate of about 300 kb/s. On the contrary, the cooperating users do not obtain any improvement. Instead, the voluntary cooperation scheme increases the throughput of cooperating users as well. Especially, users 7 and 8 are not helped since none of the users with good quality finds a worthwhile incremental advantage in doing so.

Fig. 4.8 represents the incremental utility ψ of each user and emphasizes even more the differences between the forced and voluntary cooperation schemes. For the forced cooperation case, the utility of high index users considerably increases, though at the expense of low index users which have no reward in their cooperating behavior. When cooperation is forced by node 0, users 7-10 significantly increase their own throughput and at the same time cut in half the transmission power because retransmissions are performed by users 1-3, which in turn only suffer higher power expenditures. The voluntary cooperation scheme improves this situation, since no user worsens its incremental utility ψ. The highest index users improve their utility, even though by a smaller extent than with forced cooperation, and no user is worse off than before. Indeed, this happens because cooperation is offered even in the marginal case where the incremental utility is equal to 0; however, setting a higher requirement for cooperation would yield similar results, i.e., a utility value which is higher for some users, lower for none. In this sense, the voluntary cooperation scheme *Pareto dominates* the no cooperation scheme [49]. Moreover, the figure suggests that the voluntary cooperation scheme achieves a more fair distribution of the utility function among the users. Finally, Fig. 4.8 validates the analy-

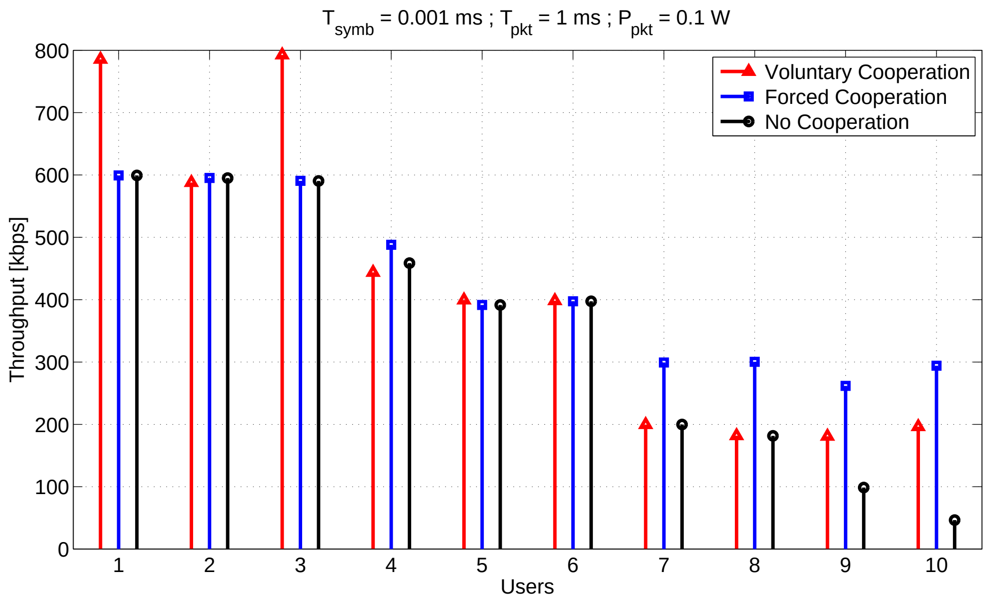


Figure 4.7. *Asymptotic throughput of each user*

sis carried out in Section 4.5. In fact, even though the incentives to cooperative users are calculated using the approximate equations (4.10), the *real* throughput gain for cooperative users is just enough to compensate the *real* additional energy consumption to relay the packets of the other users, as we wanted.

To obtain general results, not constrained over a particular network topologies and channel realization, we ran a simulation campaign over many network topologies drawn at random with a variable number of users, and averaged the results. Fig. 4.9 represents the average throughput increase of the whole network thanks to cooperation, for both forced and voluntary cooperation schemes. The values are normalized to the total throughput obtained in the no cooperation scenario. Both forced and voluntary cooperation schemes obtain a significant gain; for 50 users, they improve the total throughput by more than 25% and 35%, respectively. Remarkably, voluntary cooperation performs better than forced cooperation; this is due to the better redistribution of additional resources gained through cooperation, which in the forced cooperation scheme are given just to the users with bad channel quality, while in the voluntary cooperation scheme are distributed more evenly. It is also worth noting that the cooperation gain increases in the number of users, which is due to multi-user diversity, i.e., with more users it is just more likely to find a suitable relay. However, the voluntary cooperation scheme does better in this sense, i.e., it increases more rapidly in the number of users, in fact it is more likely to find a suitable relay which is also willing to cooperatively participate in the retransmissions.

Fig. 4.10 shows the Jain index related to the throughput vector, i.e., $J(BR)$. Clearly, the no

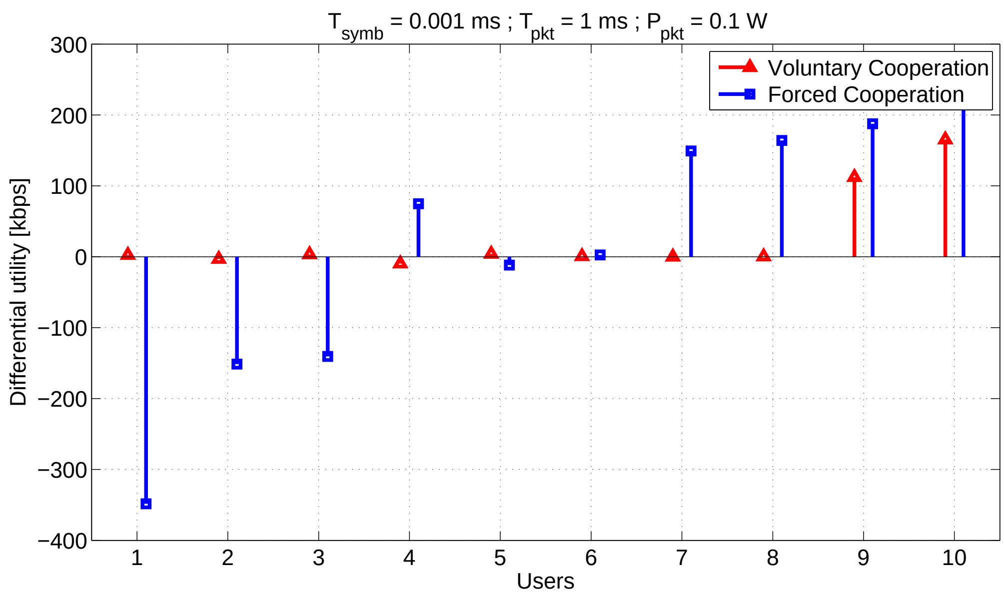


Figure 4.8. *Incremental utility ψ of each user*

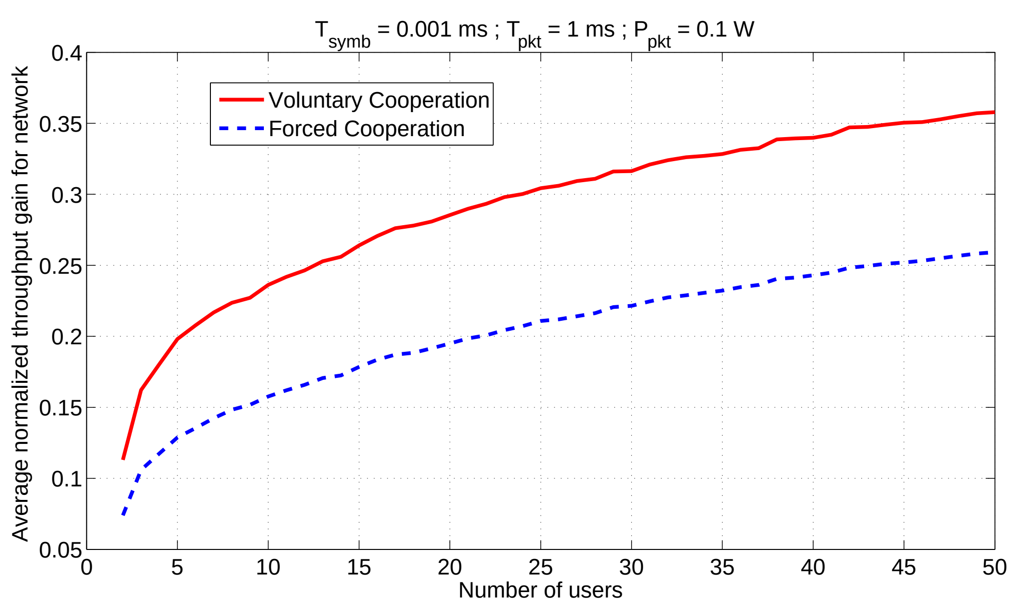


Figure 4.9. *Average throughput gain normalized to the no cooperation scenario*

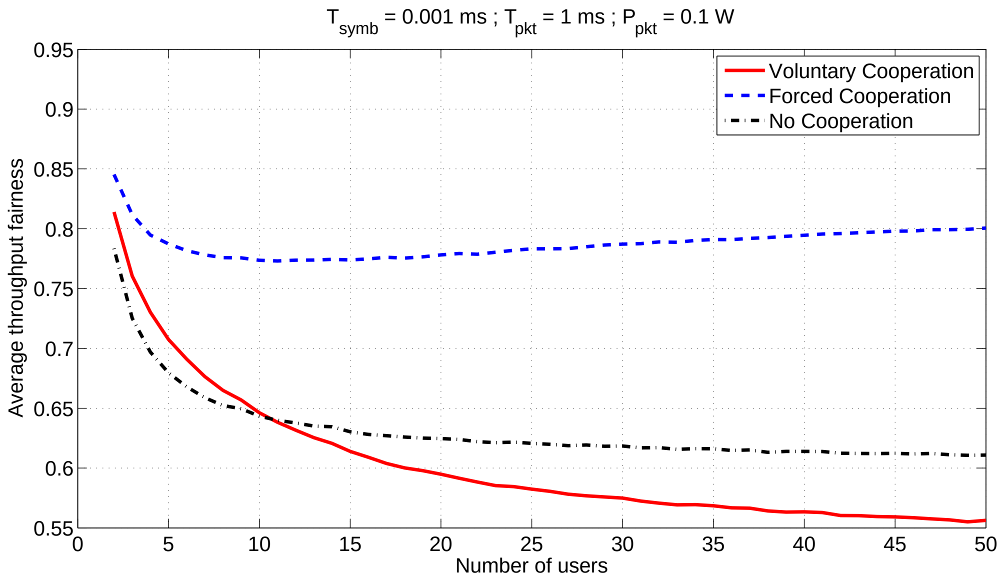


Figure 4.10. *Average throughput fairness*

cooperation case just reports what is the average situation for what concerns fairness in the considered scenario if no cooperation is applied. Apparently, the forced cooperation scheme achieves the best value of fairness for throughput. In fact, users with lower throughput are helped by collaborative relays which have no other choice, therefore throughput gaps are smoothed out. After an initial decrease, the Jain index becomes even larger as the number of users increases. In fact, the higher the number of users, the higher the probability of finding a suitable relay (not necessarily a willing one, since cooperation is forced). The fairness decreases quite rapidly for the voluntary cooperation scheme. This is due to the fact that users with good channel conditions, which already have a higher throughput than others, are rewarded by the access point if they cooperate, which means that they further increase their throughput. This pulls fairness even below the no cooperation case. However, it is worth noting that, although fairness is decreased, throughput is never decreased for anybody. Moreover, evaluating fairness over throughput just gives a very partial picture. Even though users with good channel increase their throughput, they also have to pay this gain in terms of power consumption, since they retransmit packets on behalf of bad users (which in turn can save energy); even their reward in terms of increased scheduling probabilities also implies more transmissions and therefore higher energy consumption.

Fig. 4.11 shows the Jain index related to the utility vector, i.e., $J(\mathbf{\Psi})$. The situation is inverted with respect to the preceding case. As the number of users increases, the fairness rapidly decreases for the forced cooperation scheme. This is due to the fact that a small subset of users, i.e., those

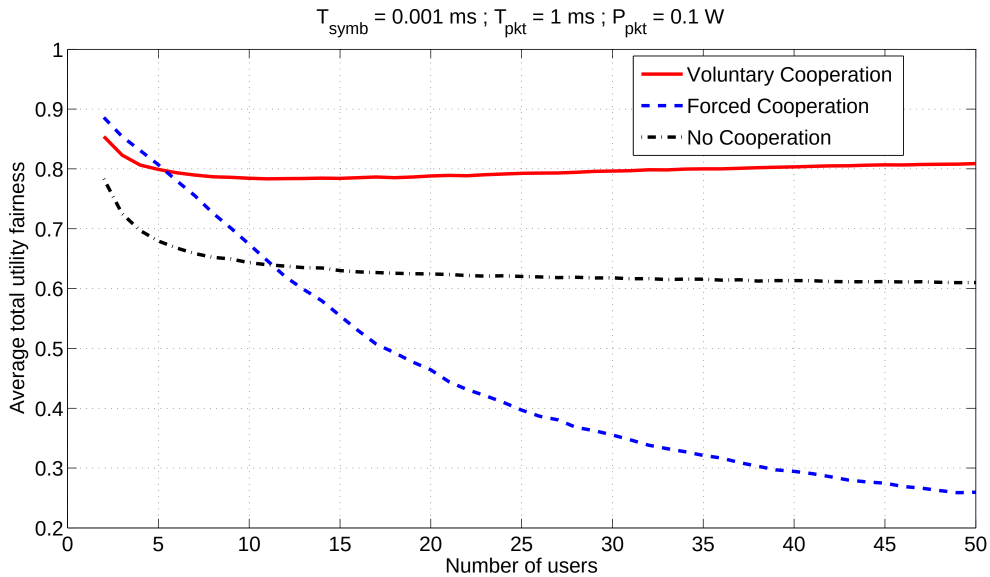


Figure 4.11. *Average utility fairness*

having a very good channel quality and able to act as relays for a large area, are more and more forced to cooperatively relay packets. This pulls their utility much more below the utility of users that are exploiting them as relays, decreasing the total fairness of the network to values even below the no cooperation case. On the other hand, in the voluntary cooperation scheme users acting as relays do not experience a decrease in their utility while helped users can increase their own utility, which results in smoother utility gaps. Note that, if the utility fairness is considered as the social welfare metric, Fig. 4.11 gives a representation of the Price of Anarchy, defined as the ratio between the overall system welfare in the worst Nash equilibrium and in the best Pareto efficient case. In fact, the highest value of the utility fairness is 1, obtained when the users' utilities are equal, while the worst Nash equilibrium coincides with the unique equilibrium of the game under consideration.

To sum up, the comparison between the three schemes shows that voluntary cooperation is able to significantly improve the network performance over the case without cooperation. In all the comparisons, the forced cooperation scheme is to be regarded as a theoretical upper bound, as it implies a centralized scheduling determined a priori with full system knowledge, to which all the users adhere. Conversely, the voluntary cooperation scheme may be applied dynamically (based on transmission outcomes) and in a distributed manner, since each user decides freely whether to cooperate or not. The goal of the coordinator is just to set the system in an NE, for which the exchange of information required is rather limited and the convergence is pretty fast. Note also that the forced cooperation scheme does not operate in a stable point, i.e., at a NE. Thus, with the same system conditions of rational decision and distributed action the forced cooperation scheme will become identical to the no cooperation scheme. On the contrary, the voluntary cooperation scheme is robust toward strategic and self-interested users. Moreover, the performance of the voluntary cooperation scheme can be regarded as an improvement not only over the basic case without cooperation, but even over the forced cooperation scheme, especially since it achieves a higher total throughput and a more fair overall utility distribution.

4.7 Discussions and Future Works

The results obtained in this chapter have been derived considering a simplified model of a wireless communication network. In this section we discuss possible relaxations of some hypotheses we have made.

First, we consider the time invariant channels and the perfect channel state knowledge hypotheses, that allow to calculate the performance of each user and of the system by means of an analysis based on renewal process theory. If channels are time varying, the asymptotic performance is not longer equivalent to the statistical mean. However, for slowly varying channels, there is enough time for the physical quantities under investigation (i.e., throughput and energy consumption) to approach the statistical means, as Fig. 4.6 confirms. Hence, our formulation can be applied to the slowly varying channels scenario as well, by considering adaptive estimates. This work can also be extended to highly varying channels and imperfect channel state knowledge, assuming that the entities involved aim at maximizing the statistical mean of their performance, which might not coincide with their asymptotic performance. In this case, the statistics of the channel evolution and of the channel estimates are needed.

As frequently considered in many game theoretic studies, we assumed that *every* user is self-interested and strategic. In a network there might be some users that act individually or cooperatively independently of their personal advantage. Our framework and results can be easily extended assuming that a mix of no cooperation and forced cooperation nodes are present in the network of voluntary cooperation nodes. The former might receive the cooperation of the other users, but never offer their cooperation. Thus, the indices of such nodes do not belong to set W_i and do not appear in the summation and multiplication of Eq. (4.10). The latter always offer their cooperation, hence, there is no need to give them incentives by increasing their access opportunities, i.e., their cooperation weights can be set to 0. Thus, the indices of such nodes belong to set W_i and appear in the summation and multiplication of Eq. (4.10). It is straightforward to demonstrate that Theorems 1 and 2 and Corollary

1 are still valid, excluding the no cooperation and forced cooperation nodes from the sub-game (they do not play a game since their actions are fixed).

Another aspect which may be worth looking at is the evaluation of the overhead introduced by the forced and voluntary cooperation schemes with respect to the no cooperation scheme. This point is key to translate the theoretical framework proposed in this chapter into an effective and realistic MAC protocol. However, it can be shown through simple computations that such an additional overhead is minimal and can be neglected. We do not consider the overhead for the estimation and communication of the channel states (which is needed in every scheme) and the computation of the cooperation weights (which is needed for the voluntary cooperation scheme), as these operations are performed sparsely since channels are slowly varying. Instead, we investigate the overhead to schedule different users, to identify eligible relays and to select one of them.

For the no cooperation scheme, at the beginning of each time slot, we assume that node 0 broadcasts a short packet indicating the user scheduled in that slot. Such a user, after a short time interval[7], sends the data packet. Finally, after another short time interval, node 0 sends an ACK to the user if it has received the packet correctly.

We modify such a simple MAC protocol to support the forced cooperation and voluntary cooperation schemes. In this case, during the scheduling phase, node 0 has to indicate not only the packet to transmit, but also who has to perform such a transmission, in case a relay service is required. Moreover, the user that transmits the packet adds, at the end of the packet data, a series of bits, one for each node, to communicate to node 0 the users for which it is available to act as a relay. This MAC protocol is not suitable if there are some users that are scheduled rarely, as in this case node 0 might not be updated about the relay opportunities offered by such users. In this case another option should be considered to inform node 0 about relay opportunities, e.g., a short contention window can be added after the ACK.

The additional overhead introduced in the considered MAC protocol can be easily quantified. Consider a time slot $T_{pkt} = 1$ ms, a symbol period of $T_{sym} = 1$ μs and a network of 50 users. Hence, the additional number of bits needed in the scheduling packet is equal to 6 while the additional number of bits needed in the data packet is equal to 50. Assuming, in the worst case, a $BPSK$ modulation, the additional overhead is equal to 56 μs over 1 ms, i.e., about 5%, that is very low compared to the throughput gain of the forced cooperation and voluntary cooperation scheme that are equal to 25% and 35% in such a scenario (see Fig. 4.9).

[7] In the 802.11 $g/n/ac$ standards the SIFS (short inter-frame space), defined as the sum of the RX/TX turnaround time, MAC processing delay and total receive delay from the antenna, is equal to 16 μs

Finally, in this chapter we have not considered the cost incurred by every node to listen and store the transmission of all the other nodes. Even though the power spent in reception is typically lower than the transmission power, such an effect might become predominant for a high number of users. Moreover, in the worst case each user might have to store up to $n - 1$ additional packets, requiring a large buffer. These problems might be counteracted considering a simplified version of the proposed schemes, where we limit, for each user, the number of users to ask for a relay service and to cooperate with. Such a simplified version is motivated by the high gain that the voluntary cooperation scheme is able to obtain for a high number of users, as shown in Fig. 4.9. Such a potential gain might not be completely exploited if we limit the relay opportunities, but at the same time the scheme becomes more practical as the number of users increases. We will take into consideration the study of such a scheme in our future work.

4.8 Conclusions

We tackled the problem of promoting cooperative relaying in a wireless network with coordinated time-division access, by giving the following contributions. First, we outlined mathematical models, based on Markov chains and renewal theory, to quantify the achievable throughput. Moreover, we modeled the cooperation option of the single users through game theory and we proposed an incentive scheme for voluntary cooperation that gives transmission resources to cooperating users when they retransmit a packet on behalf of other users. We modeled this access scheme as a Stackelberg game, where a network unit plays the role of access coordinator. We presented a constructive approach to determine the NE of the sub-game, proven to be unique. We also proved the existence of a Stackelberg equilibrium, which results in the best incentive strategy that the coordinator can adopt.

Finally, we numerically compared the three schemes of no cooperation, forced cooperation, and voluntary cooperation. A careful analysis of these results justifies the voluntary cooperation scheme as a valid solution to increase the network performance in a viable manner from an implementation standpoint.

Chapter 5

Inter-Network Cooperation exploiting Game Theory and Bayesian Networks

In this chapter[1] we analyze a scenario where two wireless ad hoc networks are willing to share some of their nodes, acting as relays, in order to gain benefits in terms of lower packet delivery delay and reduced loss probability. Bayesian Network analysis is exploited to compute the correlation between local parameters and overall performance, whereas the selection of the nodes to share is made by means of a game theoretic approach. Our results are then validated through use of a system level simulator, which shows that an accurate selection of the shared nodes can significantly increase the performance gain with respect to a random selection scheme.

5.1 Introduction

We consider two wireless multi–hop networks deployed in the same region, but operated by different entities, that are willing to share some of their nodes, acting as relays for the other network. In such a scenario, cooperation can leverage the benefits of multi–path diversity, since more paths connecting two nodes will be available, obtaining a considerable gain in the efficiency of shared resources. Sharing the whole set of nodes provides the highest number of paths available for each of

[1]The material presented in this chapter has been published in:

[C5] G. Quer, F. Librino, **L. Canzian**, L. Badia, and M. Zorzi, "Using Game Theory and Bayesian Networks to Optimize Cooperation in Ad Hoc Wireless Networks," in *Proc. IEEE ICC*, Ottawa, Canada, Jun. 10-15, 2012

[J2] G. Quer, F. Librino, **L. Canzian**, L. Badia, and M. Zorzi, "Inter-Network Cooperation exploiting Game Theory and Bayesian Networks," *Submitted to IEEE Trans. Commun.*

the two networks. However, this comes at the cost of increased traffic that should be handled by some of the shared nodes. In a realistic environment, an operator may not be willing to share too many nodes to improve the traffic of another operator, e.g., for security or privacy reasons. Therefore, both operators may decide to share only a limited number of nodes. If this is the case, an efficient choice of the shared nodes, according to certain criteria, is needed. Indeed, some nodes deployed in crucial positions may be particularly suited for helping the other network; on the contrary, nodes placed close to the network border are likely to be less useful or even useless. Furthermore, sharing a node implies that a higher amount of traffic will be routed through it, which results in a higher latency for the traffic of its own network.

We assume that each node of each network is sending packets to every other node in the same network. In the case of no cooperation, the two coexisting networks perform their operations separately: each network only uses its own resources to deliver the data packets generated by its nodes. Clearly, since they are assumed to share the same spectrum resources, cross–network interference may limit the overall performance. For such a scenario, we select a set of local parameters: some of them are directly observable (i.e., we can assume that each network knows their values), and depend only on the topology of the network (topological parameters), like the number of neighbors at a given node. Some other parameters are not observable and depend on the link characteristics and on the traffic load (performance parameters). We exploit Bayesian network analysis to estimate the joint probability distribution of this set of parameters, and to predict, given the evaluation of the observable parameters, the values of the other parameters that will be used to calculate a cost metric. Then we use this information to model the interaction between the two networks through game theory and to select the best nodes to be used as relays, assuming that both networks are interested in optimizing their performance.

5.1.1 Related work

In multi–hop wireless networks, the use of relays can be seen as a form of cooperation, since they create new multi–hop routes. Several protocols have been designed to balance the enhanced link reliability and the increased number of transmissions [74–77]. Coded cooperation is developed in [74] and [76], whereas an implementation based on hybrid automatic repeat request is introduced in [77]. The use of relays shows how cooperation can be also exploited for routing purposes, as investigated in [78–80]. The choice of the best relay, based on the channel conditions, is discussed in [78], whereas several relays, chosen according to topological criteria, simultaneously cooperate in forwarding a packet in the scheme described in [79]. Finally, a cross-layer approach, where cooperation is exploited in ad hoc networks together with the opportunistic routing paradigm, has been shown in [80].

Although a wide literature is available about cooperation among terminals of the same network, fewer works have instead been focused on cooperation between different networks. In most of them, the idea behind a cooperative behavior of two coexisting networks is to share the spectrum resources. Such a paradigm, known as spectrum sharing, is exploited by primary/secondary cognitive radio networks: an unlicensed network is allowed to exploit the same spectrum assigned to a licensed one, provided that a given QoS is guaranteed to the latter. The spectrum can be shared through strategies exploiting different levels of awareness and coordination, whose performance has been analyzed and discussed in [81] and [82]. In [83], the authors investigated the case where two cellular networks share their own spectrum resources and cooperate in order to minimize the mutual interference, observing a gain inversely proportional to the cardinality of the networks. Also infrastructure sharing has been considered as a promising cooperation technique for cellular networks; in [84] the sharing of some parts of the network structure is described from a business and regulatory perspective.

To enable the use of cooperation, it is necessary to infer the network gain and cost in advance, thus choosing whether or not it is worth to perform cooperation. Other choices, which require some knowledge about the network, must be made, like which nodes to select as relays. An effective tool to exploit the available information and make a real-time estimation of the expected performance is given by probabilistic graphical models [85]. The use of this probabilistic tool is very promising for wireless network optimization, and it has been recently exploited, e.g., in [86] where a Bayesian Network approach is adopted for predicting the occurrence of congestion in a multi-hop wireless network. The use of Bayesian prediction in a game theoretic framework to allow cooperation is discussed in [87].

In spite of the considerable gain allowed by cooperative transmission, modeling the involved agents as selfish decision-makers usually leads to inefficient non-cooperative outcomes. In this chapter we formulate the problem as a repeated game, in which agents must account for the consequences of their current actions on the evolution of the game, and cooperation is obtained by punishing deviating users in subsequent stages. Repeated interactions have already been applied to the study of cooperative relaying. A packet forwarding mechanism balancing the relaying opportunities that each node gives to and receives from other nodes is proposed in [17]. A virtual currency and a mechanism to charge/reward a player that asks/provides a relay service are introduced in [18] and [19]. Finally, [20] considers a reputation mechanism, where a user gains reputation acting as relay and can choose not to serve users having low reputation.

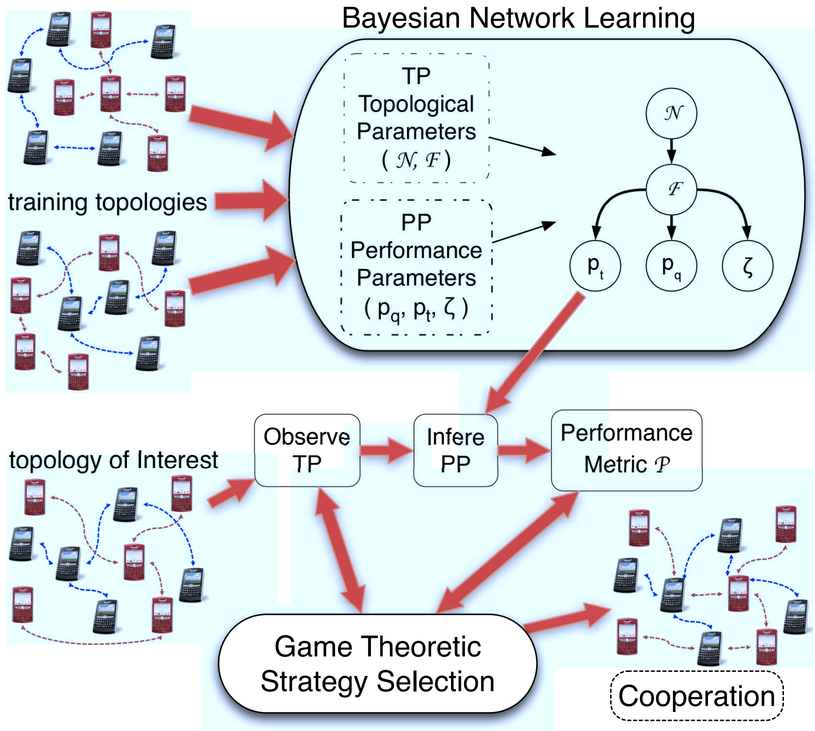


Figure 5.1. *Logical structure of the proposed approach.*

5.1.2 Problem statement and outline of the proposed approach

In the scenario proposed in this chapter, two multi–hop networks share the same wireless resources and compete to access the channel. Each network can share with the other network a limited number of nodes for packet relaying, with the goal to minimize a given cost metric.

The logical structure of the proposed approach is detailed in Fig. 5.1. During a learning phase, we observe some local *Performance Parameters* (PP) of the two networks in many different training topologies, each of them characterized by some local *Topological Parameters* (TP). We use the observed data to build the probabilistic relationships among all the parameters, summarized in a Bayesian Network (BN). Then we consider the scenario of interest, we observe the TP in such a scenario, and we use the BN to infer the PP. Through our game theoretic approach we promote the cooperation among networks and we choose the best nodes to be shared in order to minimize the chosen cost metric that is obtained from the PP. We measure through simulation the performance improvement due to cooperation. Note that we do not need to repeat the learning phase every time the topology changes, since the BN learned from the observation of the training topologies can be reused for every topology of interest. This makes our approach suitable to be implemented also in the presence of a fast changing topology, since it allows to choose the best nodes to be shared based only on observable TP, without the need for an initial setup.

In brief, the main contributions of this chapter are:

- the use of BN theory to learn the probabilistic relationships among a set of parameters in the network, in order to infer the network performance from the observable topological parameters;
- the definition of the cooperation problem between two networks sharing the same spectrum resources as a strategic game;
- the implementation of the BN predictor and the strategic game in an actual wireless network simulator that evaluates the network behavior at the physical, MAC and network layers;
- a performance comparison showing the effectiveness of our algorithm, which achieves the same performance of a fully cooperative approach by sharing only few selected nodes.

The rest of the chapter is divided as follows. In Section 5.2 we introduce the BN approach. In Section 5.3 we describe our network scenario. In Section 5.4 we define three performance metrics and we detail how to compute them. In Section 5.5 we describe the considered game theoretic approach. In Section 5.6 we present the simulation setup and show the main results. Section 5.7 concludes the chapter.

5.2 Bayesian Networks Preliminaries

A Bayesian Network is a probabilistic graphical model [85] describing conditional independence relations among a set of M random variables through a Directed Acyclic Graph (DAG), which is composed of vertices and directed edges. A vertex i in the graph represents a random variable x_i, while a directed edge from vertex i to vertex j represents a direct probabilistic relation between the corresponding variables x_i and x_j. In this case, we say that i is a parent of j, and we write $x_i \in \text{pa}(x_j)$. The absence of a direct edge between two variables implies that the variables are independent, given certain conditions on the other variables.

Learning the DAG is equivalent to calculating an approximate structure of the joint probability distribution among M variables. This structure is used to calculate the parameters of such joint probability distribution with a limited number of samples, see [85] for further details. The technique to learn the approximate joint probability distribution through a BN is divided into two phases, the structure learning and the parameter learning phases.

5.2.1 Structure learning

This is a procedure to define the DAG that represents the qualitative relationships between the random variables, i.e., the presence of a direct connection between a couple of variables, not conditioned by other variables. We follow a score based method [88], i.e., we do not assume any a priori knowledge on the data, but we just analyze the realizations of the variables and we score each possible DAG with the Bayesian Information Criterion (BIC) [89] that we have chosen as a score function. The BIC is easy to compute and is based on the maximum likelihood criterion, i.e., how well the data suits a given structure, and penalizes DAGs with a higher number of edges. If each variable is distributed according to a discrete probability distribution, i.e., it has a finite number of possible outcomes, then the BIC becomes very simple to compute, involving only summations for all possible outcomes of the variables and all possible outcomes of the parents of each variable, see [88]. As an example, suppose that we apply the BIC score based method to a limited number of realizations of the variables x_h, x_i and x_j, and we obtain a DAG such that h is a parent of i and i is a parent of j. Using this approximation, the joint probability of the corresponding variables can be written as

$$P(x_h, x_i, x_j) = P(x_h)P(x_i|x_h)P(x_j|x_i)$$

that is simpler than a general joint probability among three variables.

5.2.2 Parameter learning

This phase consists in estimating the parameters of the simplified joint distribution according to the probability structure defined by the DAG chosen in the structure learning phase. To obtain the joint distribution, it suffices to estimate the probability of each variable conditioned by the variables that correspond to its parent nodes in the graph. Coherently with the choice of the BIC as a scoring function, we use the maximum likelihood estimation technique also to determine all the conditional probabilities for each variable considered.

5.3 System Model

In this section, we describe the network scenario under investigation from the physical up to the routing layer. In our scenario, two ad hoc wireless networks coexist and share the common spectrum resource. Each network consists of n terminals randomly deployed, and each node is a source of traffic, which generates packets according to a Poisson process with intensity λ packets/s/node. The end destination is chosen at random, for each packet, among the other nodes in the network. Furthermore, time is divided in slots and slot synchronization is assumed across the whole network.

5.3.1 Physical layer

At the physical layer Code Division Multiple Access (CDMA) with fixed spreading factor is employed to separate simultaneous transmissions, since both networks share the same spectrum resources, and a training sequence for channel estimation is added at the beginning of each transmission. The receiving node, $D^{(0)}$, uses a simple iterative interference cancellation scheme to retrieve the desired packet when M simultaneous communications, namely $T^{(1)}, \ldots, T^{(M)}$, are received. We define the Signal to Interference plus Noise Ratio (SINR) at $D^{(0)}$ for the incoming transmission $T^{(i)}$ from node $D^{(i)}$ as

$$\Gamma^{(i)} = \frac{S_f P^{(i)}}{N_0 + \sum_{j \neq i} P^{(j)}}$$

where N_0 is the noise power and S_f is the spreading factor. $P^{(j)}$ indicates the incoming power due to $T^{(j)}$, i.e., for all $j = 1, \ldots, M$:

$$P^{(j)} = \frac{P_T |h_{D^{(j)},D^{(0)}}|^2 d_j^{-\alpha}}{\chi}$$

where P_T is the transmission power, which is considered to be the same for all the nodes in the network, χ is a fixed path-loss term, d_j is the distance between the receiving node and the source of $T^{(j)}$, α is the path loss exponent, and $h_{D^{(j)},D^{(0)}}$ is a complex zero mean and unit variance Gaussian random variable, which represents the effect of multi-path fading. More precisely, in our scenario, we consider a time correlated block fading. Therefore, for the channel between nodes $D^{(j)}$ and $D^{(0)}$, the multi-path fading coefficient in time slot t is

$$h_{D^{(j)},D^{(0)}}(t) = \rho \;\; h_{D^{(j)},D^{(0)}}(t-1) + \sqrt{1-\rho^2}\, \xi$$

where ρ is the time-correlation factor and ξ is an independent complex Gaussian random variable with zero mean and unit variance. The iterative interference cancellation scheme works as follows:

- the destination node $D^{(0)}$ sorts the M incoming transmissions according to the received SINR, in decreasing order (for simplicity, assume $\Gamma^{(1)} \geq \cdots \geq \Gamma^{(M)}$);
- starting from transmission $T^{(1)}$, $D^{(0)}$ tries to decode the corresponding packet, with a decoding probability that is a function of $\Gamma^{(1)}$ and of the modulation scheme;
- if the packet is correctly received, its contribution is subtracted from the total incoming signal;

- $D^{(0)}$ attempts to decode the transmission with the next highest SINR, $T^{(2)}$, and goes on until the transmission being decoded is the packet of interest.

5.3.2 MAC layer

At the MAC layer, we implement a simple transmission protocol based on a Request-To-Send/Clear-To-Send handshake. Every time node $D^{(i)}$ wants to send a packet to node $D^{(j)}$, it checks the destination availability by sending an Request-To-Send packet; if $D^{(j)}$ is not busy, it replies with a Clear-To-Send packet so that $D^{(i)}$ can start transmitting the packet. Correct reception is acknowledged by means of an ACK packet. In the case of decoding failure, after a random backoff time, node $D^{(i)}$ schedules a new transmission attempt, unless the maximum number of retransmissions M_{tx} has been reached, in which case it discards the packet. Signaling packets are very short, i.e., they are transmitted within a single time slot, and are protected by a simple repetition code of rate $1/2$. Instead, data packets may span several time slots, so error detection coding is used to verify their correct reception, i.e., redundancy bits are added at the end of each packet.

5.3.3 Network layer

The source and destination nodes are not necessarily within coverage range of each other, so we consider multi-hop transmissions. Two nodes can communicate directly if their distance is less than or equal to the transmission range r. To transmit to destinations that are not within coverage, nodes use static routing tables, which are built using Optimized Link State Routing (OLSR) [**?**]. Each time a node generates a new packet, or receives a packet to be forwarded, it puts it in the node queue, with first-in-first-out policy. The buffer size b is fixed and equal for all nodes. If a new packet arrives when the buffer is full, it is discarded.

5.4 Definition and Estimation of the Network Performance

In this section, we define three different cost metrics that can be used as performance indicators by the two networks and we show how to compute such cost metrics starting from link parameters, which in turn can be decomposed in local PP that can be estimated, through a Bayesian approach, from observable TP. In Section 5.5 the cost metrics are used to build a game theoretic model for a careful selection of the sharing nodes and to provide an incentive for both networks to cooperate.

5.4.1 Cost metrics

We consider three different cost metrics because we do not focus on a particular network application, thus, the three cost metrics can be thought as the performance indicators of three different scenarios. Moreover, we want to remark that the approach we use is transparent to the considered performance metric, and different metrics can be easily accommodated.

Given the path from $D^{(i)}$ to $D^{(j)}$, we first define the delivery delay $\zeta^{(i,j)}$ as the average end-to-end delay of a packet sent along the path, given that the packet is received, and the packet loss probability $p_{pl}^{(i,j)}$ as the probability that a packet is lost along the path. Notice that no end-to-end packet retransmission mechanism is implemented in our network. These link parameters are taken into account by each of the three cost metrics. In fact, ignoring lost packets (i.e., computing the delay statistics only on correctly delivered packets) may lead to an optimistic evaluation of the network performance under heavy traffic, where few packets actually reach the destination. In this case, a high-loss path might end up being considered better than a more reliable path with a slightly higher delivery delay. The other extreme, i.e., defining the delay contribution of a lost packet as infinite, makes the delay evaluation meaningless. Clearly, neither option is desirable in our case. In the following, we describe the three cost metrics considered, that give a finite bias to the average delay in case of a packet loss.

Weighted delivery delay: $\mathcal{P}_{WD}$

In this metric, when a packet is lost in the path from $D^{(i)}$ to $D^{(j)}$, we increase the delay of the following packet in the same path by the time to generate another packet routed on that path[2]. This additional delay is given by $\tau = (n-1)/\lambda$, i.e., the inverse of the per-path average traffic intensity[3]. Accordingly, we recursively define the average *weighted delivery delay* of a packet sent via multi-hop transmission by node $D^{(i)}$ to node $D^{(j)}$ as:

$$\mathcal{P}_{WD}(i,j) = \left(1 - p_{pl}^{(i,j)}\right) \zeta^{(i,j)} + p_{pl}^{(i,j)} \left(\tau + \mathcal{P}_{WD}^{(i,j)}\right)$$

In this calculation, the channel and interference conditions, and thus the loss probability, are assumed to be independent for different packets. This is due to the fact that the time between two subsequent

[2]Equivalently, we assign to lost packets a delay contribution equal to the interarrival time and to received packets the actual delay incurred; then we divide the sum of all contributions by the number of correctly received packets only.

[3]Each packet generated at $D^{(i)}$ has a randomly chosen destination among the remaining nodes of the network, so that the per-node traffic λ needs to be divided by the number of possible destinations, $n-1$. Notice that it would be easy to extend our model considering different traffic intensities for different paths, however, this would lead to a more cumbersome notation without adding any relevant aspect to the final results.

packet transmissions over the same path is deemed to be long enough. From (5.1) we obtain:

$$\mathcal{P}_{WD}^{(i,j)} = \tau \frac{p_{pl}^{(i,j)}}{1 - p_{pl}^{(i,j)}} + \zeta^{(i,j)}$$

Lost or not in-time packet rate: $\mathcal{P}_{IT}$

In many applications, the packets are relevant if they are delivered within a given maximum delay, d_{max}. If a packet successfully reaches the destination after a delay longer than d_{max}, it is considered obsolete and discarded. In this scenario, to calculate a cost metric we must estimate the probability $\hat{p}_{IT}^{(i,j)}$ of in-time delivery of a packet in the path from $D^{(i)}$ to $D^{(j)}$, given that the packet is correctly received. Considering K successful transmissions, with packet delivery delay $\zeta_k^{(i,j)}$, $k = 1, \ldots, K$, we can estimate

$$\hat{p}_{IT}^{(i,j)} = \frac{\sum_{k=1}^{K} \mathbb{1}\left(\zeta_k^{(i,j)} \leq d_{\text{max}}\right)}{K}$$

where $\mathbb{1}(\cdot)$ is the indicator function. Thus, the in-time packet arrival rate is

$$\lambda_{IT} = \left(1 - p_{pl}^{(i,j)}\right) \hat{p}_{IT}^{(i,j)} \frac{\lambda}{n-1}$$

and the lost or not in-time packet rate can be written as:

$$\mathcal{P}_{IT}^{(i,j)} = \left(p_{pl}^{(i,j)} + \left(1 - p_{pl}^{(i,j)}\right)\left(1 - \hat{p}_{IT}^{(i,j)}\right)\right) \frac{\lambda}{n-1}$$

Information obsolescence: $\mathcal{P}_{IO}$

In a monitoring application, we assume that each node is tracking a specific signal and we are interested in calculating the average time interval since the last correctly received packet was generated, i.e., the average obsolescence of the information from node $D^{(i)}$ at the receiving node $D^{(j)}$. We recursively define it as:

$$\mathcal{P}_{IO}^{(i,j)} = \left(1 - p_{pl}^{(i,j)}\right)\left(\zeta^{(i,j)} + \frac{\tau}{2}\right) + p_{pl}^{(i,j)}\left(\tau + \mathcal{P}_{IO}^{(i,j)}\right)$$

where the two terms account for the obsolescence of the information in case of correctly received and lost packets, respectively. In the case of a packet correctly received, we consider that the obsolescence of the last correctly received packet linearly varies from $\zeta^{(i,j)}$ at the moment in which the packet is received, to $\zeta^{(i,j)} + \tau$, immediately before the next packet is received. Thus, the average information obsolescence is given by $\zeta^{(i,j)} + \tau/2$. In the case of a packet loss, an additional time interval τ is added to the information obsolescence every time a packet is lost.[4] Similarly to (5.1), we can write:

$$\mathcal{P}_{IO}^{(i,j)} = \tau \frac{p_{pl}^{(i,j)}}{1 - p_{pl}^{(i,j)}} + \zeta^{(i,j)} + \frac{\tau}{2} \tag{5.1}$$

Notice that the expressions of $P_{WD}^{(i,j)}$ and $P_{IO}^{(i,j)}$ are similar, however, we believe it is worth to describe both metrics because they can be applied in different scenarios. Nevertheless, in Section 5.6.3, we discuss only the results obtained considering $P_{WD}^{(i,j)}$, since with $P_{IO}^{(i,j)}$ we would obtain the same performance gains.

We define the cost metric of the whole network, $\overline{\mathcal{P}}$, as the average of a cost metric (chosen among $\mathcal{P}_{WD}$, $\mathcal{P}_{IT}$ and $\mathcal{P}_{IO}$) over all the couples of nodes belonging to the network. The aim of each network is to (selfishly) adopt a cooperation action that minimizes its cost metric $\overline{\mathcal{P}}$, this issue is addressed in Section 5.5. In the following subsections, we propose a method to decompose $\zeta^{(i,j)}$ and $p_{pl}^{(i,j)}$, needed for the computation of $\overline{\mathcal{P}}$, into local PP, and to estimate the PP from TP which can be easily observed.

5.4.2 Computation of $\zeta^{(i,j)}$ and $p_{pl}^{(i,j)}$

The delivery delay $\zeta^{(i,j)}$ is determined by the number of retransmissions in each link on the path. Indeed, for multi-hop routes, a packet has to wait at each relay node until all the packets ahead in the queue have been sent. The loss of a packet can be caused either by an excessive number of retransmissions, which lead to a packet drop, or by a buffer overflow, i.e., the packet is discarded if the next relay has a full queue. Thus, both the delivery delay $\zeta^{(i,j)}$ and the loss probability $p_{pl}^{(i,j)}$ depend on the channel and interference conditions in each link of the path, that in turn depend on the nodes that the routing protocol selects as relays.

In a static network, it is possible to estimate $\zeta^{(i,j)}$ and $p_{pl}^{(i,j)}$ during a training period, which on the other hand is impractical if the network is dynamic (mobile nodes or time-varying traffic statistics). We propose a different way of estimating the delay and the loss probability, based only on instantaneous topological and routing information. Since a packet sent over a multi-hop path has to traverse a number of nodes before reaching the destination, we decompose the overall path delivery delay and the overall path loss probability into contributions given by the various traversed nodes, and we assume that such contributions are independent. More precisely, the overall delivery delay is given by the sum of the average delays required to traverse every single node (time in queue plus transmission time), whereas the overall loss probability is obtained from the loss probabilities at every

[4]Notice that in our network scenario the packets are received at the destination node in the same order they are transmitted.

node (probability of too many transmission failures and probability of buffer overflow). If $\mathcal{R}^{(i,j)}$ is the set of nodes belonging to the path between $D^{(i)}$ and $D^{(j)}$ (excluding $D^{(i)}$ and $D^{(j)}$), we have:

$$\zeta^{(i,j)} = \zeta^{(i)} + \sum_{h \in \mathcal{R}^{(i,j)}} \zeta^{(h)}$$

where $\zeta^{(h)}$ is the average time between the arrival of a packet at node $D^{(h)}$ and its reception at the next hop. This delay depends on the next relay; indeed, while the time needed for traversing the queue is the same for all packets, the time required for a successful transmission depends on the channel condition, and hence on the next hop chosen. We consider $\zeta^{(h)}$ as averaged over all the packets sent by node $D^{(h)}$ to the next-hop relays.

The packet loss in the multi-hop path is calculated in a similar way, i.e.,

$$p_{pl}^{(i,j)} = 1 - (1 - p_{tf}^{(i)})(1 - p_{qo}^{(j)}) \prod_{h \in \mathcal{R}^{(i,j)}} (1 - p_{tf}^{(h)})(1 - p_{qo}^{(h)})$$

where $p_{tf}^{(h)}$ is the probability that a transmission from node h to the next hop fails because the maximum number of retransmissions is reached, and $p_{qo}^{(h)}$ is the probability that a packet correctly received at node $D^{(h)}$ is discarded due to buffer overflow. Furthermore, we notice that $p_{qo}^{(h)}$ depends on the queue of the receiving node $D^{(h)}$, while $p_{tf}^{(h)}$ depends also on which node is used as next hop. For this reason, similarly to $\zeta^{(h)}$, we consider a value averaged over all the neighbors of $D^{(h)}$.[5]

The parameters $\zeta^{(i)}$, $p_{tf}^{(i)}$, and $p_{qo}^{(i)}$ are the PP we need to estimate to compute the cost metric $\overline{\mathcal{P}}$ of the whole network.

5.4.3 A Bayesian network approach to infer PP from TP

We want to use some TP, that can be easily observed at each node $D^{(i)}$, to estimate the PP $\zeta^{(i)}$, $p_{tf}^{(i)}$, and $p_{qo}^{(i)}$. We decide to consider the number of neighbors $\mathcal{N}^{(i)}$ and the number of flows $\mathcal{F}^{(i)}$, that can be easily calculated from the routing table. The BN approach can be summarized in the following three steps: (1) we measure TP and PP for each node in simulations run over several training topologies, as a function of the traffic load λ; (2) we build a DAG with nodes $\mathcal{N}$, $\mathcal{F}$, ζ, p_{tf}, and p_{qo}, describing qualitatively the probabilistic relationships among them (see Subsection 5.2.1); and (3) we estimate the joint distribution according to the probability structure defined by the DAG (see Subsection 5.2.2).

[5] The underlying assumption is that the probabilities $p_{qo}^{(h)}$ and $p_{tf}^{(h)}$, with $h \in \mathcal{R}^{(i,j)}$, are all independent. This is a reasonable assumption since there are multiple flows that contribute to the queue length in each node, and the fading considered is spatially uncorrelated.

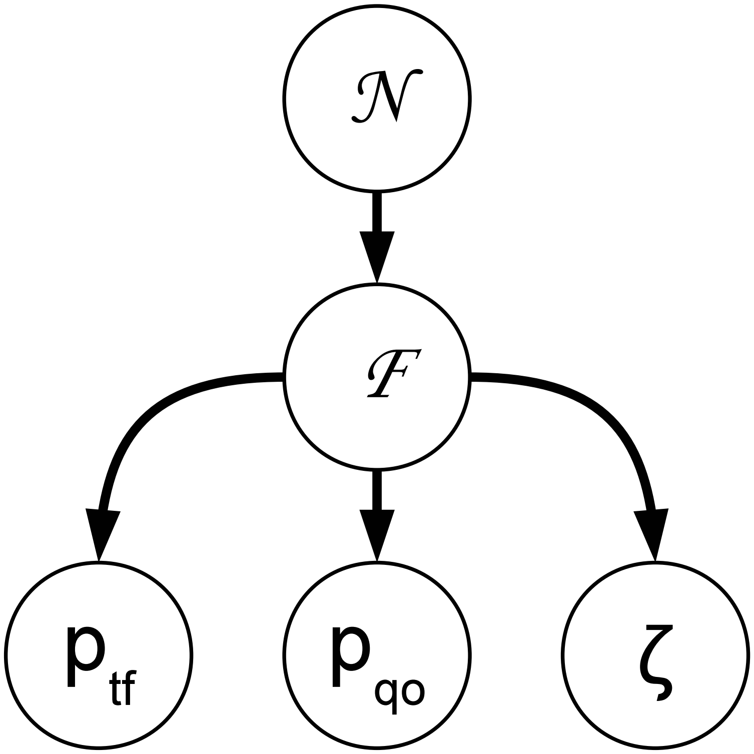


Figure 5.2. *Bayesian Network showing the probabilistic relationships among the* 5 *parameters of interest:* ζ*,* p_t*,* p_q $\mathcal{F}$*, and* $\mathcal{N}$*.*

We remark that this procedure is different from using a training period to directly derive the parameters in the scenario of interest. In fact, in this case a training period would be needed every time the topology changes, so as to evaluate their value for each specific node or path. On the contrary, with our procedure we can estimate the general joint probability distribution among these parameters, that does not depend on the specific topology.

The DAG results the same for all values of λ, and is represented in Fig. 5.2, while quantitatively the probabilistic relationships change with λ. Note that $\mathcal{N}$ does not influence, to a first approximation, the values of the three PP, once the value of $\mathcal{F}$ is observed. In other words, once we calculate from the routing table the value of $\mathcal{F}$, we can estimate the PP ζ, p_t, and p_q, and from these estimated parameters we can calculate also the overall cost for the network, $\overline{\mathcal{P}}$.

5.5 Game Theoretic Approach

In spite of the social gain allowed by cooperation, each network has to individually decide if it is beneficial for it to cooperate, and possibly to accurately select the set of nodes to share. To do this, each network can estimate, through the framework introduced in Section 5.4, the performance obtainable for each cooperation possibility, and *play a game* with the other network.

For the time being we model the interaction of the two networks as a *static game with complete information*. We label the nodes of the networks from 1 to $2n$, where the nodes in the sets $S_1 = \{1, ..., n\}$ and $S_2 = \{n+1, ..., 2n\}$ belong to network 1 and 2, respectively. We formally define the game $\Gamma = (N, A_1, A_2, U_1, A_2)$, in which the players are the two networks, $N = \{1, 2\}$, and the actions $a_1 \in A_1$ and $a_2 \in A_2$ represent the set of nodes network 1 and 2 want to share. In general, an operator may not be willing to share too many nodes or some important nodes (e.g., for security or privacy reasons), thus the action sets A_1 and A_2 are a subset of the power set of S_k, $A_k \subseteq 2^{S_k}$, $k = 1, 2$. The utility $U_k : A_1 \times A_2 \to \Re$ can be any decreasing function of $\overline{\mathcal{P}^k}(a_1, a_2)$, $k = 1, 2$, which denotes the cost metric referred to network k given the shared nodes a_1 and a_2. Given a_1 and a_2 the routing tables calculated via OLSR change accordingly, the number of flows for each node can be computed, $\overline{\mathcal{P}^k}(a_1, a_2)$ can be estimated, and finally the utility $U_k(a_1, a_2)$ can be obtained. In particular, $U_k(\emptyset, \emptyset)$ is the utility of network k when no nodes are shared.

We say that an action a_k is *non trivial* is the shared nodes are exploited by the other network to obtain more efficient paths. Except for the *no cooperation action* $a_k = \emptyset$, we consider only non trivial actions. In fact, a trivial action is perfectly equivalent to the no cooperation action $\emptyset$.

Proposition 3. $a_k = \emptyset$ *is a strictly dominant action of the game* Γ*, for each network* $k = 1, 2$.

Proof. Given the strategy of the other network, network k strictly prefers not to share any node. In fact, shared nodes strictly increase the traffic handled by the network, which in turns strictly increases the cost metric and strictly decreases the utility, with respect to the no cooperation case. □

Corollary 2. *The unique NE of the game* Γ *is* $a_1^{NE} = a_2^{NE} = \emptyset$.

In the static game formulation it is not possible to provide incentives for the networks to cooperate because, whatever the other network decides to do, a network never wants to manage additional flows of packets belonging to the other network. However, we argue that the static formulation is not a proper model for the scenario we have in mind, in which the interaction among the networks is sustained over the time. In this case, a *repeated game* formulation seems more reasonable.

5.5.1 Repeated game

We define the infinitely repeated game Γ^R in which the two networks play the stage game Γ infinitely, obtaining the average utility

$$U_k^R = (1-\delta) \lim_{T \to +\infty} \sum_{t=1}^{T} \delta^{t-1} U_k^{(t)}$$

where $U_k^{(t)}$ is the utility obtain by network i at stage t and $\delta \in (0,1)$ is the discount factor.

We want to design a *cooperation strategy profile* $s^* = (s_1^*, s_2^*)$ in which both network have the incentive to cooperate. The key idea is the adoption of a *trigger strategy* in which the two networks adopt by default the *cooperation action profile* $a^* = (a_1^*, a_2^*)$ and, as soon as one of the two networks deviates from this action profile, the other network *punishes* it by adopting the no cooperation action $\emptyset$ forever.[6] An issue related to this approach is the selection of an appropriate a^*. In fact, the two networks have in general different preferences: each of them would like to choose an a^* that allows it to obtain the highest gain, that does not usually coincide with the a^* in which the other network obtains the highest gain. Inspired by the Nash bargaining solution [49], we select a cooperation action profile a^* as a solution of the following problem

$$\begin{aligned} &\operatorname*{argmax}_{a \in A} \left(U_1(a) - U_1^{(}\emptyset,\emptyset) \right) \left(U_2(a) - U_2(\emptyset,\emptyset) \right) \\ &\text{subject to:} \\ &U_k(a) - U_k(\emptyset,\emptyset) > 0, \ k = 1,2 \end{aligned} \tag{5.2}$$

This corresponds to the solution that an impartial arbitrator would recommend to increase in a fair way the utilities of both networks. We obtain the following results.

Proposition 4. *If (5.2) has no solution, there exist no cooperation action profile $a^* \neq (\emptyset,\emptyset)$ and trigger strategy s^* such that s^* is a Nash equilibrium of Γ^R.*

Proof. Let assume (5.2) has no solution and there exists $a^* \neq (\emptyset,\emptyset)$ and a trigger strategy s^* such that s^* is a NE of Γ^R. Since (5.2) has no solution, there exist a network k such that $U_k(a^*) - U_k(\emptyset,\emptyset) \leq 0$. Without loosing generality we assume that $k = 1$. If both networks adopt the trigger strategy the

[6] More complex strategies in which the networks synchronously change, from stage to stage, the cooperative action profile are possible. Though these strategies may achieve better theoretical results, we argue that they are very complex and computationally expensive to implement in practice, since they require frequently updates of the routing tables and introduce the problem of readdressing packets that were transmitted along paths which do not exists anymore. Thus, we prefer to consider more simple strategies.

average utility for network 1 is $U_1^R(s_1^*, s_2^*) = U_1(a^*)$. If network 2 adopts the trigger strategy and network 1 always adopts the no cooperation action $\emptyset$ the average utility for network 1 is

$$U_1^R(\emptyset, s_2^*) = (1-\delta)U_1(\emptyset, a_2^*) + (1-\delta)\lim_{T\to+\infty}\sum_{t=2}^{T}\delta^{t-1}U_1(\emptyset,\emptyset) > U_1(\emptyset,\emptyset) \geq U_1^R(s_1^*, s_2^*)$$

where the first inequality, i.e., $U_1(\emptyset, a_2^*) > U_1(\emptyset,\emptyset)$, is valid because network 1 can exploit the node shared by network 2 to find better paths (remember that we consider non-trivial actions), and the last inequality is valid for hypothesis. Hence, network 1 has the incentive to deviate from the trigger strategy s_1^* and adopt always the no cooperation action $\emptyset$, contradicting the initial hypothesis that s^* is a NE of Γ^R. □

Since a Subgame-Perfect Equilibrium (SPE) is a refinement of a NE, if (5.2) has no solution neither a trigger strategy SPE exits. In this case we assume that the networks never cooperate ($(\emptyset,\emptyset)$ is a NE of the stage game Γ, hence it is also a SPE of Γ^R). Notice that (5.2) is without solution if it does not exist an action profile $a \neq (\emptyset,\emptyset)$ such that both network can benefit from cooperation. This possibility happens very rarely (precisely, when for each sharing choice one network would not exploit a lot the shared nodes of the other network and, at the same time, the other network would exploit a lot its shared nodes), and corresponds to situations in which cooperation does not provide a high gain.

Proposition 5. *If a^* is a solution of (5.2) and δ is close enough to 1, then the trigger strategy s^* is a subgame-perfect equilibrium of Γ^R.*

Proof. We need to show that the strategy s_1^* is a best response to the strategy s_2^*, in each subgame of Γ^R (if so, for symmetry s_2^* will be a best response to s_1^*). Assume network 2 adopts s_2^*. Network 1 knows that, if the outcome ever differs from (a_1^*, a_2^*), network 2 will play $\emptyset$ forever. Thus, from that point on, also for network 1 is optimal to play $\emptyset$ forever. So s_1^* is a NE in all the subgames of Γ^R in which a deviation from (a_1^*, a_2^*) has occurred in the past. Now let consider the subgame of Γ^R in a generic stage $\bar{t}$ in which a deviation has not occurred in the past (this includes also the case $\bar{t} = 1$, i.e., the subgame coincides with Γ^R). We just need to show that, at stage $\bar{t}$, it is not beneficial for network 1 to deviate from the trigger strategy s_1^*, playing an action different from a_1^*. In fact, if it does not deviate in stage $\bar{t}$, then for the same reason it will not deviate in stage $\bar{t}+1$, and so on. The past utility and the discount factor at the instant $\bar{t}$, $\delta^{\bar{t}-1}$, are constants and do not play any role in the equilibrium analysis. Hence, we can simply impose $\bar{t} = 1$ and evaluate network 1's best first move. If network 1 adopts s_1^* from the initial stage its average utility is $U_1^R(s_1^*, s_2^*) = U_1(a^*)$. Every strategy resulting in an action different from a_1^* in the first stage is dominated by the strategy in which $\emptyset$ is always played, in fact $\emptyset$ allows to obtain the highest utility possible in every stage given that user 2 plays a_2^* in the first stage and $\emptyset$ in the subsequent stages. If network 1 always play $\emptyset$ achieves an average utility of $U_1^R(\emptyset, s_2^*) = (1-\delta)\, U_1(\emptyset, a_2^*) + \delta\, U_1(\emptyset, \emptyset)$. We obtain $U_1^R(s_1^*, s_2^*) \geq U_1^R(\emptyset, s_2^*)$ if and only if $\delta \geq \dfrac{U_1(\emptyset, a_2^*) - U_1(a^*)}{U_1(\emptyset, a_2^*) - U_1(\emptyset, \emptyset)}$. Notice that $U_1(\emptyset, a_2^*) > U_1(a^*) > U_1(\emptyset, \emptyset)$, where the first inequality is valid because network 1 can exploit the node shared by network 2 to find better paths (remember that we consider non-trivial actions), and the last inequality is valid because a^* is a solution of (5.2). Thus, the threshold on δ is lower than 1. □

Notice that the trigger strategy can be substituted with a strategy in which, as soon as a deviation from the cooperation action profile is detected, a network adopts the punishment action $\emptyset$ only for a finite amount of stages. The duration of the punishment must be set so that the gain obtained during the deviating stage does not compensate the loss incurred during the subsequent stages.

5.6 Results

5.6.1 Simulation setup

To assess the effectiveness of our approach, we developed a network simulator which encompasses the layers from physical to routing, as described in Section 5.3. The system parameters are reported in Table 5.1. Each simulation run is performed with randomly generated connected networks, and lasts for 10000 time slots. With the given parameters setup, we first identified, through simulation, the value λ_t of packet generation intensity which results in an end–to–end packet loss probability of 0.1. This can be seen as a threshold value between a lightly loaded and an overloaded network. Different values of the normalized traffic generation intensity $\lambda_n = \lambda/\lambda_t$ were considered, from $\lambda_n = 0.4$ up to $\lambda_n = 2$. For each value, 500 simulation runs were performed to collect the data required for the BN inference (training topologies). Based on this information, the empirical distributions and the average values of ζ_q, p_{tf} and p_{qo}, conditioned on $\mathcal{F}$, were derived.

In the subsequent steps, a new set of 500 simulation runs was performed for each value of λ_n. In each run, two networks are again randomly deployed. We investigate the average performance of the networks when (1) no nodes are shared, namely *No Coop*; (2) 2 nodes randomly chosen are shared, namely *2 Rand*; (3) 2 nodes selected through the proposed game theoretic approach are shared, namely *2 GT*; (4) all nodes are shared, namely *Full Coop*. To adopt the game theoretic approach we assume that the utility function of each network is the reciprocal of the average cost for that network,

Table 5.1. *Simulation parameters*

Number of nodes per network	10
Network area side [m]	200
Transmission range, r[m]	75
Transmission power [dBm]	24
Chip rate [chip/s]	7.5×10^6
Noise floor [dBm]	-103
Path loss exponent	4
Path loss fixed term	1000
Fading correlation factor, ρ	0.9
Modulation type	BPSK
Time slot duration [ms]	1
Spreading factor S_f	32
Packet length [bit]	4096
Packet transmission time [slots]	6
Transmission rate, λ [pkts/s/node]	1 to 5
Buffer size b [pkts]	16
Maximum number of MAC retransmissions	5
Initial backoff window [slots]	16
Routing algorithm	OLSR
Simulation duration [slots]	10000

$U_k(a_1, a_2) = [\overline{\mathcal{P}^k}(a_1, a_2)]^{-1}$, δ is close enough to 1, and the networks can share either no nodes or exactly 2 nodes. Although our approach can be extended to a larger number of cooperating nodes, our results show that a large fraction of the available cooperation gain is already achieved with this choice.

5.6.2 Bayesian network estimation

Exploiting the stochastic estimation of local parameters we can evaluate the expected value of the three parameters of interest, namely the average delivery delay ζ_q, the probability of buffer overflow p_{qo} and the probability of transmission failure p_{tf}, as a function of the number of flows $\mathcal{F}$ passing through the node and of the normalized traffic intensity λ_n. The expected values of ζ_q, p_{qo}, and p_{tf} are shown in Figs. 5.3, 5.4 and 5.5, respectively. The highest number of flows through a single node is reached when that node becomes the only connection among five separate clusters of nodes.[7] If these groups have similar cardinalities, we have that the maximum number of flows through a single node is

$$N_f < 2(n-1)\left(n - \frac{n-1-N_c}{N_c}\right)$$

where N_c is the number of clusters in which the two networks are divided, and n is the number of nodes in each network. In our case, due to the small number of nodes in each network ($n = 10$), we reasonably assume that in the worst case the nodes can be divided in three separate clusters of nodes, thus $N_f < 144$. This explains why the number of flows $\mathcal{F}$ is limited in the figures. We also observe in Fig. 5.3 that for very high values of $\mathcal{F}$ and λ_n, the average delivery delay decreases. We conjecture that this happens for two reasons: (1) the queue of these nodes are always almost full, so the time to traverse them cannot grow much further, and (2) a node traversed by a high number of flows is often chosen as receiver by most of its neighbors. For these reasons, when it transmits, a lower number of communications can interfere, thus leading to a lower time needed to deliver a packet to the next hop.

5.6.3 Performance

In Fig. 5.6, we present the actual gain, in terms of delay reduction for the metric $\mathcal{P}_{WD}$, offered by the considered scenarios. The curves are obtained by averaging over 500 random topologies, each consisting of two networks with $n = 10$ nodes each. The system parameters are reported in Tab. 5.1.

[7] A single node can be the only connecting node of no more than 5 clusters of nodes, since in a plane it is impossible to have more than 5 points with distance less than or equal to r from a central point, such that each couple of points have a distance bigger than r.

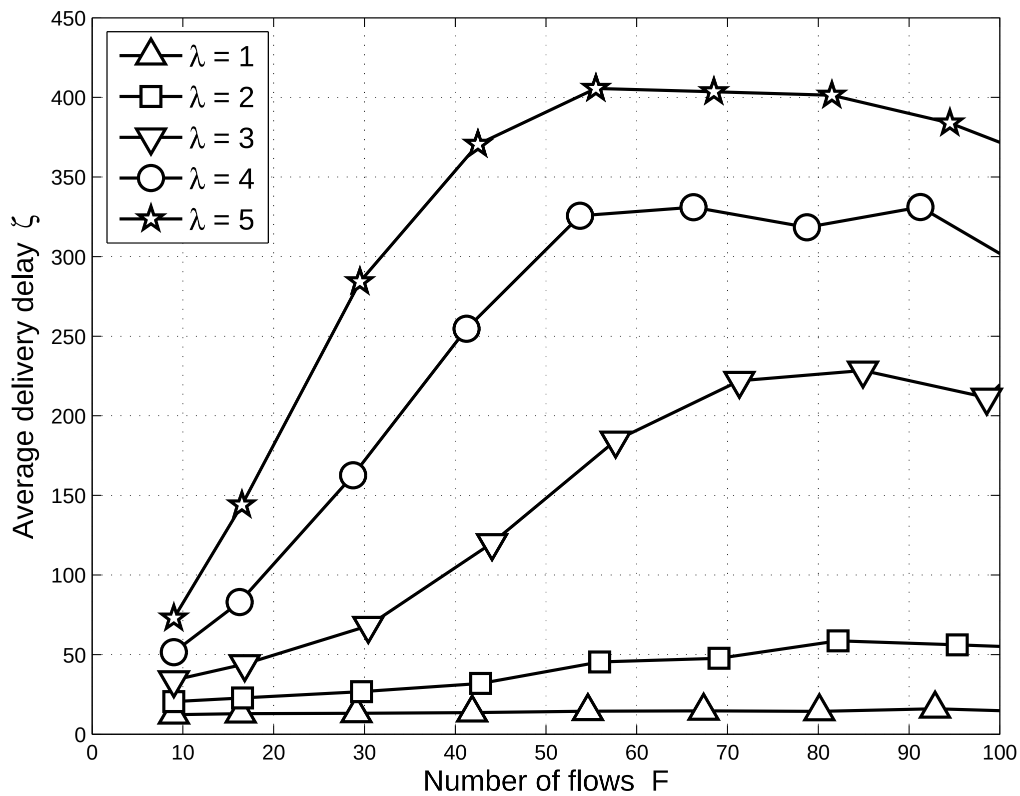


Figure 5.3. *BN estimation of the average delivery delay ζ as a function of the number of flows $\mathcal{F}$ passing through the node.*

It can be observed that, as intuition suggests, full cooperation grants the highest benefits, due to the higher spatial diversity. Hence, this is the maximum achievable gain for the scenario investigated. This gain is more pronounced when the networks are heavily loaded, since congested paths are more frequent, and adding new routes becomes more advantageous. When only two nodes can be shared, the choice of the shared nodes makes the difference. In fact, Fig. 5.6 shows that a careful selection of the resources to be shared can significantly increase the achievable gain when compared to a blind random selection. A random selection can not offer a significant gain for lightly loaded networks, while, for heavily loaded networks, it can offer only one third of the gain granted by full cooperation. On the contrary, if the shared nodes are chosen by means of our game-theoretic approach, the maximum achievable gain is fully obtained for lightly loaded networks and closely approached for heavily loaded networks.

The same performance gains are obtained also by using the cost metric $\mathcal{P}_{IO}$, the information

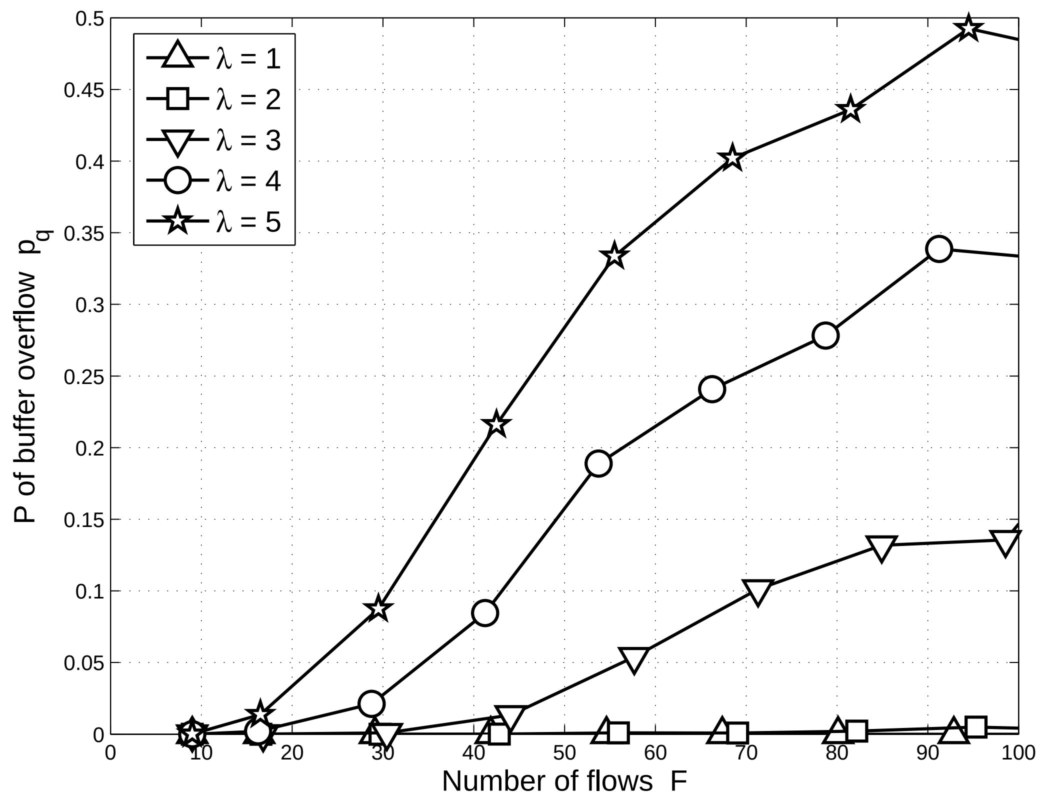


Figure 5.4. *BN estimation of the probability of buffer overflow* p_{qo} *as a function of the number of flows* $\mathcal{F}$ *passing through the node.*

obsolescence, since $\mathcal{P}_{WD}$ and $\mathcal{P}_{IO}$ differ only by a constant term $\tau/2$, see (5.1) and (5.1).

In Fig. 5.7 and in Fig. 5.8 we adopt the cost metric $\mathcal{P}_{IT}$ and we show the performance of the four cooperation strategies in terms of in-time packet arrival rate, λ_{IT}. In Fig. 5.7-(a) we show λ_{IT} as a function of the normalized packet generation intensity λ_n for a maximum allowed delay $d_{\max} = 100$ slots, and in Fig. 5.7-(b) we show λ_{IT} for $d_{\max} = 600$ slots. We notice that also in this case, adopting the cost metric $\mathcal{P}_{IT}$, an accurate choice of the cooperating nodes made by our cooperation strategy, 2 GT, allows to reach the same performance of the case in which all nodes are shared, namely Full Coop. Instead, the random choice of the nodes to share, 2 Rand, provide only a third or less of the total gain achievable with full cooperation.

In Fig. 5.8, adopting again the cost metric $\mathcal{P}_{IT}$, we show λ_{IT} as a function of the maximum allowed delay $d_{\max}$ for a packet generation intensity $\lambda_n = 1.2$ and $\lambda_n = 2$, in Fig. 5.8-(a) and in Fig. 5.8-(b), respectively. We observe that varying the maximum allowed delay $d_{\max}$ with our

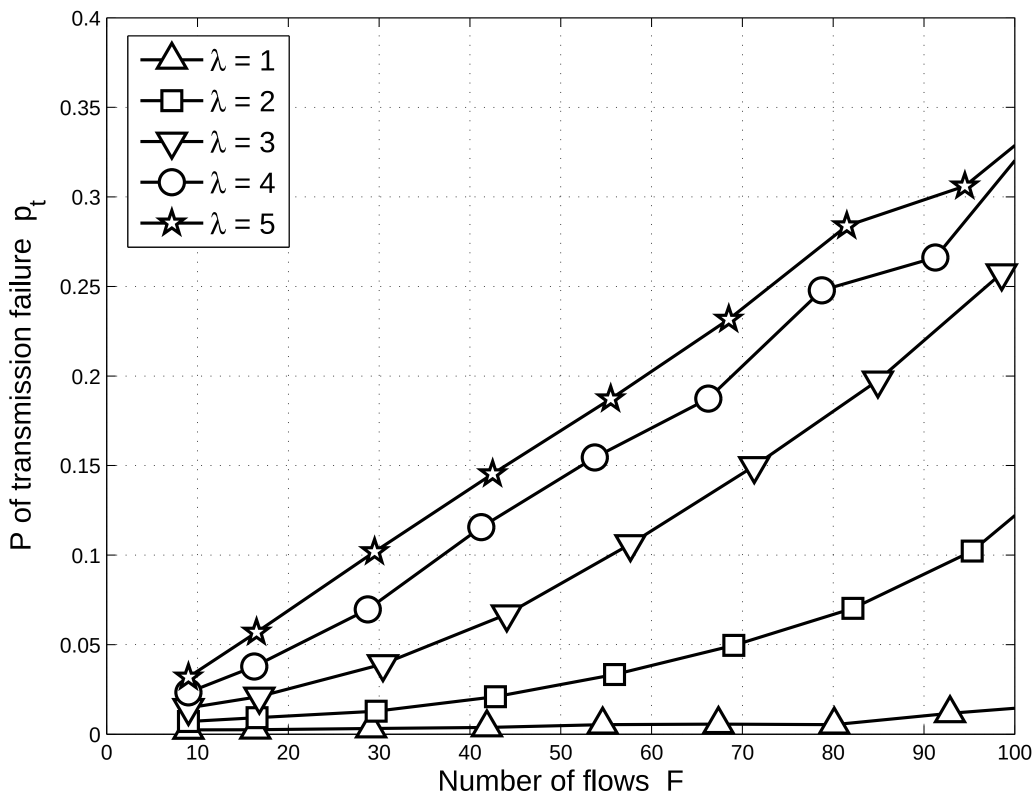


Figure 5.5. *BN estimation of the probability of transmission failure p_{tf} as a function of the number of flows $\mathcal{F}$ passing through the node.*

cooperation strategy we obtain the same gain as with full cooperation, while with a random choice of the cooperative nodes we obtain less than a third of the total gain achievable with full cooperation.

5.7 Conclusions

In this chapter we develop a framework which can be used to select the cooperation strategy between two coexisting wireless networks sharing some of their nodes. To sum up, our framework is represented in Fig. 5.1 and follows the following steps:

(1) we learn the network behavior by measuring the TP and PP of interest over several random training topologies;

(2) we use the BN method to infer the joint distribution among TP and PP;

(3) in the scenario of interest we observe the TP, we infer the PP, and we estimate the utility function

of each network for all possible choices of the sharing nodes;

(4) we select the nodes to be shared based on a game theoretic approach in which each network shares the nodes only if it obtains a benefit in doing that, incentives towards cooperation are provided through a simple trigger strategy which takes into account the actions adopts by the another network in the past.

Finally, we develop a wireless network simulator showing that, even when only a small fraction of the nodes is shared, we obtain a significant gain. In particular, both for lightly and heavily loaded scenarios, the selection scheme based on game theory can achieve almost the same performance as a full cooperation scheme, for all the three performance metrics considered.

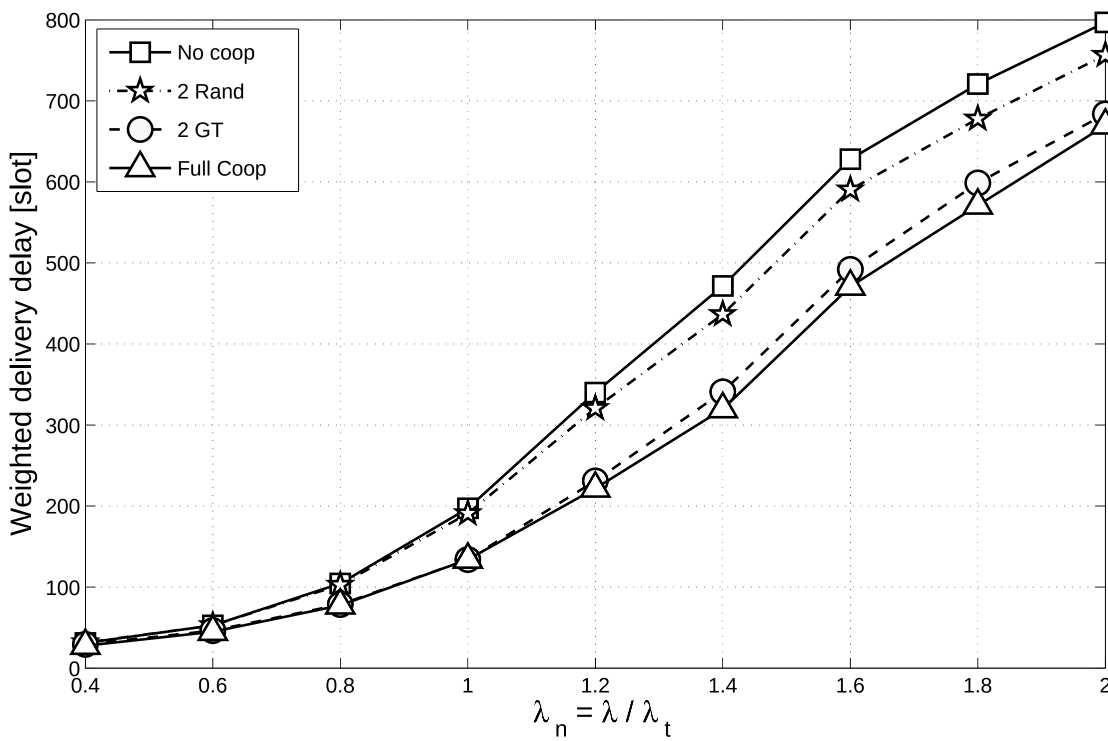
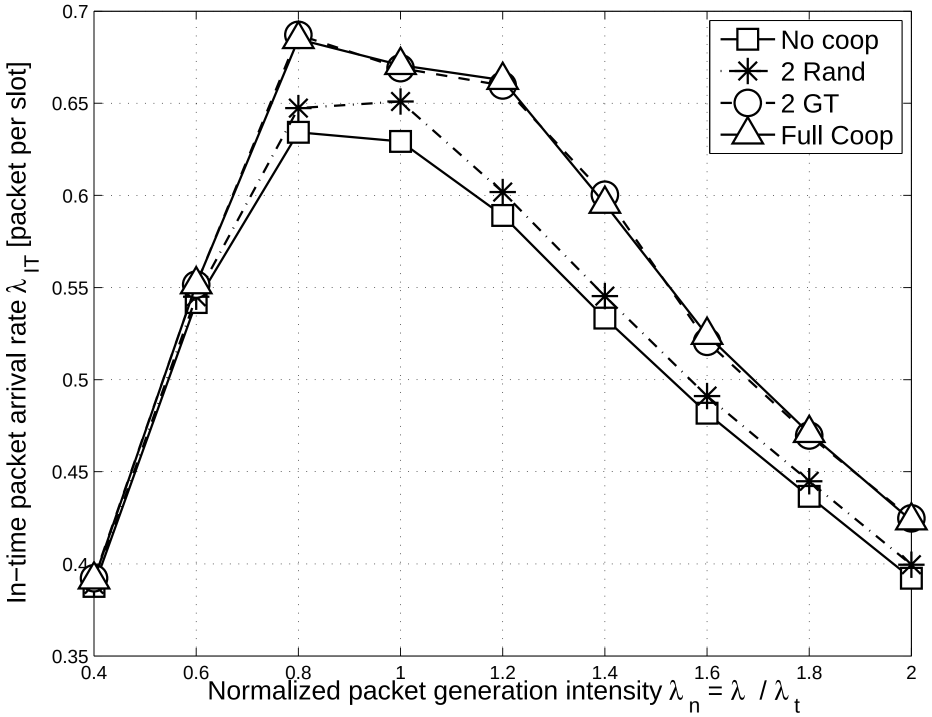
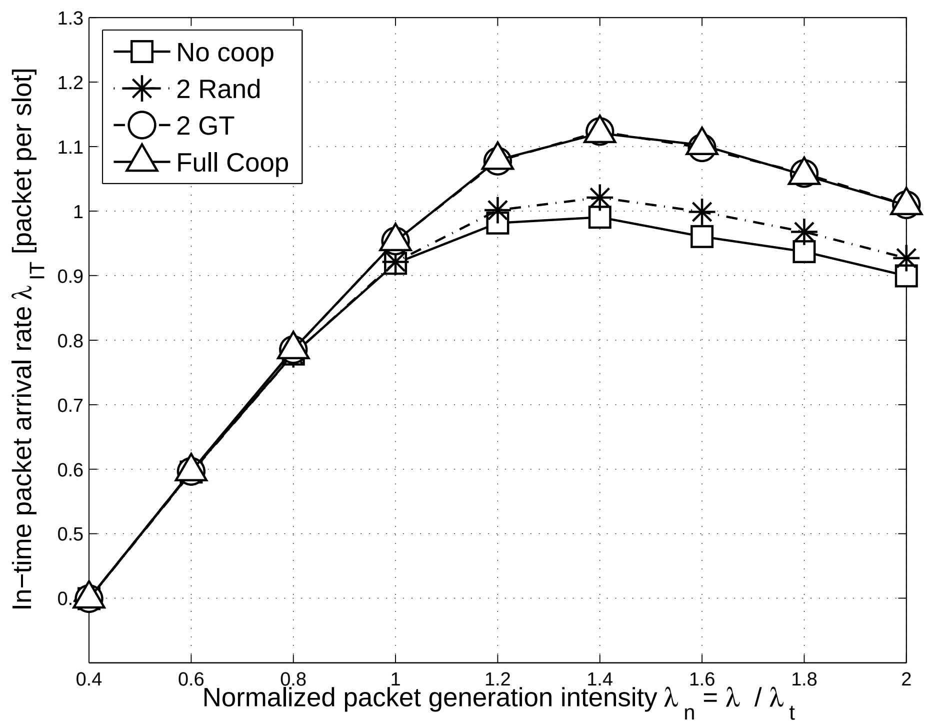


Figure 5.6. *Weighted delivery delay $\mathcal{P}_{WD}$ as a function of the normalized packet generation intensity $\lambda_n = \lambda/\lambda_t$, for the four compared scenarios.*

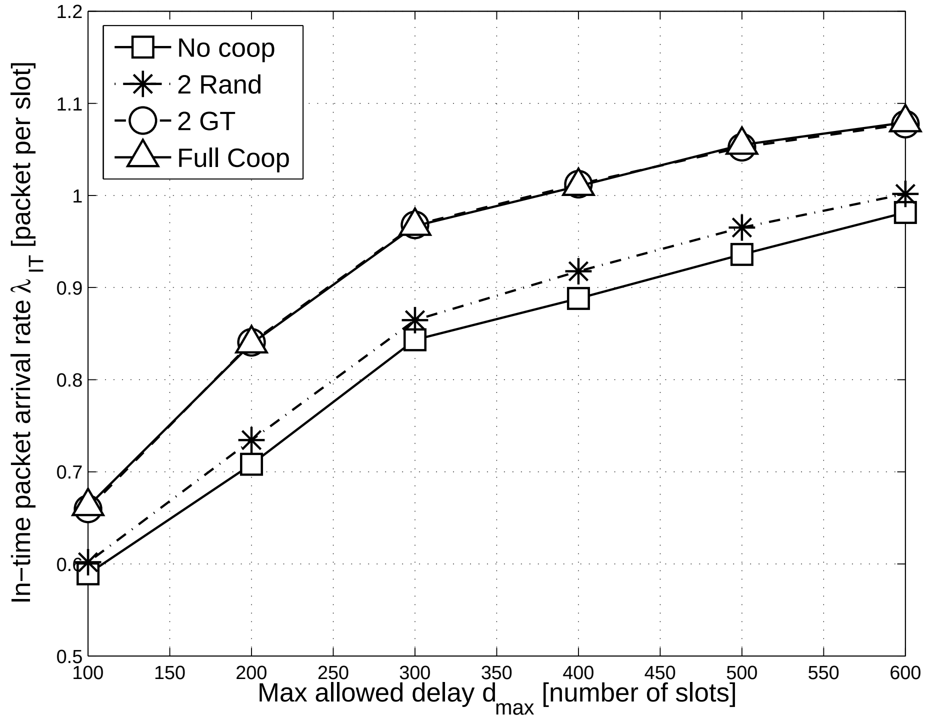


(a)

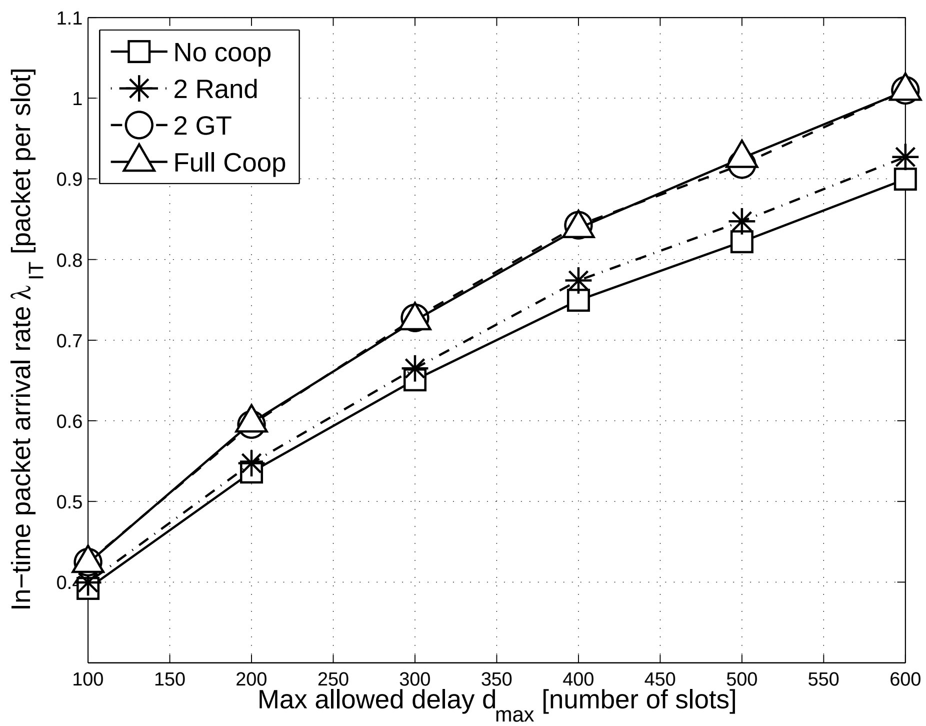


(b)

Figure 5.7. *In-time packet arrival rate λ_{IT} as a function of the normalized packet generation intensity λ_n for a value of the maximum allowed delay (a) $d_{max} = 100$ and (b) $d_{max} = 600$, in number of slots.*



(a)



(b)

Figure 5.8. *In-time packet arrival rate λ_{IT} as a function of the maximum allowed delay d_{max} for a normalized packet generation intensity (a) $\lambda_n = 1.2$ and (b) $\lambda_n = 2$.*

Chapter 6

Designing and Selecting MAC Protocols With Selfish Users

In this chapter[1] we consider a number of users who compete to gain access to a channel, a slotted-Aloha like random access protocol and two incentive schemes: *pricing* and *intervention*. We provide some criteria for the designer of the protocol to choose one scheme between them and to design the best policy for the selected scheme, depending on the system parameters. Our results show that intervention can achieve the maximum efficiency in the *perfect monitoring* scenario. In the *imperfect monitoring* scenario, instead, the performance of the system depends on the information held by the different entities and, in some cases, there exists a threshold for the number of users such that, for a number of users lower than the threshold, intervention outperforms pricing, whereas, for a number of users higher than the threshold pricing outperforms intervention.

6.1 Introduction

In wireless communication networks, multiple users often share a common channel and contend for access. Many distributed Medium Access Control (MAC) protocols, some of them being used in current international standards (e.g., IEEE 802.11 a/b/g/n), have been designed assuming that users are compliant with the protocol rules. Unfortunately, a self-interested and strategic user might ma-

[1]The material presented in this chapter has been published in:

[J3] **L. Canzian**, Y. Xiao, M. Zorzi, and M. van der Schaar, "Game Theoretic Design of MAC Protocols: Pricing and Intervention in Slotted-Aloha," *Submitted to IEEE/ACM Trans. Networking* nipulate the protocol in order to obtain a larger share of the channel resource at the expense of that of the other users.

We consider a slotted-Aloha like random access protocol, where each user transmits within a slot according to some user-chosen probability. Without any further mechanism, self-interested users would implement the *always transmit* strategy, resulting in the network collapse. To make the network robust to selfish users, it is fundamental to design a scheme that provides to the users the incentives to adopt a better (from the network designer point of view) strategy.

In the past decade a lot of research was dedicated to the development of such incentive schemes for slotted-Aloha like random access protocols. Some of this research, such as [35–39], adopts pricing schemes that charge the users for their resource usage.[2] In this way, it is in the self-interest of each user to limit its access probability. Such pricing schemes may achieve the goal of efficient use of network resources. However, they suffer from the following drawbacks: (1) the designer has to know how the prices affect the users' utilities to design an efficient scheme; (2) it is not clear what do to with the collected money, unless the network is managed by a profit-making enterprise; (3) a secure infrastructure to collect the money is needed.

Recently, a new incentive scheme, called *intervention*, has been proposed in [66] and has been applied to MAC problems [12, 42]. In this scheme, an *intervention device* is placed in the network. Such a device can monitor the users' behavior and intervene affecting the users' resource usage. The action of the intervention device depends on the actions of the users. The intervention device provides the incentives for the users to obey a given access probability rule by threatening *punishments* if users disobey. Intervention is more robust than pricing because users cannot avoid intervention as long as they use network resources, but they might be able to avoid monetary charges. The implementation of an intervention scheme requires to place an additional device, i.e., the intervention device, in the network.

Repeated games can also encourage cooperative behaviors [90]. In this case users are forced to take into account how their current actions can influence the future actions of the other users. A cooperative behavior is induced by punishing deviating users in the future. Differently from the previously considered methods, this scheme does not require the presence of a central entity. However, it requires a repeated interaction among users and the users must keep track of their past observations

[2] Notice that in the literature pricing schemes may refer also to distributed schemes in which the users are cooperative and *fictitious* prices are used to obtain an efficient distributed algorithm. In our case, we consider strategic and selfish users, thus, to be effective, the pricing scheme requires the users to pay real money.

and be able to detect deviations and to coordinate their actions in order to punish deviating users. We exclude incentive schemes based on repeated games because of these difficulties.

In this chapter we provide the tools to design pricing and intervention schemes to make a random access protocol robust against strategic users. As in most of the previous works in pricing and intervention, we consider only *linear intervention* and *linear pricing* schemes, because they are simple to implement and yet efficient enough to achieve high performance (or even optimality in some cases). Simple rules are important in particular for pricing schemes, because the users might not accept to pay for their resource usage following complex rules. It is difficult to argue between different incentive schemes in general: depending on the particular deployment scenario, the performance criterion, and the implementation issues, each one of the incentive schemes can be better than the others. The problem of the network designer is to identify the scheme that best fits its requirements and to design the best policy for the selected scheme.

The complexity of the design process and the performance achievable depend on various features of the system, such as the number of users, the users' heterogeneity, the capability of monitoring the users' actions and the information held by the designer and the users. To the best of our knowledge, this is the first work that compares intervention and pricing in terms of the network environment, the knowledge of the designer and the knowledge of the users. We focus on a simple MAC protocol, slotted-Aloha, because it makes it possible to formulate a simple game in which the outcomes can be computed analytically, to highlight the consequence of not taking into account the strategic nature of some users when designing a MAC protocol, and to obtain important insights about possible solutions to such a problem. For these features slotted-Aloha is widely used in game theoretic studies [8, 12, 35–39, 42]. The extension of this work to more realistic MAC protocols will be considered in future works.

This chapter is divided into two main parts. In the first part, we consider the *perfect monitoring* scenario, i.e., we assume that the users' actions are estimated without errors. We show that intervention can achieve the maximum efficiency, i.e., the maximum social welfare, while pricing is able to reach an efficient use of the network resources but the positive payments subtracted from the users' utilities prevent it to achieve the maximum social welfare

In the second part, we consider an *imperfect monitoring* scenario, assuming that a uniformly distributed noise term is added to the estimated actions. We derive the optimal pricing and intervention schemes and quantify the performance achievable in this scenario, assuming that (1) neither the designer nor the users are aware of the estimation errors (i.e., they believe that the designer is able to observe the users' actions perfectly), (2) only the designer is aware of the estimation errors, and

(3) both the designer and the users are aware of the estimation errors. In the imperfect monitoring scenario, the performance of the intervention scheme degrades considerably as the number of users increases and the information held by the designer and the users plays an important role. In particular, for case (3) there exists a threshold for the number of users such that, for a number of users lower than the threshold, the intervention scheme outperforms the pricing scheme, while for a number of users higher than the threshold the pricing scheme outperforms the intervention scheme. In the other cases intervention allows to obtain higher performance than pricing. The analysis in this chapter can serve as a guideline for a designer of a MAC protocol to select between pricing and intervention and to design the best policy for the selected scheme, depending on some system parameters such as the number of users, the statistics of the monitoring noise and the information held by the designer and the users.

Despite its practical importance, very few works address the impact of the monitoring errors and the information heterogeneity on the design and performance of an incentive scheme. To the best of our knowledge, no prior work on pricing considers the issue of imperfect monitoring on users' actions. As to the intervention scheme, both [12] and [42] consider the imperfect monitoring scenario. [12] adopts the same noise model we use, but it simplifies the analysis limiting the users' action space, whereas [42] considers a different type of imperfect monitoring, whose distribution depends on the length of the time the intervention device takes to estimate users' actions. However, in both works it is assumed that the designer and the users are aware of the imperfect monitoring model. In our work we analyze the effect of the information heterogeneity, considering also the cases in which nobody is aware of the estimation errors and in which only the designer is aware of the estimation errors. This provides understanding on how robust the considered incentive schemes are with respect to the heterogeneity of information.

The remainder of this chapter is organized as follows. In Section 6.2 we describe the considered MAC protocol. We introduce the games that model the interaction between strategic users and we formulate the problem of designing efficient incentive schemes in Section 6.3. In Section 6.4 we derive the optimal pricing and intervention schemes to adopt in the perfect monitoring scenario and we quantify the performance achievable. We consider the imperfect monitoring scenario in Section 6.5, for three different cases, depending on who is aware of the imperfect monitoring model. Section 6.6 concludes with some remarks.

6.2 System Model

We consider a wireless network of n users that share a common channel and we make the following assumptions for the contention model:

- Time is slotted and slots are synchronized;
- Users always have packets to transmit in every slot;
- If a packet is received, the receiver immediately sends an acknowledgment (ACK) packet;
- The transmission of a packet and the (possible) corresponding ACK is completed within a slot;
- A packet is received successfully if and only if it does not collide with other transmissions;
- Each user i selects a transmission probability $a_i \in [0\,1]$ at the beginning of the communication and will transmit with the same probability a_i in every time slot, i.e., there are no adjustments in the transmission probabilities. This excludes coordination among users, for example, using time division multiplexing.

Notice that ACK packets are always successfully received because they are transmitted over idle channels.

Denoting with $a = (a_1, \ldots, a_n)$ the transmission probability vector, the average throughput (in packets per slot) of user i is given by

$$T_i(a) = a_i \prod_{j=1, j \neq i}^{n} (1 - a_j)$$

The resource usage of user i is therefore proportional to i's transmission probability.

We assume that the utility of user i is given by

$$U_i(a) = \theta_i \ln T_i(a) \tag{6.1}$$

where the parameter $\theta_i > 0$ allows to differentiate between different classes of users. The higher θ_i, the higher user i's valuation for the throughput. The logarithm makes the utility a concave function, which models the fact that the users usually have more desire to increase their own throughput when it is low than when it is high.

We define the social welfare of the network as the sum of all users' utilities:

$$U(a) = \sum_{i=1}^{n} U_i(a) \tag{6.2}$$

Finally, the network is said to operate optimally if the users choose the transmission probabilities that maximize Eq. (6.2). It is straightforward to check that the Hessian of $U(a)$ is a diagonal matrix with strictly negative diagonal entries, therefore it is negative definite. Imposing the partial derivatives equal to 0, the unique transmission probability vector $a^* = (a_1^*, \dots, a_n^*)$ that maximizes Eq. (6.2) is given by

$$a_k^* = \frac{\theta_k}{\sum_{i=1}^n \theta_i} \quad , \quad k = 1, \dots, n \tag{6.3}$$

$U(a^*)$ represents the maximum social welfare achievable.

In order to adopt the optimal transmission probability, the users need to know the sum of the valuations θ_i of the other users. This information must be spread in the network at the beginning of the communication. This can be done either in a distributed way or in a centralized way. In particular, in the last case an entity (e.g., a predetermined user or the access point) might collect the users' valuations and broadcast to all users the value $\sum_{i=1}^n \theta_i$. Once the users have this information, they can locally compute their optimal transmission probabilities according to Eq. (6.3) and adopt them.

6.3 Game Model and Design Problem Formulation

While the network optimal transmission policy a^* is easy to compute, the actual transmission probability selected by each user depends on the objective of that user. If the users are compliant with the optimal policy, then they compute and adopt a^* and the network operates optimally. However, if the users are self-interested and strategic, instead of complying with the optimal policy they will adopt the transmission probabilities that optimize their own utility. Since the interests of individual users are different from the interests of the group of users as a whole, the network might (and usually will) operate inefficiently.

To analyze the interaction between strategic decision-makers, we define the contention game

$$\Gamma = (N, A, \{U_i(\cdot)\}_{i=1}^n)$$

where $N = \{1, 2, \cdots, n\}$ denotes the set of users, $A = \times_{i=1}^n [0, 1]^n$ denotes the action space and $U_i : A \rightarrow \Re$ is the utility of a generic user i, defined by Eq. (6.1). The action for user i represents the transmission probability a_i chosen by user i. Throughout the chapter, we will use the terms action and transmission probability interchangeably, and similarly for action profile and transmission probability vector.

The NEs of the contention game Γ can be easily characterized considering the following cases.

1) Assume that all user, except for user i, adopt a transmission probability strictly smaller than 1. Then the utility of user i is increasing in a_i: the higher the transmission probability chosen by i the higher i's throughput. Thus, i chooses $a_i = 1$.

2) Assume that there is at least a user $j \neq i$ that adopts a transmission probability equal to 1. Then the channel is always busy and user i obtains a throughput equal to 0, regardless of its transmission probability.

In case 1) $a_i = 1$, in case 2) $a_j = 1$ Thus, a is a Nash Equilibrium of the contention game Γ if and only if at least one user adopts a transmission probability equal to 1. Notice that $a_i = 1$ is a *weakly dominant strategy* for every user i, i.e., $u_i(1, a_{-i}) \geq u_i(a)$, for every action profile a. In our contention game each user has an incentive to adopt the *always transmit* strategy, resulting in network collapse.

Here we ask if it is possible to design the network to make it robust against strategic users. We want to introduce some mechanism to deter the users from adopting high transmission probabilities. The incentive schemes we consider belong to two classes:

- Pricing: users are charged depending on their transmission probabilities

- Intervention: the users' resource usage is affected by the intervention device, in a way that depends on the users' transmission probabilities

The interaction between the designer, the users and the system can be roughly summarized into three stages, (1) the design stage, (2) the information exchange stage, and (3) the transmission stage.

In the *design stage* the designer designs the pricing or intervention scheme. Specifically, the designer predicts strategic users' actions given any pricing or intervention scheme, and chooses the pricing or intervention scheme that results in the most desired outcome. This is done once, then the designer leaves the system forever. Notice that, to efficiently design these schemes, the designer has to know how pricing or intervention affect the users' utilities. This might be easier for the intervention scheme, in which the users' throughput is altered. In this case the designer has to know only the relation between the throughput and the utility of each user. Differently, in the pricing schemes users are charged for their resource usage. Hence, the designer has to know how throughput and payments are connected to the utility of each user. In this work we implicitly assume that the designer knows these dependencies, because we focus on a particular relation between the utilities, the throughput, and the payments.

In the *information exchange stage* some useful information is collect and, possibly, distributed. The intervention device (or the device that manages the payments in the pricing scheme) has to identify the users that are connected to the network, has to inform them about the adopted intervention or pricing scheme, and has to learn the action they select. For the latter point, as an example, it can count the number of correct transmissions of each user in a certain time interval. However, since this time interval must be finite, the estimation might be affected by errors. To consider the impact of this imperfect estimation we will denote by $\hat{a}_i$ the estimated action of user i, by $\hat{a}$ the estimated action profile and by $\pi_i(\hat{a}_i \mid a_i)$ the probability density function of i's estimated action, given that i's action is a_i. We say that the monitoring is *perfect* if the users' actions are estimated without errors, i.e., $\hat{a}_i$ coincides with a_i.[3] We say that the monitoring is *imperfect* if the estimates are affected by errors, i.e., there is a positive probability that $\hat{a}_i$ is different from a_i.

In the *transmission stage* the users transmit the packets adopting the same transmission probability and, in the meantime, they have to pay for their resource usage based on the pricing scheme, or their resource usage is affected based on the intervention scheme.

In this chapter we play the role of a benevolent designer that seeks to design the pricing and intervention rules to maximize the social welfare of the system in the transmission stage. We neglect the social welfare obtained in the information exchange stage because we assume that the transmission stage length is much longer than that of the information exchange stage.

6.3.1 Pricing

Pricing schemes use monetary charges to deter users' greediness. If i's payment is increasing in i's resource usage, user i might find it convenient to limit its transmission probability. In general, user i is charged according to the *pricing rule* $f_i^P : [0,\, 1] \rightarrow \Re$, which is a function of i's estimated action $\hat{a}_i$. Assuming that the payments affect additively the users' utilities, i's expected utility is given by

$$U_i^P(a) = \mathbb{E}\left[\theta_i \ln T_i(a) - f_i^P(\hat{a}_i)\right] = \theta_i \ln T_i(a) - \int_0^1 \pi_i(\hat{a}_i \mid a_i) f_i^P(\hat{a}_i) \partial \hat{a}_i \tag{6.4}$$

where $\mathbb{E}\left[\cdot\right]$ is the expectation operator.

Once a pricing scheme is selected and communicated to the users, the interaction among users can be modeled through the game

$$\Gamma^P = \left(N, A, \left\{U_i^P(\cdot)\right\}_{i=1}^n\right) \tag{6.5}$$

[3] In this case $\pi_i(\hat{a}_i \mid a_i)$ might be thought as a Dirac delta function centered in a_i.

Among all the possible pricing rules, there is one class of rules that is particularly interesting, namely, the class of *linear pricing rules*, in which users are charged linearly with respect to their transmission probabilities, i.e.,

$$f_i^P(\hat{a}_i) = c_i \hat{a}_i$$

where $c_i \geq 0$ is the unit price. We restrict our attention to the linear pricing rules, as done in most of the pricing literature, because they are computationally simple to implement and we do not lose much, in term of performance, in doing so.

Once the prices $c = (c_1, \ldots, c_n)$ are fixed, since we will prove the existence and uniqueness of the NE of the game Γ^P, the social welfare can be uniquely determined. The goal of the designer is to choose the unit prices $c = (c_1, \ldots, c_n)$ to maximize the social welfare, i.e., it has to solve the following Pricing Design (**PD**) problem:

$$\textbf{PD} \qquad \underset{c}{\operatorname{argmax}} \sum_{i \in N} U_i^P(a^{NE})$$

subject to:

$$c_i \geq 0 \;\; , \;\; \forall\, i \in N$$

$$U_i^P(a^{NE}) \geq U_i^P(a_i, a_{-i}^{NE}) \;\; , \;\; \forall\, a_i \in [0,\,1] \;\; , \;\; \forall\, i \in N$$

6.3.2 Intervention

In the intervention framework the designer deploys in the network an intervention device that monitors the users' actions and can intervene adopting itself an action that affects the users' resource usage. In our case, we assume that the intervention device is able to correctly recognize the packets transmitted by different users and to estimate the users' actions. If the packet of a generic user i is correctly received, the intervention device may choose to jam its ACK[4] depending on the estimate of its action. Specifically, the intervention device jams the ACK sent to user i with a probability that is given by the *intervention rule* $f_i^I : [0,\,1] \rightarrow [0,\,1]$, which is a function of the estimated action $\hat{a}_i$.

The intervention level $f_i^I(\hat{a}_i)$ must be interpreted as a *punishment* to user i after having deviated from a recommended (socially-beneficial) action. Such punishments are a threat to users, and must be designed such that the users find in their self-interest to adopt the recommended actions. At the same time, when users adopt the recommended actions, the intervention level must be minimized (possibly, nullified), to avoid to decrease the users' utilities.

[4]Many works on security, such as [91–93], take into consideration the possibility of performing intelligent jamming in which the jamming signal is concentrated on control packets.

Different from pricing, intervention changes the structure of the utility of each user affecting directly their resource usage. In fact, the average throughput of user i is now given by

$$T_i^I(a) = \mathbb{E}\left[a_i\left(1 - f_i^I(\hat{a}_i)\right)\prod_{j=1, j\neq i}^{n}(1-a_j)\right] = a_i\left(1 - \int_0^1 \pi_i(\hat{a}_i \mid a_i) f_i^I(\hat{a}_i)\partial \hat{a}_i\right)\prod_{j=1, j\neq i}^{n}(1-a_j) \tag{6.6}$$

where $\int_0^1 \pi_i(\hat{a}_i \mid a_i) f_i^I(\hat{a}_i)\partial \hat{a}_i$ represents the average intervention level.

The utility of user i is modified accordingly

$$U_i^I(a) = \theta_i \ln T_i^I(a) \tag{6.7}$$

Once the intervention rules are selected and communicated to the users, the interaction among the users can be modeled through the game

$$\Gamma^I = \left(N, A, \left\{U_i^I(\cdot)\right\}_{i=1}^n\right) \tag{6.8}$$

We say that the intervention rules $f^I = \left(f_1^I, \ldots, f_n^I\right)$ *sustain* an action profile a, if a is a NE of Γ^I.

Among all the possible intervention rules, there is one class of rules that is particularly interesting, namely, the class of *affine intervention rules*. $f_i^I : [0,\, 1] \to [0,\, 1]$ is an affine intervention rule if

$$f_i^I(\hat{a}_i) = \left[r_i(\hat{a}_i - \tilde{a}_i)\right]_0^1$$

for certain parameters $\tilde{a}_i \in [0,\, 1]$ and $r_i \geq 0$, where $[\cdot]_a^b = \min\left\{\max\left\{a, \cdot\right\}, b\right\}$.

In an affine intervention rule, $\tilde{a}_i$ represents a target action for user i while r_i represents the rate of increase of the intervention level due to an increase in i's action. If the estimated action $\hat{a}_i$ is lower than or equal to the target action $\tilde{a}_i$, then the intervention level is equal to 0. If the estimated action $\hat{a}_i$ is higher than the target action $\tilde{a}_i$, then the intervention level is proportional to $\hat{a}_i - \tilde{a}_i$, until it saturates to 1.

For $r_i \to +\infty$, the intervention device jams the ACKs sent to user i whenever it detects that i is adopting an action higher than the target one. Such a rule, which we refer to as an *extreme rule*, represents the strongest punishment that the intervention device can adopt.

We restrict our attention to the affine intervention rules because they are computationally simple to implement and we do not lose much, in term of performance, in doing so (as we will see, in some cases such rules are even able to achieve the benchmark optimum).

Once the parameters $\tilde{a} = (\tilde{a}_1, \ldots, \tilde{a}_n)$ and $r = (r_1, \ldots, r_n)$ are fixed, and assuming that the users coordinate to the best (from the social welfare point of view) NE of the game Γ^I [5], the social

[5] The existence of NEs will be proved for the considered scenarios and it is easy to coordinate the users to the best NE. In fact, we will prove that the best NE is uniquely determined by $\tilde{a}$.

welfare can be determined. The goal of the designer is to choose the parameters $\tilde{a}$ and r to maximize the social welfare, i.e., it has to solve the following Intervention Design (**ID**) problem:

$$\textbf{ID} \qquad \underset{\tilde{a},r}{\text{argmax}} \left[\max_{a^{NE}} \sum_{i \in N} U_i^I(a^{NE}) \right]$$

$$\text{subject to:}$$

$$\tilde{a}_i \in [0,\, 1] \quad , \quad r_i \geq 0 \quad , \quad \forall\, i \in N$$

$$U_i^I(a^{NE}) \geq U_i^I(a_i, a_{-i}^{NE}) \quad , \quad \forall\, a_i \in [0,\, 1] \quad , \quad \forall\, i \in N$$

Differently from the **PD** problem, the **ID** problem requires a maximization with respect to the NEs because of the non uniqueness of the NE.

6.4 Perfect Monitoring

In this section we assume that the estimated actions are equal to the real actions, i.e., $\hat{a}_i = a_i$, for every user $i \in N$. Hence, in Eqs. (6.4) and (6.6) the integrals must be substituted, respectively, with $f_i^P(a_i)$ and $f_i^I(a_i)$. In the following we compute the optimal linear pricing scheme and affine intervention rule that a designer should adopt to maximize the social welfare if the monitoring is perfect.

6.4.1 Pricing design

Given a linear pricing scheme c_i, $i \in N$, the interaction between users in the perfect monitoring scenario adopting pricing is modeled with the game

$$\Gamma^P = \left(N, A, \{U_i^P(\cdot)\}_{i=1}^n \right) \tag{6.9}$$

where

$$U_i^P(a) = \theta_i \ln \left[a_i \prod_{j=1, j \neq i}^{n} (1 - a_j) \right] - c_i a_i \tag{6.10}$$

The goal of the designer is to design the unit prices c to maximize the social welfare in the presence of strategic users, solving the **PD** problem with the utilities given by Eq. (6.10).

Lemma 1. *The unique NE of the game* Γ^P *is* $a_k^{NE} = \dfrac{\theta_k}{c_k}$, $k \in N$.

Proof. To compute the best response function of users k, we use the first order condition. First, we check that $U_k^P(a)$ is concave in a_k (i.e., the second derivative with respect to a_k is negative). Then, we set to 0 the first derivative of $U_k^P(a)$, with respect to a_k.

$$\frac{\partial U_k^P(a)}{\partial a_k} = \frac{\theta_k}{a_k} - c_k \quad , \quad \frac{\partial^2 U_k^P(a)}{\partial a_k^2} = -\frac{\theta_k}{a_k^2} < 0 \quad , \quad \frac{\partial U_k^P(a)}{\partial a_k} = 0 \longrightarrow a_k = \frac{\theta_k}{c_k}$$

□

Proposition 6. *The optimal pricing scheme to adopt is* $c_k^* = \sum_i \theta_i$.

Proof. We want to find the unit prices c_k, $k \in N$, so that the social welfare $U(a) = \sum_{i=1}^n U_i^P(a)$ is maximized, assuming that the users adopt the NE action profile (i.e., we have to substitute c_k with $\frac{\theta_k}{a_k}$ into the expression of $U(a)$). We first prove that $U(a)$ is a (multivariable) concave function, by checking its Hessian.

$$\frac{\partial U(a)}{\partial a_k} = \frac{\theta_k}{a_k} - \frac{\sum_{i\neq k}\theta_i}{1-a_k} \quad , \quad \frac{\partial^2 U(a)}{\partial a_k^2} = -\frac{\theta_k}{a_k^2} - \frac{\sum_{i\neq k}\theta_i}{(1-a_k)^2} < 0 \quad , \quad \frac{\partial^2 U(a)}{\partial a_k dp_i} = 0 \ , \ i \neq k$$

The Hessian of $U(a)$ is negative definite (it is a diagonal matrix with strictly negative diagonal entries), so $U(a)$ is concave. Thus, the global maximizer of $U(a)$ can be obtained with the first order condition

$$\frac{\partial U(a)}{\partial a_k} = 0 \longrightarrow a_k = \frac{\theta_k}{\sum_i \theta_i} \longrightarrow c_k = \sum_i \theta_i \ , \quad k \in N$$

□

Notice that the transmission probabilities adopted by the users in the optimal pricing policy are equal to the transmission probabilities adopted by compliant users to maximize the social welfare, i.e., $a_k^{NE} = \frac{\theta_k}{c_k^*} = a^*$, where a^* is defined in Eq. (6.3).

6.4.2 Intervention design

Given an affine intervention rule r_i and $\tilde{a}_i$, $i \in N$, the interaction between users in the perfect monitoring scenario adopting intervention is modeled with the game

$$\Gamma^I = \left(N, A, \{U_i^I(\cdot)\}_{i=1}^n\right) \tag{6.11}$$

where

$$U_i^I(a) = \theta_i \ln \left[a_i \left(1 - [r_i(a_i - \tilde{a}_i)]_0^1\right) \prod_{j=1, j\neq i}^{n} (1-a_j) \right] \tag{6.12}$$

The goal of the designer is to design the intervention rule to maximize the social welfare in the presence of strategic users, solving the **ID** problem with the utilities given by Eq. (6.12).

Notice the $\tilde{a}_k = 1$ and $\tilde{a}_k = 0$ represent trivial cases. If $\tilde{a}_k = 1$ the intervention device never jams the ACK sent to user k, $\forall\, a_k$, and in this case $a_k = 1$ represents a weakly dominant strategy, as discussed in Section 6.3. If $\tilde{a}_k = 0$ user k is punished whenever it transmits with positive probability. However, the aim of the designer is to maximize the social welfare, hence, it must first guarantee a positive throughput to every user. Thus, it is always more beneficial to consider a $\tilde{a}_k$ slightly higher than 0 instead of 0. For this reason, in the following we focus on intervention rules in which $\tilde{a}_k \in (0, 1)$, $\forall\, k$.

Lemma 2. *$\tilde{a} = (\tilde{a}_1, \ldots, \tilde{a}_n)$ is a NE of the game Γ^I if and only if $r_k \geq \frac{1}{\tilde{a}_k}$, for every user $k \in N$. Moreover, once $\tilde{a}$ and $r_k \geq \frac{1}{\tilde{a}_k}$ are fixed, among all the NEs of Γ^I, $\tilde{a}$ is (individually and socially) the best.*

Proof. We can write $r_k = \frac{1}{\tilde{a}_k + \delta}$, for some constant $\delta > -\tilde{a}_k$. Then

$$U_k^I(a_k, \tilde{a}_{-k}) = \begin{cases} \theta_k \ln \left[a_k \prod_{j \neq k} (1 - \tilde{a}_j) \right] & \text{if } \; a_k < \tilde{a}_k \\ \theta_k \ln \left[\dfrac{-a_k^2 + 2\tilde{a}_k a_k + \delta a_k}{\tilde{a}_k + \delta} \prod_{j \neq k} (1 - \tilde{a}_j) \right] & \text{if } \; \tilde{a}_k \leq a_k \leq 2\tilde{a}_k + \delta \\ -\infty & \text{if } \; a_k > 2\tilde{a}_k + \delta \end{cases}$$

We study the sign of $\frac{\partial U_k^I(a_k, \tilde{a}_{-k})}{\partial a_k}$ in the interval $[0, 2\tilde{a}_k + \delta]$ to obtain the best action for user k.

$$\frac{\partial U_k^I(a_k, \tilde{a}_{-k})}{\partial a_k} = \begin{cases} \dfrac{\theta_k}{a_k} & \text{if } \; a_k < \tilde{a}_k \\ \theta_k \dfrac{2\,(\tilde{a}_k - a_k) + \delta}{a_k\,(2\tilde{a}_k - a_k + \delta)} & \text{if } \; \tilde{a}_k \leq a_k \leq 2\tilde{a}_k + \delta \end{cases}$$

If $\delta \leq 0$ (i.e, $r_k \geq \frac{1}{\tilde{a}_k}$), $U_k^I(a_k, \tilde{a}_{-k})$ is continuous, increasing in a_k for $a_k < \tilde{a}_k$ and decreasing otherwise. Thus, $\tilde{a}_k$ is the best action for user k.

If $\delta > 0$ (i.e, $r_k < \frac{1}{\tilde{a}_k}$), $U_k^I(a_k, \tilde{a}_{-k})$ is continuous, increasing in a_k for $a_k < \tilde{a}_k + \frac{\delta}{2}$ and decreasing otherwise. Thus, $\tilde{a}_k + \frac{\delta}{2}$ $(> \tilde{a}_k)$ is the best action for user k.

Hence, $\tilde{a}$ is a NE if and only if $r_k \geq \frac{1}{\tilde{a}_k}$, $\forall\, k$. Notice also that, in this situation, $\tilde{a}$ is a weakly dominant strategy: it is in the self interest of each user k to adopt $\tilde{a}_k$, independently of the strategies of the other users. Thus, the users will coordinate to such NE.

Finally, notice that other NEs of Γ^I can only be obtained when at least two users transmit with probability 1. In fact, in this situation no user can increase its utility changing its action. Actually, the utility can not decrease either: it is constant and it is the worst (individually and socially) possible utility, corresponding to the situation in which the throughput of each user is equal to 0. □

Proposition 7. *The optimal affine intervention rule to adopt is* $r_k \geq \dfrac{1}{a_k^*}$ *and* $\tilde{a}_k = a_k^*$*, for every user* k*, where* a_k^* *is defined in Eq. (6.3).*

Proof. Given the actions of the users, the utility of a user and the social welfare are decreasing as the intervention level for that user increases. However, using the intervention rule $r_k \geq \dfrac{1}{a_k^*}$ and $\tilde{a}_k = a_k^*, \forall\, k$, the users have the incentive to adopt the action profile $a = a^*$ and, at the same time, the intervention level they are subjected to is equal to 0. Thus, the outcome of the system is equal to the benchmark optimum. Finally, this implies that $r_k \geq \dfrac{1}{a_k^*}$ and $\tilde{a}_k = a_k^*$ defines an optimal affine intervention rule, and, more specifically, it defines also an optimal intervention rule within the class of all intervention rules. □

Corollary 3. *The optimal affine intervention rule is optimal in the class of all intervention rules.*

6.4.3 Comparison between pricing and intervention and some results

By adopting either pricing or intervention the designer can provide the incentive for strategic users to choose the optimal action profile of Eq. (6.3). The efficiency of the utilization of the channel resource is optimized with respect to the valuations θ_i, $i \in N$, of the users. However, there is a big difference between pricing and intervention. Intervention schemes reach this objective by threatening the users to intervene if they do not follow the recommendations, although at the equilibrium the intervention is not triggered and therefore the resource usage is not affected. Conversely, pricing schemes charge each user that transmits with a positive probability, thus affecting its utility and the social welfare. Hence, only the intervention scheme is able to achieve the optimal social welfare that can be obtained when users behave cooperatively, i.e., when they comply to a prescribed protocol that maximizes the social welfare.

In Fig. 6.1 the social welfare and the total throughput in the perfect monitoring scenario are plotted as a function of the number of users in the system, both assuming that the users behave cooperatively, and adopting the pricing and intervention schemes derived in Sections 6.4.1 and 6.4.2 to enforce the users' actions. A symmetric case is considered, i.e., $\theta_i = 1, \forall\, i \in N$. Thus, the optimal transmission policy in the cooperative scenario, defined by Eq. (6.3), is $a_k^* = \dfrac{1}{n}$, for every user k.

The results confirm the above discussion: both schemes are able to obtain the same total throughput of the cooperative case, but only the intervention scheme is able to maximize the (total) users' satisfaction. In fact, there is a finite gap, which increases as the number of users increases, between the optimal social welfare and the one achievable with the pricing scheme. Finally, notice that the social welfare always decreases as the number of users increases because there are more collisions

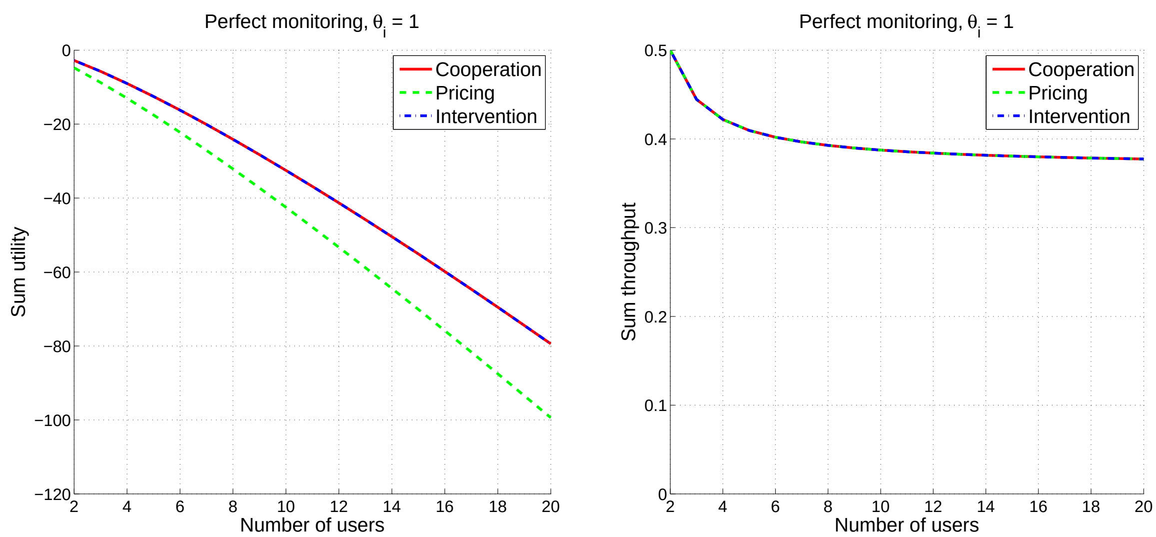


Figure 6.1. *Social welfare and total throughput vs. number of users, in the perfect monitoring scenario*

and the number of unexploited slots increases, resulting in an inefficient utilization of the channel; this is an unavoidable consequence of the lack of coordination.

6.5 Imperfect Monitoring

We now study whether the qualitative results obtained for the perfect monitoring scenario still hold for the imperfect monitoring case. In this section we will see that there is a substantial difference for the intervention scheme when the monitoring is imperfect. The intuition behind it is related to the possibility that the estimation errors trigger the intervention even though the users are adopting the recommended actions. As for the pricing scheme, if the expectations of the estimated actions are equal to the real actions, each user might be overcharged or undercharged. On average, it is charged correctly, therefore the performance is not strongly affected.

The imperfect monitoring model we consider for the estimation of user i's action is an additive noise term that is uniformly distributed in $[-\epsilon_i, \epsilon_i]$, with $0 < \epsilon_i \ll 1$, i.e.,

$$\hat{a}_i = [a_i + n_i]_0^1 \quad , \quad n_i \sim \mathcal{U}\,[-\epsilon_i, \epsilon_i]$$

In the following we compute the best linear pricing scheme and affine intervention rule that a designer should adopt to maximize the social welfare for different scenarios, depending on the information that the designer and the users have about the imperfect monitoring. In particular, we consider the following cases:

1) Nobody is aware of the estimation errors: neither the designer nor the users know about the existence of the noise, and think that the designer can estimate perfectly the users' actions.

2) The designer is aware of the estimation errors: the designer knows about the existence and the distribution of the noise, while the users think that the designer can estimate perfectly their actions.

3) Everybody is aware of the estimation errors: both the designer and the users know about the existence and the distribution of the noise.

6.5.1 Nobody is aware of the estimation errors

In this scenario both the designer and the users believe that the users' estimated actions, $\hat{a}$, are equal to the real ones, a. The additive noise n_i might be caused by a physical phenomenon which is not predicted by the designer and the users. As an example, the intervention device (or the device that manages the payments in the pricing scheme) might have, at a certain point, a malfunctioning that is not revealed and introduces noise in the measurements.

Both the designer and the users have a wrong perception of the reality: they both believe that the utilities are as in the perfect monitoring scenario even though their real utilities are affected by the noise. Since the users select their actions based on their beliefs, once the pricing and the intervention rules are fixed, their interaction can still be modeled through the games (6.9) and (6.11), as in the perfect monitoring case. Analogously, the designer designs the pricing or the intervention rule based on its beliefs. Hence, it has no reason to select rules different from the optimal (with respect to its beliefs) rules derived in Sections 6.4.1 and 6.4.2. The only difference with respect to the perfect monitoring case is that the real performance of the system is different from the one expected by the users and the designer.

Notice that both users and designer might update their beliefs observing the real performance of the system. However, this might not be easy to do due to the lack of information. On one hand, the designer designs an intervention rule and implements it in the intervention device, then it leaves the system. If the estimation errors are not correctly predicted in the design stage they affect the system, unless the designer implements a mechanism in the intervention device to reveal such errors. However, it might be difficult to discriminate between an estimation error and a real deviation of a user trying to increase its own utility.

On the other hand, the users might not be able to recognize the effect of an estimation error. As an example, in the intervention scheme the estimation error triggers, occasionally, the intervention, with the consequent decrease of the throughput of a generic user i. However, user i's throughput decreases if another user increases its transmission probability as well. Thus, i is not able to understand if its utility has decreased due to the presence of the estimation errors or for some other reasons, and is not able to update its belief correctly.

This scenario has been considered in order to analyze how robust to an unknown noise the schemes derived in Sections 6.4.1 and 6.4.2 are.

6.5.2 The designer is aware of the estimation errors

In this scenario the users are not aware about the estimation noise, while the designer knows the distribution of the noise and knows that the users' beliefs are wrong. Once the pricing and the intervention rules are given, the interaction between users can still be modeled through the games (6.9) and (6.11), in which the users act believing that their utilities are not affected by the estimation noise. When designing the pricing or the intervention rule, the designer has to take into account both that the users act strategically, according to their mismatched perceptions, and that the social welfare is affected by the noise. It has to solve the **PD** and **ID** problems using the expectation of the noisy utilities given by Eq. (6.4) and (6.7) in the maximization and using the non-noisy utilities given by Eq. (6.10) and (6.12) in the constraints. In fact, the set of constraints represents the NEs of the game played by the users, in which the users select their action to maximize the utilities they believe to receive, i.e., the non-noisy utilities; while the maximization reflects the choice of the designer, that wants to maximize the real satisfaction of the users, represented by the expectation of the noisy utilities.

Finally notice that, as described in Subsection 6.5.1, it might be difficult for the users to reveal the presence of the estimation errors by observing the real performance of the system.

6.5.2.1 Pricing design

Let $a_{k,1}$ denote the unique solution of equation

$$\theta_k a_k^3 - \left(\theta_k + 4\epsilon_k \sum_{i=1}^{n} \theta_i\right) a_k^2 + \left(4\epsilon_k\theta_k - \epsilon_k^2\theta_k\right) a_k + \epsilon_k^2\theta_k = 0$$

in $(0, \epsilon_k)$, assuming $\dfrac{\theta_k}{\sum_{i=1}^{n}\theta_i} < \epsilon_k$. Let $a_{k,2}$ denote the unique solution of equation

$$-\theta_k a_k^3 + \left(\theta_k - 4\epsilon_k \sum_{i=1}^{n} \theta_i\right) a_k^2 + \left(4\epsilon_k\theta_k + (1-\epsilon_k)^2\theta_k\right) a_k - (1-\epsilon_k)^2\theta_k = 0$$

in $(1-\epsilon_k,\,1)$, assuming $\dfrac{\theta_k}{\sum_{i=1}^n \theta_i} > 1-\epsilon_k$.

In the proof of the following result, it is shown that $a_{k,1}$ and $a_{k,2}$ exist and are unique.

Proposition 8. *The optimal unit price c_k to adopt is,* $\forall\, k \in N$,

$$c_k = \begin{cases} \dfrac{\theta_k}{a_{k,1}} & \text{if } \dfrac{\theta_k}{\sum_{i=1}^n \theta_i} < \epsilon_k \\ \sum_{i=1}^n \theta_i & \text{if } \epsilon_k \le \dfrac{\theta_k}{\sum_{i=1}^n \theta_i} \le 1-\epsilon_k \\ \dfrac{\theta_k}{a_{k,2}} & \text{if } \dfrac{\theta_k}{\sum_{i=1}^n \theta_i} > 1-\epsilon_k \end{cases}$$

Proof. See Appendix A.1. □

6.5.2.2 Intervention design

To design an intervention rule able to sustain the target action profile $\tilde{a}$, the designer has to satisfy the condition $r_k \ge \dfrac{1}{\tilde{a}_k}$, $\forall\, k$, provided by Lemma 2. The best option for the designer is to select $r_k = \dfrac{1}{\tilde{a}_k}$, $\forall\, k$, in order to sustain $\tilde{a}$ and, at the same time, to minimize the punishment adopted against k when intervention is triggered by estimation errors. Finally, the designer has to select the best $\tilde{a}_k$, for every user k.

Let $a_{k,3}$ denote the unique solution of equation

$$\left(-\theta_k - \sum_{j=1}^n \theta_j\right) a_k^2 + \left(2\theta_k - 2\epsilon_k \sum_{j=1}^n \theta_j\right) a_k + 2\epsilon_k\theta_k = 0$$

in $(0,\,\epsilon_k)$. In the proof of the following result, it is shown that $a_{k,3}$ exists and is unique.

Proposition 9. *The optimal affine intervention rule to adopt is, for every user k, $r_k = \dfrac{1}{\tilde{a}_k}$ and*

$$\tilde{a}_k = \begin{cases} a_{k,3} & \text{if } \epsilon_k > \dfrac{4\theta_k}{4\sum_{j=1}^n \theta_j - \sum_{j=1,j\neq k}^n \theta_j} \\ \dfrac{4\theta_k + \epsilon_k \sum_{j=1,j\neq k}^n \theta_j}{4\sum_{j=1}^n \theta_j} & \text{if } \epsilon_k \le \dfrac{4\theta_k}{4\sum_{j=1}^n \theta_j - \sum_{j=1,j\neq k}^n \theta_j} \end{cases}$$

Proof. See Appendix A.2. □

6.5.3 Everybody is aware of the estimation errors

In this scenario both the designer and the users are aware of the estimation errors and they know their distribution. The interaction between users must be modeled through the games (6.5) and (6.8) considering the real distribution of the noise in Eq. (6.4) and (6.6). The designer has to solve the **PD** and **ID** problems using the utilities given by Eq. (6.4) and (6.7).

6.5.3.1 Pricing design

Once the pricing scheme is given, the interaction between users can be modeled with the game in Eq. (6.5), where

$$U_i^P(a) = \theta_i \ln \left[a_i \prod_{j=1, j\neq i}^{n} (1-a_j) \right] - \frac{c_i}{2\epsilon_i} \int_{-\epsilon_i}^{\epsilon_i} [a_i + x]_0^1 \,\partial x$$

Denote

$$\mathcal{C}(\epsilon) = \left\{ x \ : \ \frac{1}{2} \leq x \leq 1-\epsilon \ \text{ and } \ x \ln x - x \geq \frac{\epsilon}{4} - 1 \right\}$$

$$\overline{a}_k = \begin{cases} -\frac{\epsilon_k}{2} + \frac{1}{2}\sqrt{\epsilon_k^2 + \frac{8\epsilon_k \theta_k}{c_k}} & \text{if } \ \frac{\theta_k}{c_k} < \epsilon_k \\ \frac{\theta_k}{c_k} & \text{if } \ \epsilon_k \leq a_k \leq \frac{1}{2} \text{ or } a_k \in \mathcal{C}(\epsilon_k) \\ 1 & \text{otherwise} \end{cases} \tag{6.13}$$

Lemma 3. *$\overline{a}_k$ is the unique NE of the game Γ^P.*

Proof. See Appendix A.3. □

Consider the following notation:

$$a_{k,4} = \frac{\theta_k(2-\epsilon_k)}{4\sum_i \theta_i} + \frac{1}{2}\sqrt{\left[\frac{\theta_k(2-\epsilon_k)}{2\sum_i \theta_i}\right]^2 + 4\frac{\theta_k \epsilon_k}{2\sum_i \theta_i}}$$
$$a_{k,5} = \max \mathcal{C}(\epsilon_k)$$

Proposition 10. *The optimal unit price c_k to adopt is, $\forall\, k \in N$,*

$$c_k = \begin{cases} \frac{2\epsilon_k \theta_k}{a_{k,4}(a_{k,4}+\epsilon_k)} & \textit{if } \ a_{k,4} < \epsilon_k \\ \frac{\theta_k}{\epsilon_k} & \textit{if } \ a_{k,4} \geq \epsilon_k \ \textit{ and } \ \frac{\theta_k}{\sum_i \theta_i} \leq \epsilon_k \\ \sum_i \theta_i & \textit{if } \ \epsilon_k \leq \frac{\theta_k}{\sum_i \theta_i} \leq \frac{1}{2} \ \textit{ or } \ \frac{\theta_k}{\sum_i \theta_i} \in \mathcal{C}(\epsilon_k) \\ \frac{\theta_k}{a_{k,5}} & \textit{otherwise} \end{cases} \tag{6.14}$$

Proof. See Appendix A.4. □

6.5.3.2 Intervention design

Once the intervention scheme is given, the interaction between users can be modeled with the game in Eq. (6.8), where

$$U_i^I(a) = \theta_i \ln \left[a_i \mathbb{E}\left[\left[r_i \left(a_i + n_i - \tilde{a}_i \right) \right]_0^1 \right] \prod_{j=1, j \neq i}^{n} (1 - a_j) \right]$$

Lemma 4. *Assume* $2\epsilon_k \leq \overline{a}_k \leq 1 - \epsilon_k$*,* $\overline{a}_k$ *is the unique NE of the game* Γ^I *if* $r_k \to +\infty$ *and* $\tilde{a}_k = \overline{a}_k + \epsilon_k$*.*

Proof. See Appendix A.5. □

Lemma 4 states that, using an extreme rule, each user k has the incentive to adopt a transmission probability $\overline{a}_k$ which is ϵ_k lower than $\tilde{a}_k$, to avoid the possibility of an intervention triggered by the estimation errors. This is true as long as $\tilde{a}_k$ is not too low, otherwise for user k it is convenient to adopt a transmission probability closer to $\tilde{a}_k$, accepting the risk of an intervention triggered by the estimation errors.

Proposition 11. *If* $a_k^* = \dfrac{\theta_k}{\sum_{i=1}^n \theta_i} \geq 2\epsilon_k$*, for every user* k*, then the intervention rule* $r_k \to +\infty$ *and* $\tilde{a}_k = a_k^* + \epsilon_k$ *is an optimal affine intervention.*

Proof. According to Lemma 4, users have the incentive to adopt $a^* = (a_1^*, \ldots, a_n^*)$. In this case the intervention level is equal to 0 because the estimation errors can not be higher than $\epsilon = (\epsilon_1^*, \ldots, \epsilon_n^*)$. Thus, the outcome of the system is equal to the benchmark optimum. Finally, this implies that $r_k \to +\infty$ and $\tilde{a}_k = a_k^* + \epsilon_k$ define an optimal affine intervention rule, and, more specifically, also define an optimal intervention rule within the class of all intervention rules. □

Corollary 4. *If* $a_k^* = \dfrac{\theta_k}{\sum_{i=1}^n \theta_i} \geq 2\epsilon_k$*, the optimal affine intervention rule is optimal in the class of all intervention rules.*

We consider the following affine intervention rule, for every user k

$$r_k \to +\infty$$
$$\tilde{a}_k = \begin{cases} a_k^* + \epsilon_k & \text{if } \; a_k^* \geq 2\epsilon_k \\ 3\epsilon_k & \text{otherwise} \end{cases} \tag{6.15}$$

Eq. (6.15) defines an optimal intervention rule if $a_k^* \geq 2\epsilon_k$, for every user k. If $a_k^* < 2\epsilon_k$, for some user k, the intervention rule might not be optimal. This rule is designed with the objective to minimize the intervention level. In fact, each user i has the incentive to adopt the action $\tilde{a}_i - \epsilon_i$, which results in an intervention level equal to 0.

6.5.4 Comparison between pricing and intervention and some results

In the following we investigate how the social welfare and the total throughput vary increasing the number of users in the system, for the imperfect monitoring scenario, adopting both the pricing and the intervention schemes. We consider the symmetric case, i.e., $\theta_i = \theta_j$ and $\epsilon_i = \epsilon_j, \forall\, i, j \in N$. Thus, the optimal transmission policy in the cooperative scenario, defined by Eq. (6.3), is $a_k^* = \frac{1}{n}$, for every user k.

First we assume that nobody, neither the designer nor the users, is aware of the estimation errors. As discussed in Subsection 6.5.1, the designer adopts the schemes derived in Section 6.4 and the users, consequently, have the incentive to adopt the action $a_k^* = \frac{1}{n}$. Fig 6.2 shows that the estimation errors have different effects in the two schemes. In the pricing scheme they do not affect the total throughput, and the social welfare is slightly affected only when the number of users exceeds $\frac{1}{\epsilon_i} = 10$ (corresponding to the condition $a_k^* < \epsilon_i$). In fact, if the number of users is less than or equal to 10, each user is (on average) charged correctly. Conversely, if the number of users exceeds 10, the expectation of the estimated transmission probability $\hat{a}_k$ is higher than the real transmission probability a_k^* and each user is (on average) slightly overcharged, resulting in a social welfare slightly lower than the one obtainable in the perfect monitoring scenario (see Fig. 6.1). In the intervention scheme the effect of the estimation errors is stronger. In fact, they occasionally trigger intervention, which decreases both the throughput and the utility experienced by each user. Nevertheless, the social welfare adopting intervention is still higher than the social welfare adopting pricing.

Now we consider the imperfect monitoring scenario assuming that only the designer is aware of the estimation errors. In this case, the designer can adopt the optimal pricing and intervention schemes derived in Subsection 6.5.2. The social welfares obtainable with both schemes, shown in Fig. 6.3, are only slightly higher than the social welfares obtainable when nobody is aware of the estimation errors, shown in Fig. 6.2 (such differences will be clearer in Figs. 6.6 and 6.7). This means that the designer can not gain much with the additional information on the presence of estimation errors, and knowing their statistics. In particular, for the pricing scheme such information is useless if the

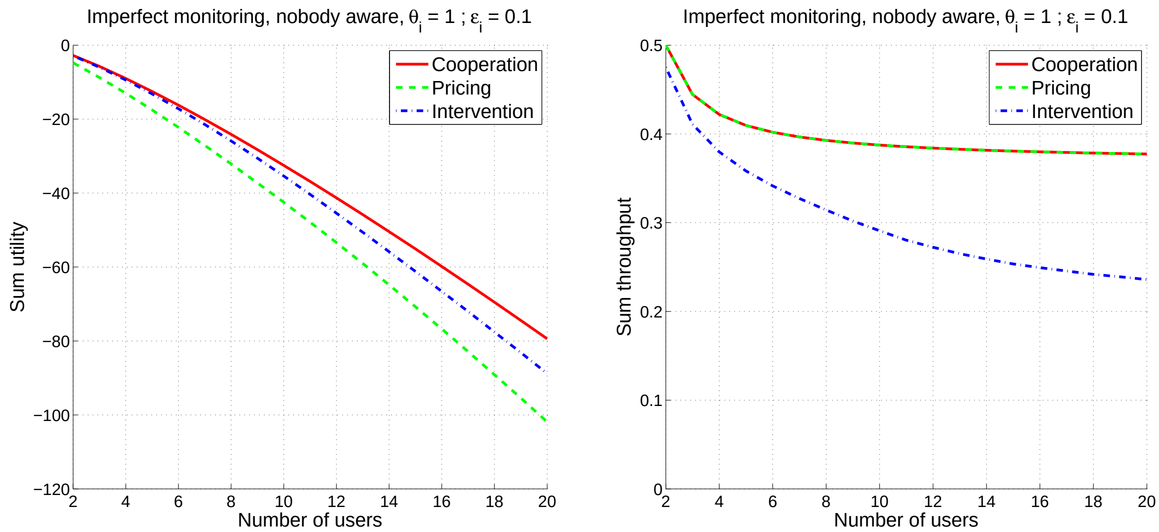


Figure 6.2. *Social welfare and total throughput vs. number of users, in the imperfect monitoring scenario, assuming nobody is aware of the estimation errors*

number of users is less than 10, because the best pricing schemes derived in Subsections 6.5.2 and 6.4.2 are identical in this situation.

Now we investigate the performance achievable in the imperfect monitoring scenario assuming that everyone is aware of the estimation errors. In this case the users, knowing that the noise might bias the payments (pricing) or the punishments (intervention), adopt a different NE action profile. Since the designer can foresee the users' behavior, it can adopt the pricing and intervention schemes derived in Subsection 6.5.3. Fig. 6.4 shows that the performance attainable with the pricing scheme is very similar to the preceding cases, only slightly worse. Conversely, the performance achievable with the intervention scheme is completely different from the preceding cases. The intervention scheme is able to achieve the optimal social welfare as long as the number of users is less than or equal to 5 (corresponding to the condition $a_k^* \geq 2\epsilon_k$), as predicted by Proposition 11). If the number of users is higher than 5, both the total throughput and the social welfare decrease rapidly as the number of users increases. This trend is a consequence of the action adopted by the users in this situation, which is constant and equal to $2\epsilon_k$ instead of scaling with the number of users. This causes a rapid increase of the number of collisions. Finally, this trend determines a threshold in the number of users such that, for a number of users lower than the threshold, intervention outperforms pricing, whereas, for a number of users higher than the threshold, pricing outperforms intervention. The value of the threshold for the considered system parameters is equal to 15.

In Fig. 6.5 the value of the threshold is plotted varying ϵ_k, the maximum intensity of the noise.

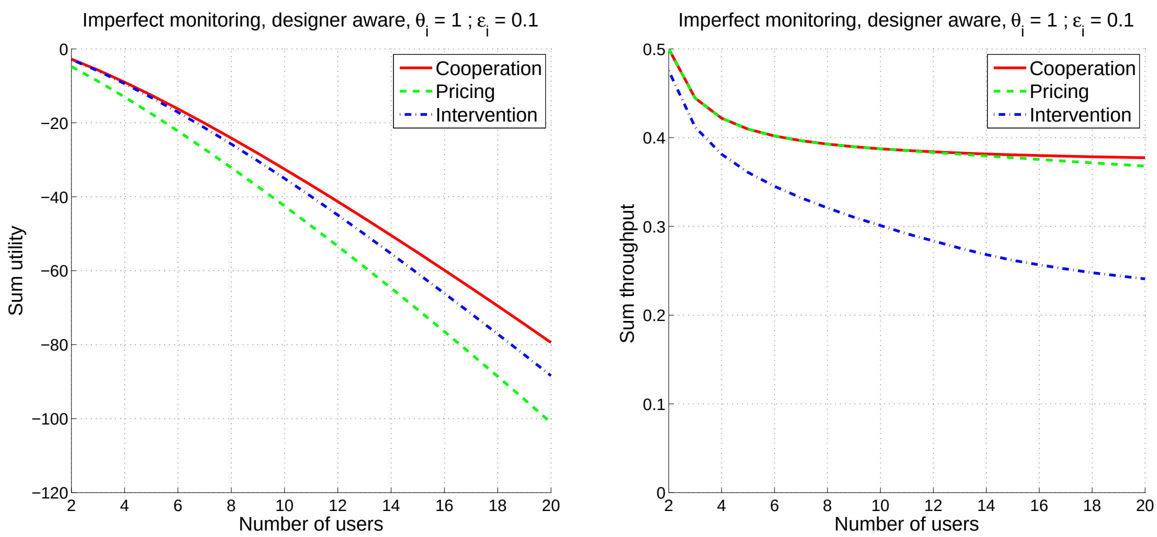


Figure 6.3. *Social welfare and total throughput vs. number of users, in the imperfect monitoring scenario, assuming that only the designer is aware of the estimation errors*

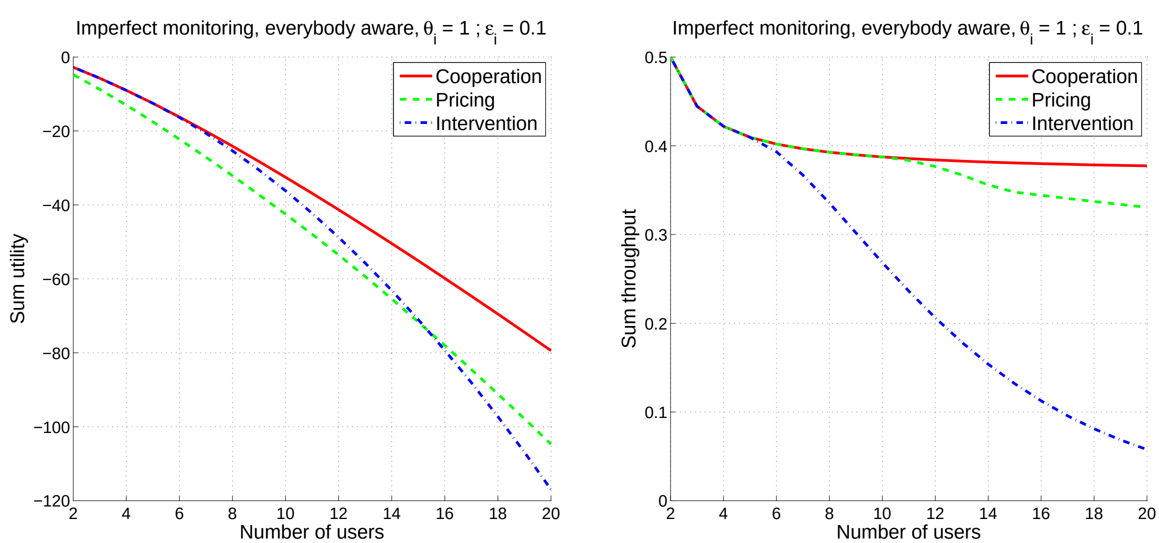


Figure 6.4. *Social welfare and total throughput vs. number of users, in the imperfect monitoring scenario, assuming everybody is aware of the estimation errors*

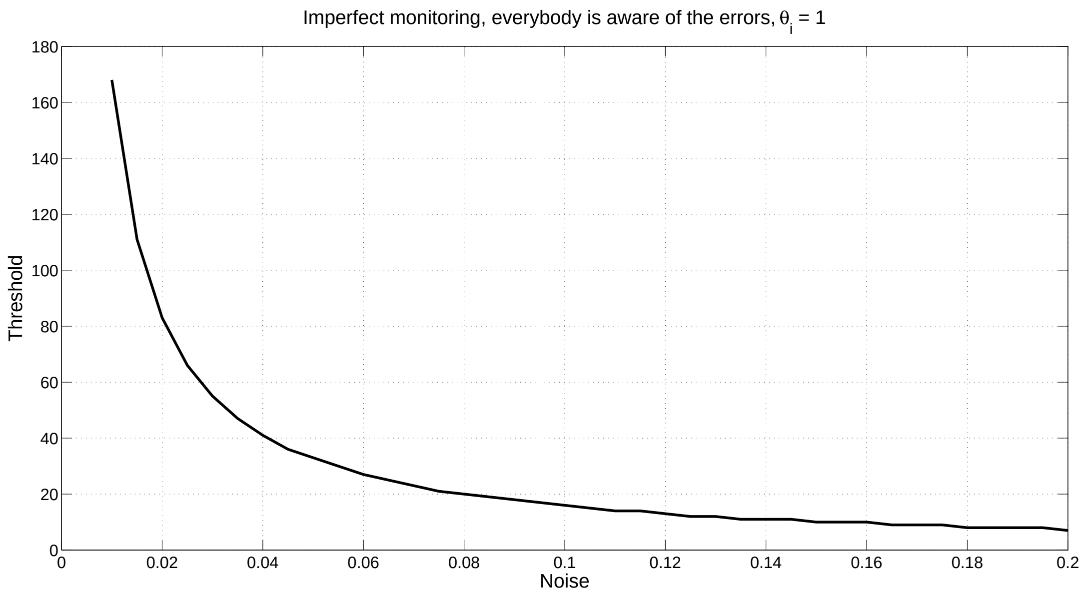


Figure 6.5. *Threshold vs. noise in the imperfect monitoring scenario, assuming everybody is aware of the estimation errors*

The threshold decreases as ϵ_k increases, because the intervention scheme is more sensitive to the estimation errors than the pricing scheme. For the highest noise considered, i.e., $\epsilon_i = 0.2$, the intervention scheme outperforms the pricing scheme as long as the number of users is less than 9.

In order to have a quantitative comparison between the different scenarios, in Figs. 6.6 and 6.7 we plot the social welfare and the total throughput achievable for all considered cases, adopting pricing and intervention respectively. In both Figures, we see that the system achieves the best performance if the monitoring is perfect. In case it is not, for the pricing scheme the best case is when only the designer is aware of the estimation errors, whereas the worst case is when also the users are aware of the estimation errors. It is not surprising that, in a strategic setting, the more information the selfish users have the worse the efficiency of the equilibrium point. Conversely, for the intervention scheme we notice that when the users are aware of the estimation errors the social welfare might be higher than when they are not. This result does not contradict the previous one, in fact it is caused by the additional information that the designer has as well: it knows that the users know that estimation errors exist, thus, it can design different intervention rules. In particular, it can adopt a more severe rule (e.g., the extreme rule, with $r_k \to +\infty$) that forces the users to keep their transmission probabilities low in order to avoid that the intervention is occasionally triggered by the estimation errors. Fig. 6.7 shows that there is a threshold in the number of users such that, for a number of users lower than the threshold, it is socially convenient that the users are aware of the estimation errors, while for a

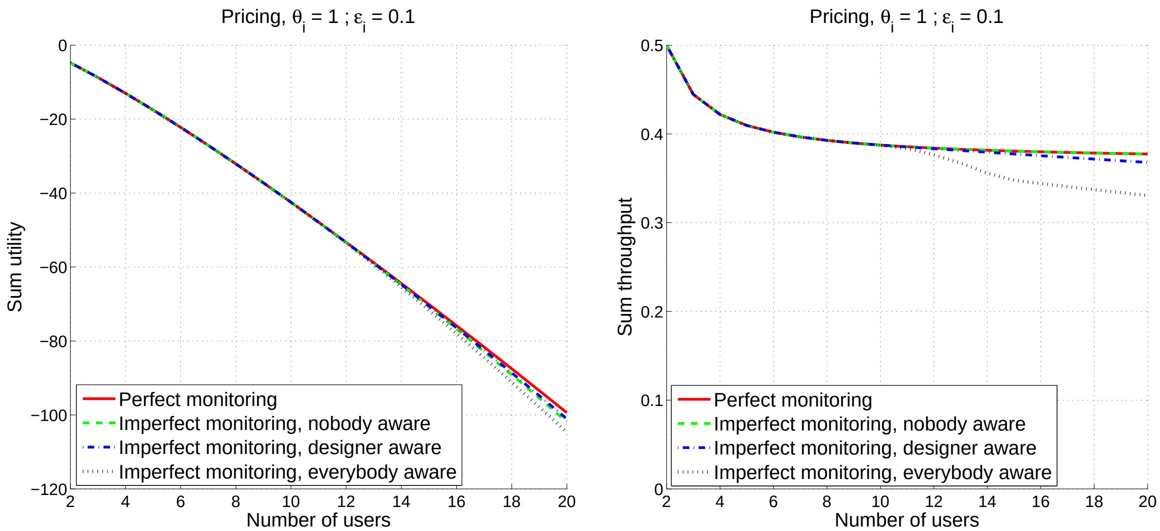


Figure 6.6. *Social welfare and total throughput vs. number of users adopting pricing, for different scenarios*

number of users higher than the threshold it is not.

Finally, in Figs. 6.8 and 6.9 we consider the imperfect monitoring scenario assuming that everyone is aware of the estimation errors, and we compare the considered intervention scheme of Eq. (6.15) with the optimal affine intervention rule. The optimal affine intervention rule is computed adopting an exhaustive search algorithm. Notice that this is possible because we consider a symmetric scenario. In asymmetric scenarios the calculation of the optimal rule through an exhaustive search algorithm would be computationally too expensive. Fig. 6.8 shows the action selected by the users and the average intervention level varying the number of users, while Fig. 6.9 shows the social welfare and the total throughput varying the number of users. Proposition 11 guarantees that the considered intervention rule is optimal for a number of users equal or lower than 5 (corresponding to the condition $a_k^* \geq 2\epsilon_k$). However, as we can see, the considered intervention rule is optimal until 9 users. If the number of users exceeds 9, it is preferable to be more aggressive with the intervention rule, using a $\tilde{a}_k$ lower than $3\epsilon_k$ and forcing the users to decrease their transmission probability as well, even though this means that the intervention is occasionally triggered.

6.6 Conclusions

In this chapter we tackle the problem of designing pricing and intervention schemes to provide incentives for the users to exploit efficiently the channel resource in a contention game. The design

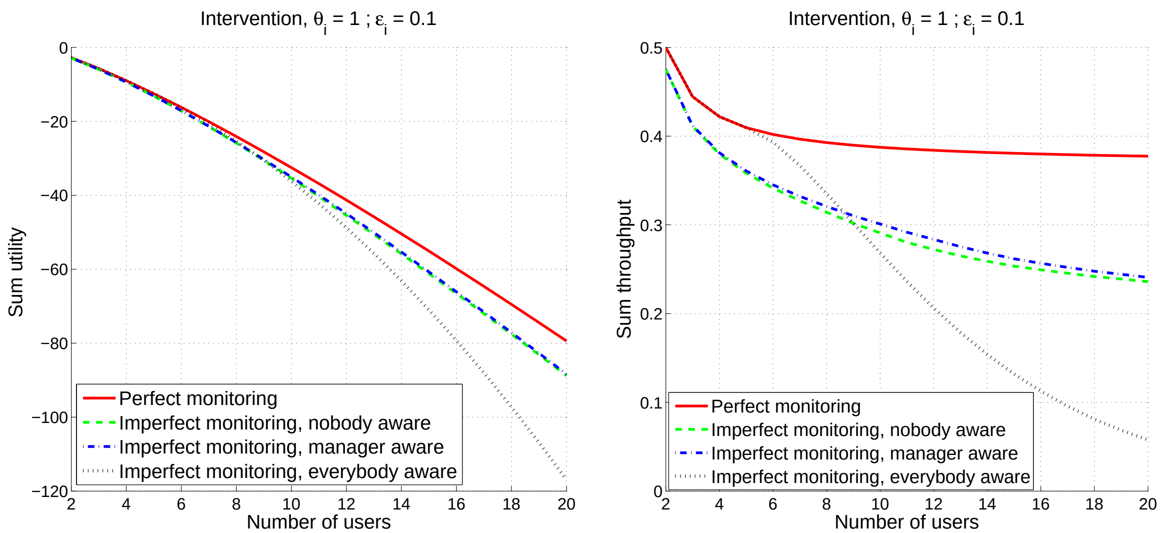


Figure 6.7. *Social welfare and total throughput vs. number of users adopting intervention, for different scenarios*

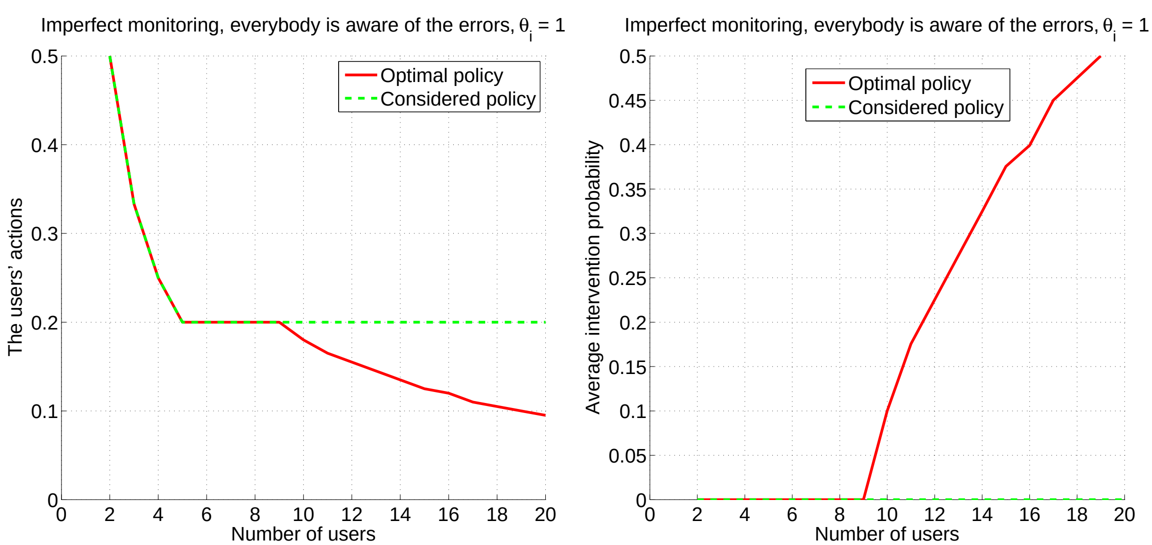


Figure 6.8. *The users' actions and the average level of intervention vs. number of users in the imperfect monitoring scenario, assuming everybody is aware of the estimation errors, adopting the considered policy and the optimal one*

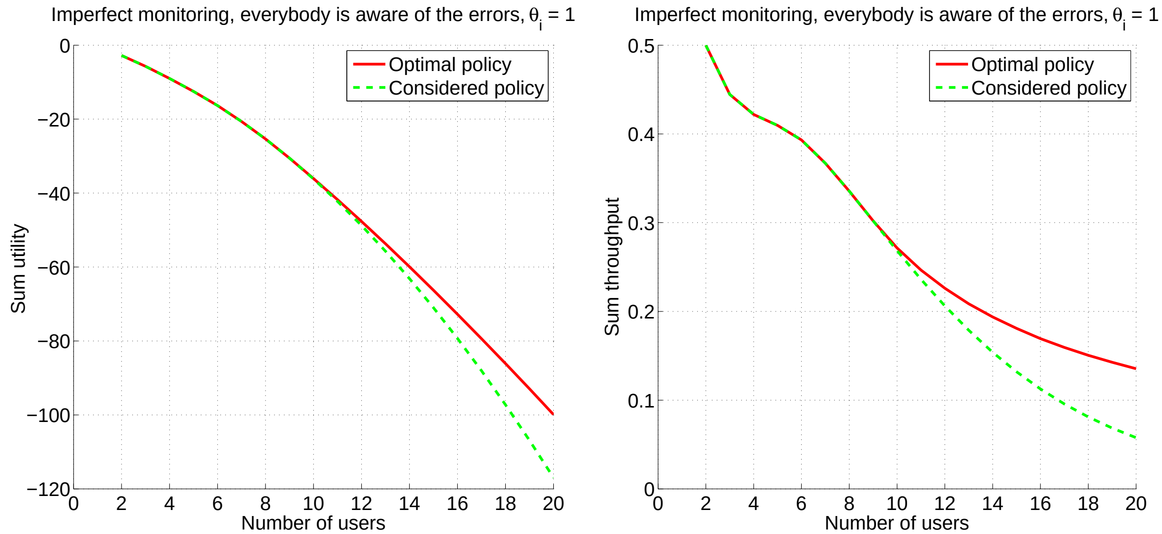


Figure 6.9. *Social welfare and total throughput vs. number of users in the imperfect monitoring scenario, assuming everybody is aware of the estimation errors, adopting the considered policy and the optimal one*

of the optimal schemes strongly depends on the parameters of the system, such as the statistics of the estimation errors, and on the information held by the designer and by the users.

In this work we have considered both the perfect monitoring and the imperfect monitoring scenarios, assuming, for the latter case, that (1) neither the designer nor the users are aware of the estimation errors, (2) only the designer is aware of the estimation errors, and (3) both the designer and the users are aware of the estimation errors. The optimal linear pricing and affine intervention schemes have been analytically computed (for the case (3), the considered intervention scheme is optimal only in some conditions).

The analysis shows that the intervention scheme, differently from the pricing scheme, is able to achieve the optimal performance in the perfect monitoring scenario. On the other hand, in the imperfect monitoring scenario intervention might be triggered even when the users adopt the recommended actions, resulting in a degradation of the system performance. Nevertheless, we noticed that intervention outperforms pricing in cases (1) and (2), while for case (3), as a rough general principle, intervention achieves greater efficiency than pricing when the number of users is small and the opposite is true when the number of users is large.

Another interesting result is related to the effect of the information held by the different entities. While it is always desirable for the designer to have as much information as possible, the effect of the information held by the selfish users is not trivial. In many cases it is preferable that the users are uninformed, but, sometimes, the information held by the users allows the designer to design better rules. In our particular case, we have seen that the intervention can achieve the benchmark optimum if the users are aware of the estimation errors and the number of users is not too high. This suggests the idea, that might be true also in other settings, of hiding some system parameters from the users in determinate conditions.

Finally, the analysis in this chapter can serve as a guideline for a designer to select between pricing and intervention and to design the best policy for the selected scheme, depending on some system parameters such as the number of users, the statistics of the monitoring noise and the information held by the designer and the users.

Chapter 7

Information Revelation and Intervention with an Application to Flow Control

In this chapter[1] we study the interaction between a designer and a group of strategic and self-interested users who possess information the designer does not have. Because the users are strategic and self-interested, they will act to their own advantage, which will often be different from the interest of the designer, even if the designer is benevolent and seeks to maximize (some measure of) social welfare. In the settings we consider, the designer and the users can communicate (perhaps with noise), the designer can observe the actions of the users (perhaps with error) and the designer can commit to (plans of) actions – *interventions* – of its own. The designer's problem is to construct and implement a *mechanism* that provides incentives for the users to communicate and act in such a way as to further the interest of the designer – *despite* the fact that they are strategic and self-interested and possess private information. To address the designer's problem we propose a general and flexible framework that applies to many scenarios. In an important class of environments, we find conditions under which the designer can obtain its benchmark optimum – the utility that could be obtained if it had all information and could command the actions of the users – and conditions under which it cannot. More broadly we are able to characterize the solution to the designer's problem, even when it does not yield the benchmark optimum. Because the optimal mechanism may be difficult to construct

[1]The material presented in this chapter has been published in:

[J4] **L. Canzian**, Y. Xiao, W. Zame, M. Zorzi, and M. van der Schaar, "Intervention with Private Information, Imperfect Monitoring and Costly Communication: Design Framework," *Submitted to IEEE Trans. Commun.*

[J5] **L. Canzian**, Y. Xiao, W. Zame, M. Zorzi, and M. van der Schaar, "Intervention with Complete and Incomplete Information: Application to Flow Control," *Submitted to IEEE Trans. Commun.*

and implement, we also propose a simpler and more readily-implemented mechanism that, while falling short of the optimum, still yields the designer a "good" result. We then apply our framework and results to design a flow control management system, in both the complete and the incomplete information scenarios. Illustrative results show that the considered schemes can considerably improve the efficiency of the network.

7.1 Introduction

We study the interaction between a group of users and a designer. If the users are compliant or the designer can command the actions of the users, then the designer is faced with an optimal control problem of the sort that is well-studied. Little changes if the users have private information (about themselves or about the environment) that the designer does not have but the designer can communicate with the users, because the designer can simply ask or instruct the users to report that information. However, a great deal changes if the users are not compliant but rather are self-interested and strategic and the designer can not command the actions and reports of the users. In that case, the users may take actions and/or provide reports that are in their own self-interest but not necessarily in the interest of the designer. The objective of this work is to understand the extent to which the designer can provide incentives to the users to take actions and provide reports that further the objectives of the designer, be those selfish or benevolent. (The case of a benevolent designer is probably the one of most interest, but the problem faced by a benevolent designer is no easier than the problem faced by a selfish designer: the goal of a benevolent designer is to maximize some measure of social welfare – which might include both total utility and some measure of fairness – but the goal of an individual user is to maximize its own utility; hence the incentives of the designer and of the individual user are no more aligned when the designer is benevolent than when the designer is selfish, so the same incentives to misrepresent and misbehave are present in both circumstances. Such incentives frequently lead to the over-use of resources and to substantial inefficiencies [9, 11].) Here, we are specifically interested in settings in which the users can send reports to the designer and the designer in turn can send messages to the users before the users act, after which the designer may take actions of its own – *interventions*. Our use of intervention builds on [12, 16, 42, 66], but we go beyond that work in considering private information, imperfect monitoring and costly communication – in addition to intervention.

Our work has something in common with the economic theory of *mechanism design* in the tradition of [94–98]. Indeed, our general framework builds on that of [99], and the abstract theory of mechanism design – in particular the revelation principle – does play a role. However, [99] does not solve any of our problems because after we use the revelation principle to restrict our attention to incentive compatible direct mechanisms we must still *construct* the optimal mechanism, which is a non-trivial undertaking.[2] Moreover, when we admit physical (and other) constraints, noisy communication and imperfect monitoring, the revelation principle does not help because it entirely obscures all of these complications. Finally, the revelation principle simply does not hold when communication is costly.

We treat settings in which the users have private information but (perhaps limited and imperfect) communication between the users and the designer – more precisely, the *device* employed by the designer – is possible. The users have the opportunity to send reports about their private information, and the device can in turn send messages to the users; in both cases, we allow for the possibility that communication is noisy so that the report/message sent is not necessarily the report/message received. After this exchange of information, the users take actions. Finally, the device, having (perhaps imperfectly) observed these actions, also has the opportunity to act. Generalizing a construction of [99], we formalize this setting as a *communication mechanism*. The device the designer employs plays two roles: first to *coordinate* the actions of the users *before* they take them and second to *discipline* the users *afterwards*. Because users are self-interested and strategic, their reports and actions will only serve the interest of the designer if they also serve their own interests. Thus we are interested in strategy profiles for the users that each user finds optimal, given the available information, the strategies of others and the nature of the given device; we refer to these as *communication equilibria*. Note that the device is not strategic – it is a device after all – but *the designer behaves strategically in choosing the device*. Because we focus here on the problem of the designer, we are interested in finding devices that support equilibria that the designer finds optimal. (If the designer is benevolent – i.e., intends to maximize social welfare, perhaps constrained by some notion of fairness – these devices will also serve the interests of the users as a whole, but if the designer is self-interested they may not.) We are particularly interested in knowing when the designer can find a device so as to achieve his benchmark optimum – the outcome he could achieve if he knew all relevant information and users were fully compliant – despite the fact that information is in fact private and users are in fact self-interested. For a class of environments that includes many engineering environments of interest (e.g., power control [15, 16], medium access control (MAC) [12, 25], and flow control [24–28]) we find

[2] Proposition 1 in [99] shows that the problem of choosing the optimal incentive compatible direct mechanism is a linear programming problem *provided* that type sets and action sets are finite *and fixed* and that the designer can send arbitrary messages – but in our context the action sets may not be finite, the action set of the designer is definitely not fixed, and the designer's choice of messages may be constrained, so our problem is different, and much more complicated.

conditions under which there exist mechanisms that achieve the benchmark optimum and conditions under which such mechanisms do not exist. In case they do not exist, we find conditions such that the problem of finding an optimal protocol can be decoupled. Because the optimal protocol may still be difficult to compute, we also provide a simple algorithm that converges to a protocol that, although perhaps not optimal, still yields a 'good' outcome for the designer. We then apply our framework and results to design a flow control management system, in both the complete and the incomplete information scenarios. First, we analytically compute the NE and the BNE of the complete and incomplete information flow control games without intervention, quantifying its inefficiency. Then, we apply the intervention scheme in the complete information setting and we design the device that can achieve the optimal performance. Finally, we apply our theoretical framework to design schemes able to deal with the incomplete information scenario as well. Illustrative results show that the considered schemes can considerably improve the efficiency of the network.

Throughout, we assume that the designer can *commit* to a choice of a device that is pre-programmed to carry out a particular plan of action *after* the reports and actions of the users. In mechanical terms, such commitment is possible precisely because the designer deploys a device – hardware or software or both – *and then leaves*. Indeed, the desire of the designer to commit is one reason that it employs a device. Although other assumptions are possible, this assumption seems most appropriate for the settings we have in mind, in which the designer is a long-lived and experienced entity who has learned the relevant parameters (user utilities and distribution of user types) over time, but the users are short-lived, come and go but do not interact repeatedly: in a particular session they are not playing a repeated game and are not forward-looking.

The remainder of this chapter is organized as follows. Section 7.2 introduces our framework of devices and mechanisms and the notion of equilibrium. Section 7.3 presents an example to illustrate how private information, information revelation and intervention all matter. Section 7.4 asks when some devices achieve the benchmark optimum. Section 7.5 studies the properties of the optimal devices and Section 7.6 offers a constructive procedure for choosing devices that are simple to compute and implement – if not necessarily optimal. Section 7.7 introduces the flow control problem. Section 7.8 studies flow control games without incentive schemes and show their inefficiency. Section 7.9 designs the incentive schemes for the complete and incomplete information scenarios and quantifies the improvement in the network efficiency. Section 7.10 concludes with some remarks.

7.1.1 Related work

There is by now a substantial communication engineering literature that addresses the problem of providing incentives for strategic users to obey a particular resource allocation scheme. Such incentives might be provided in a number of different ways. [12, 16, 42, 66] provide incentives via intervention in the absence of private information. A rather different literature, including [37, 39, 41, 100], adopts literal pricing schemes: users are required to make monetary payments for resource usage.[3] Literal pricing schemes require the designer to have specific knowledge (the value of money to the users) and require a technology for making monetary transfers, which is missing in many settings, such as wireless communication. Moreover, it does not necessarily solve the problem of a benevolent designer since monetary payments are by definition costly for the user making them and hence wasteful. An additional difficulty in employing literal payment schemes is that it is debatable whether users would agree to a pricing scheme that dynamically varies with the state of the system, in particular if users have to pay for a service that had hitherto been free. A smaller literature [38, 102, 103] addresses environments in which users have private information – their private monetary valuations for access to the resource – and uses ideas from mechanism design and auction theory [104] to create protocols in which users are asked to report their private monetary valuations, after which access to the resource is apportioned and users make monetary payments according to their access and the reports of valuations. For very detailed comparison of pricing, intervention and other approaches, see [105].

7.2 Framework

We consider a *designer* and a collection of *users*. The designer chooses an *intervention device* and then leaves – the designer itself takes no further actions. In a single *session* the device interacts with a fixed number of users n, labeled from 1 to n. We will write $N = \{1, \ldots, n\}$ for the set of users. We think of the users in a particular session as drawn from a pool of potential users, so users may be (and typically will be) different in each session. We allow for the possibility that users are drawn from different pools – e.g., occupy different geographical locations or utilize different channels.

User i is characterized by an element of a set T_i of *types*, which encodes all relevant information

[3] A different literature, which includes [13, 15, 101] but is quite far from the work here, uses pricing in scenarios where users are compliant, rather than self-interested and strategic. In those scenarios, however, the function of pricing is decentralization: prices induce utility functions for the users that lead them to take the desired actions without the need for centralized control. In these scenarios pricing is figurative rather than literal: monetary payments are not actually required.

about the user. Write $T = T_1 \times \ldots \times T_n$ for the set of possible type profiles. Users know their own type; users and the designer know the distribution of user types π (a probability distribution on T). In each session, user i chooses an *action* from the set A_i of *actions*. We write $A = A_1 \times \ldots \times A_n$ for the set of possible action profiles and (a_i, a_{-i}) for the action profile in which user i chooses action $a_i \in A_i$ and other users choose the action profile $a_{-i} \in A_{-i} = A_1 \times \ldots \times A_{i-1} \times A_{i+1} \ldots \times A_n$; we use similar notation for types, etc.

The designer is characterized by its utility function and the set $\mathcal{D}$ of *devices* it might use. A device – which might consist of hardware or software or both – has four features: 1) it can receive communications from users, 2) it can send communications to users, 3) it can observe the actions of users, and 4) it can take actions of its own. As in [66], we interpret the actions of the device as *interventions*. We formalize a device as a tuple $D = \langle (R_i), (M_i), \mu, X, \Phi, \epsilon^R, \epsilon^M, \epsilon^A \rangle$, where:

- R_i is the set of *reports* that user i might send; write $R = R_1 \times \ldots \times R_n$ for the set of report profiles;
- M_i is the set of *messages* that the device might send; write $M = M_1 \times \ldots \times M_n$ for the set of message profiles;
- $\mu : R \to \Delta(M)$ is the *message rule*, which specifies the (perhaps random) profile of messages to be sent to the users as a function of the reports received from all users; if r is the profile of observed reports we write μ_r for the corresponding probability distribution on M, and $\mu_r(m)$ for the probability that the message m is chosen when the observed report is r;[4]
- X is the set of *interventions* (actions) the device might take;
- $\Phi : R \times M \times A \to \Delta(X)$ is the *intervention rule*, which specifies the (perhaps random) intervention the device will take given the received reports, the transmitted messages and the observed actions; if r are the observed reports, m the transmitted messages and a the observed actions, we write $\Phi_{r,m,a}$ for the corresponding probability distribution on X;
- $\epsilon^R : R \to \Delta(R)$ encodes the noise in receiving reports: users send the report profile r but the designer observes a random profile $\hat{r}$ distributed according to ϵ^R_r;
- $\epsilon^M : M \to \Delta(M)$ encodes the noise in receiving messages: the device sends the message profile m but users observe a random profile $\hat{m}$ distributed according to ϵ^M_m;

[4] If the message m is always chosen given the observed report profile r, μ_r is point mass at m, i.e., $\mu_r(z) = 1$ if $z = m$, $\mu_r(z) = 0$ otherwise. However, in this case we usually prefer to abuse notation and write $\mu(r) = m$. Below, we will make similar notational abuses without further comment.

- $\epsilon^A : A \to \Delta(A)$ encodes the error in monitoring actions: the users choose an action profile a but the device observes a random profile $\hat{a}$ distributed according to ϵ^A_a.

The set of all conceivable devices is very large, but in practice the designer will need to choose a device from some prescribed (perhaps small) subset $\mathcal{D}$, so we assume this throughout. In this generality, reports and messages could be entirely arbitrary but typically reports will provide (perhaps incomplete) information about types, and messages will provide (perhaps incomplete) recommendations for actions, and we will frequently use this language.

If the report spaces are singletons then reports are meaningless, so singleton report spaces R_i express the absence of reporting. Similarly, a singleton message space M expresses the absence of messaging and a singleton intervention space X expresses the absence of intervention. The absence of noise/error with regard to reports, messages or actions can be expressed by requiring that the corresponding mapping(s) be the identity; e.g, ϵ^R_r is point mass at r and so $\hat{r} = r$ for all report profiles, etc. However, in any of these cases we would usually prefer to abuse notation and omit the corresponding component of the tuple that describes the device.

The utility $U_i(a, t, r_i, x)$ of user i depends on the actions a and types t of all users, the report r_i chosen by user i, and the intervention x of the designer. The utility $U(a, t, r, m, x)$ of the designer depends on the actions a and types t of all the users, on the reports r, the messages m and intervention x of the designer. The dependence of utility on reports and messages allows for the fact that communication may be costly. Note that the utility of a user depends only on the report that user sends, but the utility of the designer depends on the messages it sends *and* on the reports of the users. If this seems strange, keep in mind that if the designer is benevolent and seeks to maximize social utility, he certainly cares about the reporting costs of users.

A *communication mechanism*, or *mechanism* for short, is a tuple $\mathcal{C} = \langle N, (T_i, A_i, U_i), \pi, U, D \rangle$ that specifies the set N of users, the sets T_i of user types, the sets A_i of user actions, the utility functions U_i of users, the distribution π of types, the utility functions U_i of users, the utility function U of the designer, and the device D. We view the designer as choosing the device, which is pre-programmed, but otherwise taking no part: the users choose and execute plans and the device carries out its programming.

The operation of a communication mechanism $\mathcal{C}$ is as follows.

- users make reports to the device;

- the device “reads” the reports (perhaps with error) and sends messages to the users (perhaps depending on the realization of the random rule);

- users "read" the messages[5] (perhaps with error) and take actions;
- the device "monitors" the actions of the users (perhaps imperfectly) and, following the rule, makes an intervention (perhaps depending on the realization of the random rule).

A *strategy* for user i is a pair of functions $f_i : T_i \to R_i$, $g_i : T_i \times M_i \to A_i$ that specify which report to make, conditional on the type of user i, and which action to take, conditional on the type of user i and the message observed. We do not specify a strategy for the device because the device is not strategic; its behavior is completely specified by the message rule and the intervention rule – but the *designer behaves strategically in choosing the device*. Given a profile (f, g) of user strategies, and the intervention device D, the expected utility of a user i whose type is t_i is obtained by averaging over all random variables involved, i.e.,[6]:

$$EU_i(f, g, t_i, D) = \sum_{t_{-i} \in T_{-i}} \pi(t \mid t_i) \sum_{\hat{r} \in R} \epsilon_r^R(\hat{r}) \sum_{m \in M} \mu_{\hat{r}}(m) \sum_{\hat{m} \in M} \epsilon_m^M(\hat{m}) \sum_{\hat{a} \in A} \epsilon_a^A(\hat{a}) \sum_{x \in X} \Phi_{\hat{r},m,\hat{a}}(x)\, U_i(a, t, r_i, x)$$

where $r_j = f_j(t_j)$ and $a_j = g_j(t_j, \hat{m}_j)$, $\forall\, j \in N$, are the reports sent and the actions taken by users. Similarly, the expected utility of the designer is

$$EU(f, g, D) = \sum_{t \in T} \pi(t) \sum_{\hat{r} \in R} \epsilon_r^R(\hat{r}) \sum_{m \in M} \mu_{\hat{r}}(m) \sum_{\hat{m} \in M} \epsilon_m^M(\hat{m}) \sum_{\hat{a} \in A} \epsilon_a^A(\hat{a}) \sum_{x \in X} \Phi_{\hat{r},m,\hat{a}}(x)\, U(a, t, r, m, x)$$

The strategy profile (f, g) is an *equilibrium* if each user is optimizing given the strategies of other users and the device D; that is, for each user i we have

$$EU_i(f_i, f_{-i}, g_i, g_{-i}, t_i, D) \geq EU_i(f_i', f_{-i}, g_i', g_{-i}, t_i, D)$$

for all strategies $f_i' : T_i \to R_i$, $g_i' : T_i \times M_i \to A_i$.[7] We often say that the device D *sustains* the profile (f, g). We remark that the existence of such an equilibrium is not always guaranteed without additional assumptions and needs to be explicitly addressed in the specific case at hand.

Note that the action of the device is fixed and not strategic – in particular, the interventions planned by the device but not executed with positive probability – that is, threats that are not carried out – need

[5]Note that we assume that each user i can only read its own message m_i. However, our framework is suitable to model also situations in which user i is able to hear the message m_j intended for user j. In this case it is sufficient to focus on devices in which the message sent to user j is part of the message sent to user i.

[6]We have tacitly assumed that all the probability distributions under consideration have finite or countably infinite support – which will certainly be the case if the spaces under consideration are themselves finite or countably infinite; in a more general context we would need to replace summations by integrals and to be careful about measurability, etc.

[7]The notion of equilibrium defined here is that of a Bayesian Nash equilibrium of the Bayesian game induced by the communication mechanism. For simplicity, we have restricted attention to equilibrium in *pure strategies*; we could also allow for equilibria in *mixed strategies*.

not be optimal. This reflects our assumption that the designer can commit to using the device. Again: the device does not behave strategically, *the designer behaves strategically in choosing the device*.

The designer seeks to optimize his own utility by choosing a device D from some prescribed class $\mathcal{D}$ of physically feasible devices. Because users are strategic, the designer must assume that, whatever device D is chosen, the users will follow some equilibrium strategy profile (f, g). Since the designer will typically recommend actions, we assume that, if more than one equilibrium strategy profile exists, the users choose (because they are are coordinated to) the equilibrium that the designer most prefers (in case of a benevolent manager, it usually coincides with the equilibrium that the users prefer). Hence, the designer has to solve the following Optimal Device (**OD**) problem[8]:

OD $\quad \underset{D \in \mathcal{D}}{\operatorname{argmax}}\, max_{f,g} EU(f, g, D)$

subject to:

$$EU_i(f_i, f_{-i}, g_i, g_{-i}, t_i, D) \geq EU_i(f_i', f_{-i}, g_i', g_{-i}, t_i, D)$$

$$\forall\, i \in N \ , \ \forall\, t_i \in T_i \ , \ \forall\, f_i' : T_i \to R_i \ , \ \forall\, g_i' : T_i \times M_i \to A_i$$

We say that a solution D of the above problem is an *optimal device*. To maintain parallelism with some other literature, we sometimes abuse language and refer to the designer's problem as choosing an *optimal mechanism* – even though the designer only chooses the device and not the types of users, their utilities, etc. Note that optimality is relative to the prescribed set $\mathcal{D}$ of considered devices. Moreover, the expected utility the designer obtains choosing the optimal device must not be confused with the *benchmark optimum* utility the designer could achieves if users were compliant, which is in general higher. If they coincide, we say that the device D is a *maximum efficiency device*.

7.2.1 Null reports, messages and interventions

In many (perhaps most) concrete settings, it is natural to presume that users might sometimes choose not to make reports and that the device might sometimes not send messages or make an intervention. The easiest way to allow for these possibilities is simply to assume the existence of null reports, null messages and null actions. In particular, we can assume that for each user i there is a distinguished report r_i^* which is to be interpreted as 'not sending a report'. (On the device side, observing r_i^* should be interpreted as 'not receiving a report'.) Because not making a report should

[8]Because the utility functions of users depend on reports, and the utility function of the designer depends on messages and reports, which are parameters of the device chosen, this tacitly assumes that utility functions are defined on a domain sufficiently large to encompass all the possibilities that may arise when any device $D \in \mathcal{D}$ is chosen.

be costless, we should assume that – fixing types, reports of others to the device, actions by the users and intervention by the device – r_i^* yields utility at least as great as any other report: $U_i(a,t,r_i^*,x) \geq U_i(a,t,r_i,x)$ and $U(a,t,r_i^*,r_{-i},m,x) \geq U(a,t,r_i,r_{-i},m,x)$, for all a,t,x,r_i,r_{-i},m. Given this assumption, and using utility when sending the report r_i^* as the baseline, we can interpret the differences $U_i(a,t,r_i^*,x) - U_i(a,t,r_i,x)$ and $U(a,t,r_i^*,r_{-i},m,x) - U(a,t,r_i,r_{-i},m,x)$ as the cost of sending the report r_i to the user i and to the designer, respectively. In this generality, the cost of sending a report might depend on all other variables. We remark that this cost does not take into consideration the impact of the communication on the interaction among the users and the intervention device: in deciding whether or not to send a report, a user must take into account the fact that sending a report may alter the messages sent by the device and hence the actions of the users and the intervention of the device. So sending a report may well lead to higher utility because it influences the strategic choices of others.

Similarly, we could assume that for each user i there is a distinguished message m_i^* that the device might send but which we interpret as 'not sending a message'. (On the user side, we interpret receipt of the message m_i^* as 'not receiving a message'.) Because not sending a message m_i^* should be costless, we assume that $U(a,t,r,m_i^*,m_{-i},x) \geq U(a,t,r,m_i,m_{-i},x)$ for all a,t,r,m_{-i},x,m_i, and so interpret the difference $U(a,t,r,m_i^*,m_{-i},x) - U(a,t,r,m_i,m_{-i},x)$ as the cost of sending the message m_i, which might depend on all other variables.

Finally, we could assume that there is a distinguished intervention x^* that we interpret as 'not making an intervention'. If (as we usually do) we want to interpret an intervention as a *punishment*, we should assume that x^* yields utility at least as great as any other intervention for each user and the designer: $U_i(a,t,r_i,x^*) \geq U_i(a,t,r_i,x)$ and $U(a,t,r,m,x^*) \geq U(a,t,r,m,x)$ for all i,a,t,r,m,x, and we interpret the differences $U_i(a,t,r_i,x^*) - U_i(a,t,r_i,x)$ and $U(a,t,r,m,x^*) - U(a,t,r,m,x)$ as the cost of the intervention to the user i and to the designer, respectively, which might depend on all other variables.

If the sets of reports (respectively, messages, interventions) are singletons, then by default there are no possible reports (respectively, messages, interventions).

If D is a device for which 'not making an intervention' is possible and (f,g) is an equilibrium with the property that $\Phi_{\hat{r},m,\hat{a}}(x^*) = 1$, for all type profiles t, observed reports $\hat{r}$, sent messages m and observed actions $\hat{a}$ (with $\hat{r}$, m and $\hat{a}$ occurring with positive probability), we say that D sustains (f,g) *without intervention*. The most straightforward interpretation is that the device threatens punishments for deviating from the recommended actions and that the threats are sufficiently severe that they do not need to be executed. Again, this is natural in context: by using the intervention device, the designer *commits* to meting out punishments for deviation, even if those punishments are costly for the designer as well as for the users.

7.2.2 Direct mechanisms

To be consistent with [99], we say that the mechanism $\mathcal{C}^d$ is a *direct mechanism* if $R_i = T_i$ for all i (users report their types, not necessarily truthfully) and $M = A$ (the device recommends action profiles), there are no errors, and reports and messages are costless (i.e., utility does not depend on reports or messages). If $\mathcal{C}^d$ is a direct mechanism we write $(f^*, g^*) = (f_1^*, \ldots, f_n^*, g_1^*, \ldots g_n^*)$ for the strategy profile in which users are honest (report their true types) and obedient (follow the recommendations of the device); that is, $f_i^*(t_i) = t_i$ and $g_i^*(t_i, a_i) = a_i$ for every user i, type t_i, and recommendation $a_i = \mu_i(t_i)$. If (f^*, g^*) is an equilibrium, we say that $\mathcal{C}^d$ is *incentive compatible*. If a device is such that the resulting mechanism is an incentive compatible direct mechanism, we say that the device is incentive compatible.

Incentive compatible direct mechanisms play a special role because of the following generalization of the revelation principle. (We omit the proof, which is almost identical to the proof of Proposition 2 in [99].)

Proposition 12. *If $\mathcal{C}$ is a mechanism for which reports and messages are costless and (f, g) is an equilibrium of the mechanism $\mathcal{C}$, then there is an incentive compatible direct mechanism $\mathcal{C}^d$ with the same action and intervention spaces for which the honest and obedient strategy profile (f^*, g^*) yields the same probability distribution over outcomes as the profile (f, g).*

As we shall see later (this version of) the revelation principle is useful but its usefulness is limited for a number of reasons. The first reason is that, although it restricts the class of mechanisms over which we must search to find the designer's most preferred outcome, we still have to find the optimal device in this class, which is not always an easy task. The second reason is that in practice there will often be physical limitations on the devices that the designer can employ (because of limits to the device's monitoring capabilities, for instance) and hence limitations on the communication mechanisms that should be considered, but these may not translate into limitations on a corresponding direct mechanism. For instance, in a flow control scenario, it will often be the case that the device can observe total flow but not the flow of individual users and can only observe this flow with errors; no such restrictions occur in direct mechanisms. Finally, as noted before, the revelation principle does not hold when communication is costly.

7.2.3 Special cases

The framework we have described is quite general so it is worth noting that many, perhaps more familiar, frameworks are simply special cases:

- If T, R, M and X are all singletons, then our framework reduces to an ordinary static game with complete information and our equilibrium notion reduces to Nash equilibrium.
- If R, M and X are all singletons, then our framework reduces to an ordinary Bayesian game and our equilibrium notion reduces to Bayesian Nash equilibrium.
- If T, R and X are all singletons, then our framework reduces to a game with a mediation device and our equilibrium notion reduces to correlated equilibrium.
- If T, R and M are all singletons, then our framework reduces to the intervention framework of [66] and our equilibrium notion reduces to intervention equilibrium.
- If there are no errors, and reports and messages are costless, then our framework reduces to a communication game in the sense of [99] and our equilibrium notion reduces to communication equilibrium.

7.3 Why Intervention and Information Revelation Matter

To illustrate our framework, we give a simple example to show that strategic behavior matters, intervention matters, and communication plus intervention matters – in the sense that they all change the outcomes that can be achieved.

We consider the problem of access to two channels A and B (e.g., two different bandwidths, or two different time slots). In each session, two users (identified as user 1 and user 2, but drawn from the same pool of users) can access either or both channels; we use A, B, AB to represent the obvious actions. Each user seeks to maximize its utility, which is the sum of its own goodput in the two channels.

Potential users are of four types: HL, ML, LM and LH; the probability that a user is of a given type is $1/4$. We interpret a user's type xy as the quality of channels A, B to that user: channel A has quality x (Low, Medium or High), channel B has quality y (Low, Medium or High). [9] The goodput

[9]Note that the quality to a user is correlated across channels: each user finds one channel to be of Low quality and the other to be of Medium or High quality. This is not at all essential – the qualitative comparisons would be unchanged if we assumed quality to a user was uncorrelated across channels – but the calculation would be much messier.

obtained by user $i = 1, 2$ from a given channel depends on the user's type and on which user(s) access the channel.

- if user i does not access the channel it obtains goodput $= 0$
- if both users access the channel they interfere with each other and both obtain goodput $= u_I$
- if user i is the only user to access channel A and its type is xy then it obtains goodput u_x (where $x = L, M, H$)
- if user i is the only user to access channel B and its type is xy then it obtains goodput u_y (where $y = L, M, H$)

We assume $2u_I < u_L < u_M < u_H$.

We consider five scenarios: (I) no intervention or communication, (II) communication but no intervention, (III) intervention but no communication, (IV) intervention and communication, and (V) the benchmark setting in which the designer has perfect information and users are obedient. For simplicity, we assume that the devices available to the designer are very restricted: reports and messages are costless, there are no errors and the actions are either $x^* =$ "take no action" or $x_1 =$ "access both channels". If the device takes no action, user utilities are as above; if the device accesses both channels then each user's goodput is u_I on each channel the user accesses.[10] The designer is benevolent and hence seeks to maximize social utility – the expected sum of user utilities.

I **No communication, No Intervention** Independently of the user's type, the other user's type, and the other user's action, it is always strictly better for each user to access both channels, so in the unique (Bayesian Nash) equilibrium both users always choose action AB , and (in obvious and suggestive notation), (expected) social utility is $EU(I) = 4u_I$.

II **Communication, No Intervention** Nothing changes from scenario *I*: no matter what the users report and the device recommends, it is strictly better for each user to access both channels, so in the unique equilibrium both users always choose action AB , and social utility is $EU(II) = 4u_I$.

[10] Note that in this model the utility obtained when accessing a channel in the presence of interference does not depend on the number of interferers present and on their channel qualities, which may not be realistic in certain scenarios. This assumption is made here in order to keep the discussion simple, but could be easily relaxed at the price of a much more cumbersome discussion in terms of notation and number of cases to be considered. In addition, in most reasonable scenarios (i.e., when the goodput obtained in the presence of any amount of interference is significantly lower than that obtained in its absence), the qualitative conclusions we draw here would be maintained.

III **Intervention, No Communication** The sets R_i of reports and M of messages are singletons, so the device obtains no information about the users and can suggest no actions to the users. The best the designer can do is to use an intervention rule that coordinates the two users to different resources; given the restriction on device actions an optimal rule is:

$$\Phi(a_1, a_2) = \begin{cases} x^* & \text{if } a_1 = A \text{ and } a_2 = B \\ x_1 & \text{otherwise} \end{cases}$$

where a_1 and a_2 are the actions adopted by the two users. Given this intervention rule, the best equilibrium strategy profile (i.e., the one that yields highest social utility) is for user 1 to access channel A and user 2 to access channel B, so that there is never a conflict.[11] Given the distribution of types, social utility is $EU(III) = (u_H/2) + (u_M/2) + u_L$.

IV **Communication, Intervention** We consider a direct mechanism in which the users report their types ($R_i = T_i$) and the device D recommends actions ($M = A$). The device uses the following message and intervention rules:

$$\mu(r_1, r_2) = \begin{cases} (A, B) & \text{if } r_1 = HL \text{ or } r_1 = ML \text{ or } r_2 = LH \text{ or } r_2 = LM \\ (B, A) & \text{otherwise} \end{cases}$$

$$\Phi(r_1, r_2, a_1, a_2) = \begin{cases} x^* & \text{if } (a_1, a_2) = \mu(r_1, r_2) \\ x_1 & \text{otherwise} \end{cases}$$

where r_1, r_2 are the reports and a_1, a_2 are the actions. This is an incentive compatible direct mechanism. To see this we must show that the honest and obedient strategy (f_1^*, g_1^*) is the most preferred strategy for all types of user 1, given that user 2 follows its honest and obedient strategy (f_2^*, g_2^*), and conversely for user 2. We will describe the calculations for user 1, from which those for user 2 can be derived by the symmetry of the problem.

Assume user 1 is of type HL. If it is honest and obedient, it obtains a utility of u_H because it accesses its preferred channel. This utility is always higher than the utility it obtains not being obedient, i.e., if it does not follow the recommendation. In fact in this case it never obtains a utility higher than $2u_I$ because the channels are interfered by the device. Now let assume user 1 is obedient but not honest. If it reports type ML it can still access its preferred channel, obtaining a utility of u_H, the same as if it were honest. If it reports type LM or LH, it accesses half of the time its preferred channel and half of the time its less preferred channel (depending

[11] This is not the only equilibrium but it is the best, both for the designer and the users. In the other equilibrium the users access both channels.

on the type of user 2), obtaining an expected utility of $(u_H + u_L)/2$ which is lower than u_H. These considerations translate mathematically in the following relations, stating that user 1 has an incentive to be honest and obedient if it is of type HL,

$$EU_1(f^*, g^*, HL, D) = \begin{cases} u_H > 2u_I \geq EU_1(f_1, f_2^*, g_1, g_2^*, HL, D) & \forall\, f_1, \text{ if } g_1(HL, a_1) \neq a_1 \\ u_H = EU_1(f_1, f_2^*, g^*, HL, D) & \text{if } f_1(HL) = ML \\ u_H > (u_H + u_L)/2 = EU_1(f_1, f_2^*, g^*, HL, D) & \text{if } f_1(HL) = LM \text{ or } LH \end{cases}$$

Analogously, if user 1 is of type ML, LM or LH we obtain:

$$EU_1(f^*, g^*, ML, D) = \begin{cases} u_M > 2u_I \geq EU_1(f_1, f_2^*, g_1, g_2^*, ML, D) & \forall\, f_1, \text{if } g_1(ML, a_1) \neq a_1 \\ u_M = EU_1(f_1, f_2^*, g^*, ML, D) & \text{if } f_1(ML) = HL \\ u_M > (u_H + u_L)/2 = EU_1(f_1, f_2^*, g^*, ML, D) & \text{if } f_1(ML) = LM \text{ or } LH \end{cases}$$

$$EU_1(f^*, g^*, LM, D) = \begin{cases} (u_M + u_L)/2 > 2u_I \geq EU_1(f_1, f_2^*, g_1, g_2^*, LM, D) & \forall\, f_1, \text{if } g_1(LM, a_1) \neq a_1 \\ (u_M + u_L)/2 = EU_1(f_1, f_2^*, g^*, LM, D) & \text{if } f_1(LM) = LH \\ (u_M + u_L)/2 > u_L = EU_1(f_1, f_2^*, g^*, LM, D) & \text{if } f_1(LM) = HL \text{ or } ML \end{cases}$$

$$EU_1(f^*, g^*, LH, D) = \begin{cases} (u_H + u_L)/2 > 2u_I \geq EU_1(f_1, f_2^*, g_1, g_2^*, LH, D) & \forall\, f_1, \text{if } g_1(LH, a_1) \neq a_1 \\ (u_H + u_L)/2 = EU_1(f_1, f_2^*, g^*, LH, D) & \text{if } f_1(LH) = LM \\ (u_H + u_L)/2 > u_L = EU_1(f_1, f_2^*, g^*, LH, D) & \text{if } f_1(LH) = HL \text{ or } ML \end{cases}$$

Notice that following this mechanism never leads to interference (users always access different channels) and users are "assigned" to the most efficient channels $7/8$ of the time. However, users are not *always* assigned to the most efficient channels: if type profiles are $(t_1, t_2) = (ML, HL)$ or $(t_1, t_2) = (LH, LM)$ then user 1 is assigned to channel A and user 2 is assigned to channel B, which is inefficient. This inefficiency is an unavoidable consequence of incentive compatibility: if user 2 were always assigned to his preferred channel A when he reported HL (for instance) then he would never be willing to report ML when that was his true type. Expected social utility under this mechanism is

$$\begin{aligned} EU(IV) &= (1/16)\,[2(u_H + u_H) + 4(u_H + u_M) + 4(u_H + u_L) + 2(u_M + u_M) + 4(u_M + u_L)] \\ &= (3u_H/4) + (3u_M/4) + (u_L/2) \end{aligned}$$

V **Benchmark Social Optimum: Public Information, Perfect Cooperation** The social optimum is obtained by assigning the user with the best channel quality to his favorite channel and never assigning two users to the same channel. Expected social utility is

$$EU(V) = (1/16)\left[2(u_H + u_H) + 4(u_H + u_M) + 6(u_H + u_L) + 2(u_M + u_M) + 2(u_M + u_L)\right] =$$
$$= (7u_H/8) + (5u_M/8) + (u_L/2)$$

Direct calculation shows that social utilities in four of the five scenarios are strictly ranked:

$$EU(I) = EU(II) < EU(III) < EU(IV) < EU(V)$$

In words: in comparison to the purely Bayesian scenario (no intervention), communication without intervention achieves nothing, intervention without communication improves social utility by dampening destructive competition, intervention with communication improves social utility even more by extracting some information and using that information to promote a more efficient coordination across types, but even intervention with communication does not achieve the benchmark social optimum under full cooperation. It is possible to show that the same conclusions would be obtained in an environment with n users and m channels (for arbitrary n, m), provided that $m \geq n$ and $mu_I < u_L < u_M < u_H$.

It is worth noting that similar comparisons across scenarios could be made in many environments and the ordering of expected social utility would be as above:

$$EU(I) \leq EU(II) \leq EU(III) \leq EU(IV) \leq EU(V)$$

In general, any of these inequalities might be strict.

7.4 Resource Allocation Games in Communication Engineering

In the following we explore the designer's problem in a class of abstract environments that exhibit some features common to many resource sharing situations in communication networks, including power control [15, 16], medium access control (MAC), [12, 25], and flow control [24–28]. We characterize the direct communication mechanisms that are optimal among all mechanisms. We provide conditions on the environment under which it is *possible* for the designer to achieve its benchmark optimum – the outcome it could achieve if users were compliant – and conditions under which it is *impossible* for the designer to achieve its benchmark optimum. Although we can characterize the optimal device, other mechanisms are also of interest, for several reasons. The optimal device may be very difficult to compute. It is therefore of some interest to consider mechanisms that are sub-optimal but easy to compute, and we provide a simple algorithm that converges to such a mechanism.

7.4.1 The considered environment

In this subsection we formalize the particular (but, at the same time, quite general) environment we consider from now on, motivating each assumption with examples of its application in resource sharing situations in communication networks.

We consider a finite and discrete type set made by real numbers $T_i = \{\tau_{i,1}, \tau_{i,2}, ..., \tau_{i,v_i}\} \subset \Re$, $v_i \in \mathbb{N}$, in which the elements are labeled in increasing order, $\tau_{i,1} < \tau_{i,2} <, ..., < \tau_{i,v_i}$. We interpret the type of a user as the valuation of a particular resource for the user (e.g., different types may represent different quality of service classes). We assume that every type profile has a positive probability to occur, i.e., $\pi[t] > 0$, $\forall\, t$. We allow the users to take actions in a continuous interval $A_i = \left[a_i^{min},\ a_i^{max}\right] \subset \Re$, which we interpret as the level of resource usage (e.g., it may represent the adopted transmission power, which is positive and upper bounded). We assume that the devices available to the designer are such that reports and messages are costless, there are no errors, and there exists the intervention action $x^* \in X$ which we interpret as "no intervention". In this case we can simply write $U_i(a,t,x)$ for the utility of user i and $U(a,t,x)$ for the utility of the designer and we can restrict our attention to incentive compatible direct mechanism. That is, we consider only the incentive compatible devices $D = \langle (T_i), (A_i), \mu, X, \Phi \rangle$ in which $x^* \in X$.

We assume that the designer's utility satisfies the following assumptions, $\forall\, t \in T$,

A1: $U(a,t,x^*) > U(a,t,x)$, $\forall\, a \in A$, $\forall\, X$, $\forall\, x \in X$, $x \neq x^*$

A2: $g^M(t) = \mathrm{argmax}_a\, U(a,t,x^*)$ is unique

A3: $g^M(t)$ is differentiable with respect to t_i and $\frac{\partial g_i^M(t)}{\partial t_i} > 0$ [12]

Assumption **A1** states that the "no intervention" action is the strictly preferred action of the designer, regardless of users' actions and types. Interpreting interventions as punishments, assumption **A1** asserts that the designer is not happy if the users are punished.

Assumption **A2** states that, for every type profile $t \in T$, the users' joint action profile that maximizes the designer's utility is unique, and by assumption **A3**, each component in g^M is continuous and increasing in the type of that user. If actions represent the level of resource usage and types represent resource valuations, assumption **A3** asserts that the higher i's valuation the higher should be i's level of resource usage.

[12] This assumption requires the designer utility to be defined over a continuous interval that includes the finite type set T.

Under these assumptions, the benchmark optimum for the designer can be easily determined

$$EU^{ben} = \sum_{t \in T} \pi(t) U(g^M(t), t, x^*) \tag{7.1}$$

For each type profile $t \in T$, we define the complete information game

$$\Gamma_t^0 = (N, A, \{U_i(\cdot, t, x^*)\}_{i=1}^n)$$

Γ_t^0 is the complete information game (users know the type of everybody) that can be derived from our general framework assuming that sets of types T_i, reports R_i, messages M_i and interventions X are singleton (in particular, $X = \{x^*\}$). It can be thought as the game that models users' interaction in the absence of an intervention device and when the type profile is known.

The strategy of user i in this context is represented by the function $g_i : T \to A_i$ (notice that we can omit the dependence on the messages), since the function $f : T \to R$ is automatically defined (users do not send reports or receive messages, or equivalently, always send the report 'no report' and receive the message 'no message'). We denote by $g^{NE^0}(t) = \left(g_1^{NE^0}(t), \ldots, g_n^{NE^0}(t)\right)$ a Nash Equilibrium (NE) of the game Γ_t^0, which is an action profile so that each user obtains its maximum utility given the actions of the other users, i.e.,

$$U_i\left(g^{NE^0}(t), t, x^*\right) \geq U_i\left(g_i(t), g_{-i}^{NE^0}(t), t, x^*\right) \quad , \quad \forall\, i \in N \;,\; g_i : T \times \{m^*\} \to A_i$$

We assume that users' utilities $U_i(a, t, x^*)$ are twice differentiable with respect to a and, $\forall\, a \in A$, $\forall\, t \in T$, $\forall\, i, j \in N$, $i \neq j$,

- **A4**: $U_i(a, t, x^*)$ is quasi-concave in a_i and there exists a unique best response function $h_i^{BR}(a_{-i}, t) = \operatorname{argmax}_{a_i} U_i(a, t, x^*)$
- **A5**: $\frac{\partial^2 U_i(a,t,x^*)}{\partial a_i \partial a_j} \leq 0$
- **A6**: There exists g^{NE^0} such that $g^{NE^0}(t) \geq g^M(t)$ [13] and $g_k^{NE^0}(t_k, t_{-k}) > g_k^*(t_k, t_{-k})$ for some user $k \in N$ and type $t_k \in T_k$

Since for **A4** the users' utilities are quasi concave (thus the game Γ_t^0 is a quasi-concave game) and the best response function $h_i^{BR}(a_{-i}, t)$ that maximizes $U_i(a, t, x^*)$ is unique, either i's utility is monotonic with respect to a_i, or it increases with a_i until it reaches a maximum for $h_i^{BR}(a_{-i}, t)$, and decreases for higher values. As a consequence, a NE $g^{NE^0}(t)$ of Γ_t^0 exists. In fact, the best response

[13] Throughout the chapter, inequalities between vectors are intended component-wise.

function $h^{BR}(a,t) = \left(h_1^{BR}(a_{-1},t),\dots,h_n^{BR}(a_{-n},t)\right)$ is a continuous function from the convex and compact set A to A itself, therefore Brouwer's fixed point theorem assures that a fixed point exists.

Assumption **A5** asserts that Γ_t^0 is a submodular game and ensures that $h_i^{BR}(a_{-i},t)$ is a non increasing function of a_j, $j \neq i$. Interpreting a_i as i's level of resource usage, this situation reflects resource allocation games where it is in the interest of a user not to increase its resource usage if the total level of use of the other users increases, in order to avoid an excessive use of the resource. Nevertheless, assumption **A6** says that strategic users use the resources more heavily compared to the optimal (from the designer's point of view) usage level.

The class of games satisfying assumptions **A1-A6** includes the linearly coupled games [25] and many resource allocation games in communication networks, such as the MAC [12,25], power control [15,16] and flow control [24–28] games, assuming that the designer's utility is increasing in the users' utilities (i.e., a benevolent designer).

7.4.2 Intervention in the complete information setting

Before analyzing the designer's problem in the general framework, we first introduce formally the special case of intervention in the complete information setting, though the main focus of this chapter is the design of a mechanism dealing with both information revelation and action enforcement. In fact, some properties of the general mechanism are linked to the properties of the complete information setting defined in this subsection.

For each type profile $t \in T$ and intervention rule $\Phi : A \to \Delta(X)$ (we can omit the dependence on reports and messages), we define the complete information game

$$\Gamma_t = (N, A, \{U_i(\cdot, t, \Phi(\cdot))\}_{i=1}^n)$$

Γ_t is the complete information game (users and designer know the type of everybody) that can be derived from our general framework assuming that sets of types T_i, reports R_i and messages M_i are singletons. Our general framework reduces in this case to the intervention framework of [66] and our equilibrium notion reduces to intervention equilibrium.

As in the game Γ_t^0, the strategy of user i is represented only by the function $g_i : T \to A_i$. However, in this case each user has to take into account the effect of the intervention action chosen following the distribution Φ_a, which depends on the adopted action profile a. Accordingly to the notions introduced in the general framework, we say that a device D, defined by the set of interventions X and the intervention rule Φ, sustains (without intervention) the strategy profile $g(t) = (g_1(t),\dots,g_n(t))$

in Γ_t if g is an equilibrium of Γ_t (and $\Phi_{g(t)}(x^*) = 1$). If there exists a device D able to sustain (without intervention) the profile g in Γ_t, we say that g is sustainable (without intervention) in Γ_t.

7.5 Optimal Devices

In this section we study the class of environments introduced in Section 7.4 with the general framework proposed in Section 7.2. In particular, we take the part of a designer seeking to maximize his own expected utility in the presence of self-interested and strategic users, choosing an optimal device in the class of available devices $\mathcal{D}$ specified in Section 7.4 .

First of all we wonder if the designer can choose a maximum efficiency device $D \in \mathcal{D}$ to obtain his benchmark utility despite the fact that the users are strategic. We characterize the existence and the computation of maximum efficiency devices based on some properties of the complete information setting. Moreover, we prove that a necessary condition for the existence of a maximum efficiency device requires the type sets to be *sufficiently sparse*.

Even for cases in which a maximum efficiency device does not exist, the designer is still interested in obtaining the best he can, choosing an optimal device. For this reason we study the problem of finding the optimal device and we prove that, under some properties of the complete information setting, the original problem can be decoupled into two sub-problems easier to solve.

7.5.1 Properties of a maximum efficiency device

In this subsection we address the problem of the existence and the computation of a maximum efficiency incentive compatible device.

The first result we derive asserts that a maximum efficiency device exists if and only if, for every type profile t, the optimal (for the designer) strategy profile $g^M(t)$ is sustainable without intervention in Γ_t, and users have incentives to reveal their real type given that they will adopt g^M and the intervention device does not intervene. If this is the case, we are also able to characterize all maximum efficiency devices.

Proposition 13. $D = \langle (T_i), (A_i), \mu, X, \Phi \rangle$ *is a maximum efficiency device if and only if,* $\forall\, t \in T$*,*

***1**: the optimal action profile* $g^M(t)$ *is sustainable without intervention in* Γ_t*;*

***2**: each user* i *having type* t_i *prefers the action profile* $g^M(t)$ *with respect to the action profile*

$g^M(t'_i, t_{-i})$*, for every alternative type* t'_i *user* i *might have, i.e,*

$$\sum_{t_{-i}\in T_{-i}} \pi[t \mid t_i] U_i\left(g^M(t), t, x^*\right) \geq \sum_{t_{-i}\in T_{-i}} \pi[t \mid t_i] U_i\left(g^M(t'_i, t_{-i}), t, x^*\right)$$
$$\forall\, i \in N,\;\; \forall\, t_i \in T_i,\;\; \forall\, t'_i \in T_i$$

***3**: the suggested action profile is the optimal action profile of game* Γ_t*, i.e.,* $\mu(t) = g^M(t)$*;*

***4**: the restriction of the intervention rule in* $r = t$ *and* $m = g^M(t)$*, i.e.,* $\Phi'_a = \Phi_{t, g^M(t), a}$*, sustains without intervention* $g^M(t)$ *in* Γ_t*.*

Proof. See Appendix B.1 □

Condition **1** is related to what is achievable by the designer in the complete information setting, condition **2** is related to the structure of the environment (which is not controllable by the designer), while conditions **3-4** say how to obtain a maximum efficiency direct mechanism once **1-2** are satisfied.

In the second result we combine condition **2** of Proposition 13 with assumptions **A3-A6** to derive a sufficient condition on the type set structures under which a maximum efficiency incentive compatible direct mechanism does not exist. We define the bin size β_k of user k's type set, T_k, as the maximum distance between two consecutive elements of T_k: $\beta_k = \max_{s\in\{1,\ldots,v_k-1\}}\left(\tau_{k,s+1} - \tau_{k,s}\right)$. We define the bin size β as the maximum among the bin sizes of all users: $\beta = \max_{k\in N} \beta_k$.

Proposition 14. *There exists a threshold bin size* $\zeta > 0$ *so that if* $\beta \leq \zeta$ *then a maximum efficiency incentive compatible direct mechanism does not exist.*

Proof. Let $k \in N$ and $t_k \in T_k$ be such that $g_k^{NE^0}(t) > g_k^M(t)$, $\forall\, t_{-i} \in T_{-i}$. We rewrite condition **2** of Proposition 13 for users k and type t_k:

$$\sum_{t_{-k}\in T_{-k}} \pi[t \mid t_k]\, U_i\left(g^M(t_k, t_{-k}), t, x^*\right) \geq \sum_{t_{-k}\in T_{-k}} \pi[t \mid t_k]\, U_i\left(g^M(t'_k, t_{-k}), t, x^*\right) \quad , \;\; \forall\, t'_k \in T_k \tag{7.2}$$

We have $h_k^{BR}(g_{-k}^M(t), t) \geq h_k^{BR}(g_{-k}^{NE^0}(t), t) = g_k^{NE^0}(t) > g_k^M(t)$, where the first inequality is valid because of the submodularity.

Let $\tilde{t}_k(t_{-k})$ be the value of user k's type so that $g^M(\tilde{t}_k(t_{-k}), t_{-k}) = h_k^{BR}(g_{-k}^M(\tilde{t}_k(t_{-k}), t_{-k}), t)$ if it exists (in this case **A3** guarantees it is greater than t_k); and $\tilde{t}_k(t_{-k}) = \tau_{k,v_k}$ otherwise. Let $\tilde{t}_k = \min_{t_{-k}} \tilde{t}_k(t_{-k})$. If $\left(t_k, \tilde{t}_k\right] \bigcap T_k \neq \emptyset$ (in particular, this is true if $\beta \leq \tilde{t}_k - t_k$), $\forall\, t'_k \in \left(t_k, \tilde{t}_k\right] \bigcap T_k$ and $\forall\, t_{-k} \in T_{-k}$ we obtain

$$U_k\left(g^M(t'_k, t_{-k}), t, x^*\right) > U_k\left(g^M(t_k, t_{-k}), t, x^*\right)$$

contradicting Eq. (7.2). □

Interpretation: when user k's type is t_k, k's resource usage that maximizes the designer's utility, $g_k^M(t)$, is lower than the one that maximizes k's utility, $h_k^{BR}(g_{-k}^M(t), t)$, $\forall\, t_{-k} \in T_{-k}$. If k reports a type t'_k slightly higher than t_k, then the intervention device suggests a slightly higher resource usage, allowing k to obtain a higher utility. Hence, k has an incentive to cheat and resources are not allocated as efficiently as possible. To avoid this situation, the intervention device might decrease the resources given to a type t'_k. In this case the loss of efficiency occurs when the real type of k is t'_k and it does not receive the resources it would deserve. These two cases are such that at least one of them corresponds to a non-zero inefficiency. Since both occur with positive probability, a positive overall inefficiency is unavoidable.

It is worth noting that we consider finite type sets and a finite intervention rule set mainly to simplify the logical exposition. However, all results might be derived also with infinite and continuous sets. In particular, if type sets are continuous Proposition 14 implies that a maximum efficiency incentive compatible direct mechanism never exists.

7.5.2 Properties of an optimal device

If a maximum efficiency device exists, the set of optimal devices in $\mathcal{D}$ coincides with the set of maximum efficiency devices in $\mathcal{D}$, that is characterized in Proposition 13. If a maximum efficiency device does not exists, the designer seeks to obtain the best he can, minimizing the loss of efficiency. He has to choose the optimal device solving the **OD** problem. However, this may be computationally hard.

In this subsection we consider some additional conditions to simplify the **OD** problem. First, we assume that the designer's utility is a function of the users' utilities (this is the case, for example, of a benevolent designer that seeks to maximize some measure of social welfare). Moreover, we suppose that, for each type profile $t \in T$, every action profile $g(t)$ lower than $g^{NE^0}(t)$ is sustainable without intervention in Γ_t. Finally, we assume that the utility of a user i adopting the lowest action a_i^{min} is always equal to 0, i.e., $U_i(a_i^{min}, a_{-i}, t, x) = 0$, $\forall\, a_{-i}, t, x$. Interpreting a_i^{min} as no resource usage, this means that, independently of types and other users' actions, a user that does not use resources obtains no utility. In particular, this last assumption implies that:

Lemma 5. *The utility of user i is non increasing in the actions of the other users.*

Proof.

$$U_i(a, t, x) = U_i(a_i^{min}, a_{-i}, t, x) + \int_{a_i^{min}}^{a_i} \frac{\partial U_i(z, a_{-i}, t, x)}{\partial z}\, \partial z = \int_{a_i^{min}}^{a_i} \frac{\partial U_i(z, a, t, x)}{\partial z}\, \partial z$$

$$\frac{\partial U_i(a,t,x)}{\partial a_j} = \int_{a_i^{min}}^{a_i} \frac{\partial^2 U_i(z,a_{-i},t,x)}{\partial z \partial a_j} \partial z \leq 0$$

where the inequality is valid because of the submodularity (see **A5**). □

Under the additional assumptions of this subsection, we can prove the following result that allows the designer to further restrict the class of mechanisms to take into consideration.

Lemma 6. *There exists an optimal device such that, for every type profile $t \in T$, the recommended action profile $\tilde{a}(t)$ is unique (i.e., μ is point mass at $\tilde{a}(t)$) and the restriction of the intervention rule in $r = t$ and $m = \tilde{a}(t)$, i.e., $\Phi'_a = \Phi_{t,\tilde{a}(t),a}$, sustains without intervention $\tilde{a}(t)$ in Γ_t.*

Proof. See Appendix B.2 □

Lemma 6 suggests the idea to decouple the original problem into two sub-problems. First, we can calculate the optimal message rule $\tilde{\mu}$ under the constraint that users adopting the recommended actions have the incentive to report their real type. Then, it is sufficient to identify an intervention rule $\tilde{\Phi}$ able to sustain $\tilde{\mu}(t)$ without intervention in Γ_t, $\forall\, t$. This is formalized in the following.

Consider the device $\tilde{D} = \langle (T_i), (A_i), \tilde{\mu}, X, \tilde{\Phi} \rangle$, where

$$\tilde{\mu} = \underset{\mu}{\operatorname{argmax}} \sum_{t \in T} \pi(t) U\left(\mu(t), t, x^*\right)$$

subject to:

$$\sum_{t_{-i} \in T_{-i}} \pi(t \mid t_i) U_i\left(\mu(t), t, x^*\right) \geq \sum_{t_{-i} \in T_{-i}} \pi(t \mid t_i) U_i\left(\mu(t'_i, t_{-i}), t, x^*\right)$$

$$\forall\, i \in N, \ \ \forall\, t_i \in T_i, \ \ \forall\, t'_i \in T_i$$

and, $\forall\, t \in T$, $\Phi'_a = \tilde{\Phi}_{t,\mu(t),a}$ sustains without intervention $\mu(t)$ in Γ_t.

Proposition 15. *$\tilde{D}$ is an optimal device.*

Proof. Lemma 6 guarantees that there exists an optimal device inside the class of devices in which, $\forall\, t$, the recommended action profile $\mu(t)$ is unique and the restriction of the intervention rule in $r = t$ and $m = \mu(t)$, i.e., $\Phi'_a = \Phi_{t,\mu(t),a}$, sustains without intervention $\mu(t)$ in Γ_t. Among all devices belonging to such class, $\tilde{D}$ is selected to maximize the designer's expected utility. Thus, $\tilde{D}$ is an optimal device. □

7.6 Algorithm that Converges to an Incentive Compatible Device

In this section we provide a practical tool for the designer to choose an efficient device. Because the optimal device may be very difficult to compute, even in the decoupled version of Proposition 15, we provide a simple algorithm that converges to an incentive compatible device D in which μ is point mass (i.e., given a report the recommended action profile is unique) and, although perhaps not optimal, still yields a ‘good’ outcome for the designer. More precisely, D will sustain *without intervention* the honest and obedient strategy profile. The algorithm has been designed with the idea to minimize the distance between the optimal action profile $g^M(t)$ and the suggested action profile $\mu(t)$, for each possible type profile t. Such algorithm is run off-line by the designer to choose an efficient device and can be used when, for every type profile t and at each step of the algorithm, the designer is able to identify a device for the complete information setting that sustains without intervention the suggested action profile $\mu(r)$ in Γ_r. (Note that the suggested action profile will never be lower than the optimal action profile $g^M(t)$ or higher than the NE action profile $g^{NE^0}(t)$ of Γ_t^0.)

Given a device D in which μ is point mass, we denote by $W_i(t_i, t_i')$ the expected utility that user i, with type t_i, obtains reporting type t_i' and adopting the suggested action, when the other users are honest and obedient and the intervention device does not intervene, i.e.,

$$W_i(t_i, t_i') = \sum_{t_{-i} \in T_{-i}} \pi(t \mid t_i) U_i(\mu(t_i', t_{-i}), t, x^*)$$

Moreover, we say that X and $\Phi_{r,m,a}$ are *induced* by μ if the device defined by X and $\Phi_a'(x) = \Phi_{r,\mu(r),a}(x)$ sustains $\mu(r)$ without intervention in Γ_r, $\forall\, r \in T$. If the designer is able to identify the device defined by X and π_a' in the complete information setting, then he can easily compute X and $\Phi_{r,m,a}$ induced by μ, obtaining a device for the general framework that, by construction, gives the users the incentive to always adopt the recommended actions (i.e., users are obedient) and does not intervene (threats of punishments do not need to be executed since users follow the recommendations).

The algorithm initializes the device D in the following way: $\mu(r) = g^M(r)$, X and Φ induced by μ. This means that, given the report profile r, the device recommends the optimal (for the designer and if user types are r) action profile $g^M(r)$ and the users will adopt it. However, this does not guarantee that the users are honest: the reported type profile may be different from the real one, i.e., $r \neq t$. To give an incentive for the users to be honest, in each step of the algorithm the recommended action profile $\mu(r)$ is modified to increase the utility the users obtain if they are honest (or to decrease the utility they obtain when they are dishonest). Whenever $\mu(r)$ is modified, also X and Φ must be modified accordingly, selecting X and Φ induced by μ such that users remain obedient.

To explain the idea behind the algorithm we exploit Fig. 7.1, where i's utility is plotted with respect to i's action, for a fixed type profile t, when all users are honest (i.e., $r = t$) and the other users are obedient (i.e., $g_j(t_j, \mu_j(t)) = \mu_j(t)$, $\forall\, j \neq i$). Each sub-picture refers to different recommended actions (i.e., different μ), and in each sub-picture four points are marked (some of which may possible coincide) representing the following cases: (1) i adopts the best (for the designer) action $g_i^M(t)$; (2) i adopts the recommended action $\mu_i(t)$; (3) i adopts the NE action $g_i^{NE^0}(t)$ (notice that it is not the best action for user i because the other users do not adopt $g_{-i}^{NE^0}(t)$); and (4) i adopts the best action $h_i^{BR}(\mu_{-i}(t), t)$.

The initialization case, in which (1) and (2) coincide, is represented by the upper-left Fig. 7.1. By assumption **A6** $g_i^M(t) \leq g_i^{NE^0}(t)$ and by assumption **A5** $g_i^{NE^0}(t) \leq h_i^{BR}(\mu_{-i}(t), t)$, because $\mu_{-i}(t) \leq g_{-i}^{NE^0}(t)$. If $W_i(t_i, t_i) \geq W_i(t_i, t_i')$, for every alternative i's reported type t_i', then user i has an incentive to report its true type t_i. If, at a certain iteration of the algorithm, this is valid for all users and for all types they may have, then the algorithm stops and we obtain a device that sustains without intervention the honest and obedient strategy profile.[14]

Conversely, suppose there exists a user i and types t_i and t_i' such that $W_i(t_i, t_i) < W_i(t_i, t_i')$, i.e., user i has an incentive to report t_i' when its type is t_i. In this case the algorithm increases the recommended action $\mu_i(t)$ by a quantity equal to ϵ_i, moving it in the direction of the best response function $h_i^{BR}(\mu_{-i}(t), t)$, for every possible combination of types t_{-i} of the other users, and X and Φ must be modified accordingly such that users remain obedient. This has the effect, as represented by upper-right Fig. 7.1, to increase the utility of user i when it is honest, $\forall\, t_{-i}$, which in turn implies that the expected utility of users i when it is honest (i.e., $W(t_i, t_i)$) increases. This procedure is repeated as long as $W_i(t_i, t_i) < W_i(t_i, t_i')$ and $\mu_i(t) \leq g_i^{NE^0}(t)$.

In case i's suggested action $\mu_i(t)$ reaches $g_i^{NE^0}(t)$ and still $W_i(t_i, t_i) < W_i(t_i, t_i')$, then the suggested action of user k, $\mu_k(t)$, is increased by a quantity equal to ϵ_k, $\forall\, k \in N$, $k \neq i$, $\forall\, t_{-i} \in T_{-i}$. As we can see from lower-left Fig. 7.1, this means to change the shape of the curve representing i's utility with respect to i's action. In particular, by assumption **A5**, the best response function $h_i^{BR}(\mu_{-i}(t), t)$ is moved in the direction of the recommended action $\mu_i(t)$.

If $\mu_k(t)$ reaches $\mu_k^{NE^0}(t)$ as well, $\forall\, k \in N$, then $\mu_i(t)$ coincides with the best response function $h_i^{BR}(\mu_{-i}(t), t)$, as represented in the lower-right Fig. 7.1. In fact, by definition, the NE is the action profile such that every user is playing its best action against the actions of the other users. Since $\mu_i(t)$

[14]Notice that, if a maximum efficiency incentive compatible direct mechanism exists, since it must satisfy the conditions of Proposition 13, then the initialization of the algorithm corresponds to a maximum efficiency incentive compatible direct mechanism and the algorithm stops after the first iteration.

coincides with $h_i^{BR}(\mu_{-i}(t), t)$, $\forall\, t_{-i} \in T_{-i}$, user i is told to play its best action for every possible combination of the types of the other users. Hence, user i cannot increase its utility reporting type t_i', i.e., it must be $W_i(t_i, t_i) \geq W_i(t_i, t_i')$.

The algorithm stops the first time every user has an incentive to declare its real type. Since at each iteration the suggested action profiles are increased by a fixed amount, the algorithm converges after a finite number of iterations. The higher the steps ϵ_i, $i \in N$, the lower the convergence time of the algorithm. On the other hand, the lower the steps, the closer the suggested action profile to the optimal one.[15]

Algorithm 1 General algorithm.

1: **Initialization**: $\mu(t) = g^M(t), \forall\, t \in T$, X and Φ induced by μ

2: **For** each user $i \in N$ and each pair of types $t_i, t_i' \in T_i$

3: **If** $W_i(t_i, t_i) < W_i(t_i, t_i')$

4: **If** $\mu_i(t) < g_i^{NE^0}(t)$ for some $t_{-i} \in T_{-i}$

5: $\mu_i(t) \leftarrow \min\Big\{\mu_i(t) + \epsilon_i,\ g_i^{NE^0}(t)\Big\}, \forall\, t_{-i} \in T_{-i}$, X and Φ induced by μ

6: **Else**

7: $\mu_k(t) \leftarrow \min\Big\{\mu_k(t) + \epsilon_k,\ g_k^{NE^0}(t)\Big\}, \forall k \in N, k \neq i, \forall\, t_{-i} \in T_{-i}$, X and Φ induced by μ

8: Repeat from 2 until 3 is unsatisfied $\forall\, i$, t_i, t_{-i}

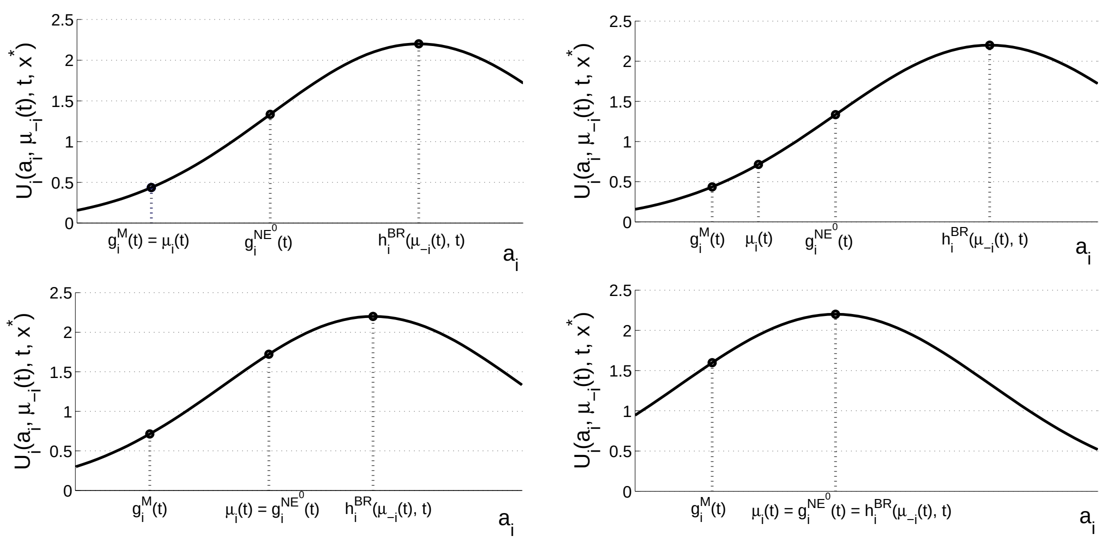


Figure 7.1. *User i's utility vs. user i's action, for different suggested actions*

[15]Notice that, since no assumption such as convexity is made for the designer's expected utility, an action profile closer to the optimal one does not necessarily imply a better outcome for the designer.

7.7 Introduction to Flow Control

In this section, we introduce the congestion problem in a store-and-forward node of a network, namely, the server. Each user connected to the server, represented by a traffic flow that enters the server, sends its packets with Poisson arrival rate. The server serves the packets, following a first-in-first-out policy, with exponentially distributed service time. The system can be modeled as an M/M/1 queue. We take into account the possibility that users belong to different classes of traffic, requiring different quality of service. The class of traffic a user belongs to is represented by the type of the user. We assume that each user can independently set its transmission rate to maximize its own utility represented by the *power*, defined in [106] as the ratio between the throughput and the delay and later extended in [107] to take into account multiple classes of traffic.

In the subsequent sections, we study the interaction between users in two different scenarios: (1) in the *complete information* scenario every user is aware of the types of the other users and their interaction can be modeled with a complete information game; (2) in the *incomplete information* scenario the users are not aware of the types of the other users, but a common probability distribution of the types of the other users exists, and the interaction can be modeled with a *Bayesian* game. We show that the self-interested and strategic nature of the users leads to the overuse of the resources and to substantial inefficiencies in both cases, which are quantified using as performance criterion the geometric mean of the users' utilities. To improve the efficiency of the network, we use a standard intervention scheme for the complete information scenario and we exploit our framework for the incomplete information scenario.

7.7.1 Related work

Flow control is a necessary operation to make a service accessible to many users. Several metrics have been considered as performance indicators. The power was proposed in [106] as a way to trade-off between throughput and delay. This concept was later extended in [107] to take into account multiple classes of traffic. To obtain distributed flow control algorithms, [108–110] model the flow control problem as a network utility maximization problem, and interpret the Lagrangian multipliers as prices. These approaches derive efficient distributed algorithms, however they assume that users are obedient in that they maximize the utilities designed by the designer, instead of their own utilities. Thus, they can not be compared with this work, in which we assume that users are strategic. The earlier applications of game theory to flow control problems were limited to the computation of the Nash equilibria of existing congestion schemes, to quantify their performance in the presence of strategic

	Obedient users	**Strategic users**	**Incentive scheme**	**Incomplete Information**	**Information revelation**
[108–110]	X				
[25,27,28,34,111]		X			
[24,30,40,41]		X	X		
[112,113]		X	X	X	
our work		X	X	X	X

Table 7.1. *Comparison among different flow control works*

users. Examples of this approach include [27,28,111] which use the power as the performance metric, and [34] that shows that most congestion control schemes used, such as TCP, encourage a behavior that leads to congestion. [25] characterizes the Nash equilibrium and the Pareto Boundary for linearly coupled communication games, leading to the same result as [28] for the particular case of the flow control game. In addition, [25] investigates the properties of an alternative solution concept named conjectural equilibrium, in which users compensate for their lack of information by forming internal beliefs about the other users. Later, with the same philosophy of our work, game theory was used to design practical schemes to deal with selfish and strategic users. [30, 40] consider pricing schemes, in which users are charged based on their resource usage, and show that if appropriate cost function and pricing mechanism are used, one can find an efficient Nash equilibrium. [41] designs the pricing scheme that maximizes the service provider's revenue instead of the users' satisfaction. [24] uses a packet-dropping scheme – a particular instance of intervention schemes – to improve the efficiency of the Nash equilibrium, allowing to arbitrarily approach the optimal social welfare. None of the above works has addressed the flow control problem in the incomplete information setting. To the best of our knowledge, the only works dealing with incomplete information, such as [112, 113], adopt a Bayesian approach, in which the expected – with respect to the unknown information – utilities are maximized. Our work differs from them in that we introduce the ideas of mechanism design [94–99] and intervention [66] to create protocols that elicit the private information of the users. Table 7.1 summarizes the differences between the described literature.

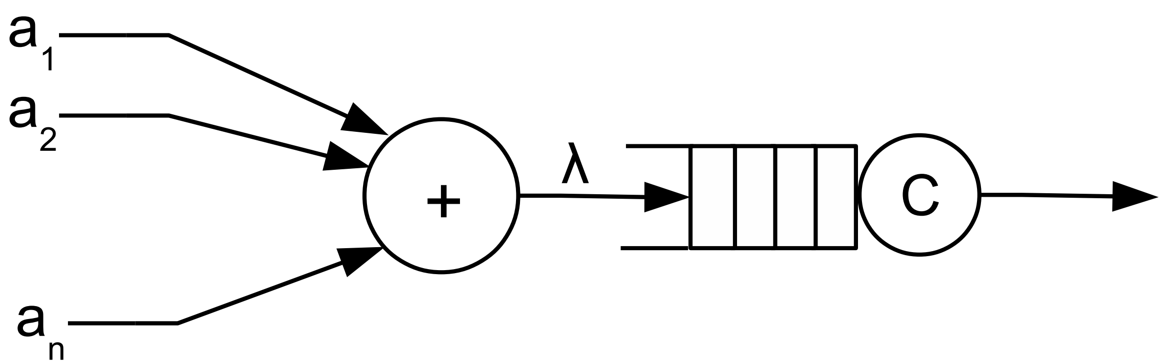


Figure 7.2. *Representation of a flow control application as an M/M/1 queue*

7.7.2 Formulation of the flow control problem

We consider n flows with Poisson arrival rates of a_1, a_2, ..., a_n that are serviced by a single server with exponentially distributed service times with mean $\frac{1}{C}$. Since we assume that all packets have the same length, we will talk interchangeably of arrival rate ($\frac{pkt}{s}$) and transmission rate ($Mbps$), and C can be seen as the channel capacity, in $\frac{pkt}{s}$, after the server.[16] We refer to each stream of packets as a user. We assume that each user i can control its own traffic (e.g., by adjusting the coding quality of its communication), i.e., it can select its transmission rate (action) $a_i \in A_i = [0,\ C]$. As represented by Fig. 7.2, the system is an M/M/1 queue with an input arrival rate $\lambda = \sum_{i=1}^{n} a_i$.

In most cases a user is faced with two conflicting objectives, i.e., to maximize its throughput[17] and to minimize its average delay. The conflict between throughput and delay is obvious since as more traffic enters the server queue the delays become larger. In order to incorporate these two measures in a single performance metric, the concept of power has been proposed in [106] and later extended in [107]. It is defined as the ratio between the throughput and the average delay, where the exponent of the throughput is a positive constant. We can therefore write i's utility as

$$U_i(a, t_i) = a_i^{t_i}\,(C - \lambda) = a_i^{t_i}\left(C - \sum_{i=1}^{n} a_i\right) \tag{7.3}$$

where $a = (a_1, \ldots, a_n)$ denotes the transmission rate (action) profile and the parameter $t_i > 0$ represents user i's type.

[16]We consider packets of the same length to keep a simple notation and because the qualitative results are not affected by this assumption. However, the model and the analysis can be easily extended to take into account packets of different lengths.

[17]Here the throughput refers to the traffic the server is able to service, i.e., the transmission rate available to the user, and does not take into account the packets lost due to physical layer transmission errors.

The value of t_i may depend, for example, on the quality of service of the application corresponding to the i-th stream of packets. As we will see in Eqs. (7.4) and (7.5), that consider compliant users and strategic users respectively, the rate adopted by a user is increasing in its type. This consideration suggests the idea that the higher the type of a user, the higher the importance of the rate, with respect to the delay, for that user. As an example, streams of packets associated to delay dependent applications should have a low type while streams of packets associated to delay tolerant applications should have a high type.

In general, the applications a server has to deal with may change over time. For this reason it is useful to define the type set T_i, for every user i, whose elements represent all the possible types user i may have. We assume that the type set is the same for all users and is finite, i.e., $T_i = T_1 = \{\tau_1, \tau_2, ..., \tau_v\}$, $v \in \mathbb{N}$, $\tau_k \in \Re$, $\tau_1 < \tau_2 < ... < \tau_v$, for every user $i \in N$. Suppose that at the beginning of a communication session the types of the users connected to the system are unknown. We assume that a common probability distribution exists and that user types are independent and identically distributed (i.i.d.) with $\pi(t_i)$ denoting the probability that a user has type t_i, $t_i \in T_1$, and $\pi(t) = \prod_{i=1}^{n} \pi(t_i)$ the probability that the type profile is t, $t \in T = T_1^n$. $\pi(t_i)$ can be thought as the average fraction of applications having type t_i that require services to the server.

The network must be designed to operate efficiently following the manager's objective, which can be quantified by a utility function. We assume that the manager's utility is the geometric mean of the users' utilities:

$$U(a,t) = \sqrt[n]{\prod_{i=1}^{n} U_i^+(a,t_i)} = (C-\lambda)^+ \prod_{i=1}^{n} a_i^{\frac{t_i}{n}}$$

where $(x)^+ = \max\{x\ ,\ 0\}$.[18] This choice allows to maintain a balance between two competing interests a benevolent manager might have: to maximize the social welfare of the network (defined as the sum utility) and to allocate resources fairly, giving to users similar utilities. Notice that maximizing $U(a,t)$ with respect to users' actions is equivalent to maximizing a proportional fairness of users' utilities, i.e., $\sum_{i=1}^{n} \ln U_i^+(a,t_i)$, and the optimal solution $g^M(t) = \left(g_1^M(t), \ldots, g_n^M(t)\right)$, as a function of users' types, is given by (see [25])

$$g_i^M(t) = \frac{t_i C}{n + \sum_{k=1}^{n} t_k} \tag{7.4}$$

[18] We consider U_i^+ instead of U_i for mathematical reasons, because utilities as defined in Eq. (7.3) may also be negative, and the geometric mean would lose meaning with negative quantities. Anyway, notice that it is in the interest of both the users and the manager to have $\lambda \leq C$, i.e., working in the sub-space of the original domain such that $U_i^+ = U_i$.

We denote by $EU_i(g, t_i)$ and $EU(g)$ the expected (with respect to the types) utilities of user i having type t_i and of the manager, where $g(t) = (g_1(t), \ldots, g_n(t))$ represents the actions adopted by the users when the type profile is t, i.e.,

$$EU_i(g, t_i) = \sum_{t_{-i} \in T_{-i}} \pi(t_{-i}) U_i(g(t), t) \quad , \quad EU(g) = \sum_{t \in T} \pi(t) U(g(t), t)$$

The benchmark optimum for the manager – the maximum expected utility he could achieve if users were compliant to a prescribed scheme – is therefore equal to $EU^{ben} = EU(g^M(t))$.

7.8 Flow Control Games Without Intervention

In this Section we compute the outcome of a flow control problem considering self-interested and strategic users, for both the complete and the incomplete information scenarios. Moreover, we quantify the loss of efficiency of the manager's utility with respect to the maximum efficiency utility.

7.8.1 Complete information scenario

We define the complete information game

$$\Gamma_t^0 = (N, A, \{U_i\}_{i=1}^n)$$

where each user i selects its action $g_i(t)$ strategically, knowing the types t_{-i} of all the other users.

We denote by $g^{NE^0}(t) = \left(g_1^{NE^0}(t), \ldots, g_n^{NE^0}(t)\right)$ a *Nash Equilibrium* (NE) of the game Γ_t^0. The unique NE $g_i^{NE^0}(t)$ of Γ_t^0 is, $\forall\, i \in N$, (see [25])

$$g_i^{NE^0}(t) = \frac{t_i C}{1 + \sum_{k=1}^n t_k} \tag{7.5}$$

Notice that strategic users use the resources more heavily with respect to compliant users, i.e., $g_i^{NE^0}(t) > g_i^M(t)$, $\forall\, i \in N$ and $\forall\, t \in T$ (excluding the trivial case $n = 1$).

The manager's expected utility in the complete information scenario is equal to $EU(g^{NE^0}(t))$.

7.8.2 Incomplete information scenario

We define the incomplete information game

$$\Gamma^0 = (N, A, T, \pi, \{U_i\}_{i=1}^n)$$

where each user i selects its action $g_i(t_i)$ strategically, knowing its own type t_i and the probability distribution of the types of the other users, $\pi(t_{-i})$.

We denote by $g^{BNE}(t) = \left(g_1^{BNE}(t_1), \ldots, g_n^{BNE^0}(t_n)\right)$ a *Bayesian Nash Equilibrium* (BNE) of the game Γ^0.

Proposition 16. *There exists a unique Bayesian Nash Equilibrium $g^{BNE}(t)$ of Γ^0 which can be obtained by solving a linear system $\mathbf{A}g^{BNE} = b$. In addition, the inverse of $\mathbf{A}$, $\mathbf{A}^{-1}$, can be computed analytically.*[19]

Proof. See Appendix B.3. □

The manager's expected utility in the incomplete information scenario is equal to $EU(g^{BNE}(t))$.

7.8.3 Illustrative results

Fig. 7.3 shows the manager's expected utility with respect to the number of users, considering $C = 5\,Mbps$ and a type set $T_1 = \{0.1,\ 1\}$ with uniformly distributed types. The upper curve represents the benchmark optimum, while the dashed and the dotted lines represent the manager's expected utility when users are strategic, in the complete and incomplete information cases, respectively. The manager's utility when users act strategically, for both the complete and the incomplete information scenarios, is far below the benchmark optimum. Notice that the manager can obtain a higher utility in the incomplete information scenario with respect to the complete information scenario, at least when there are more than three users in the system. This agrees with the results of [114, 115] where, in a strategic setting, the less closely related the agents' goals the lower the quantity of information they prefer to exchange. In our case, the objective of the manager becomes less closely related to the objective of a single user as the number of total users increases. In fact, the manager's objective is to increase the utility of all users in a fair way, while the goal of a user is to improve only its own utility, at the cost of the utility of all the other users. Hence, as the number of users increases, the selfishness of a single user has a higher negative impact on the manager's objective.

7.9 Flow Control Games with Intervention

Fig. 7.2 shows that the manager's expected utility in strategic settings (for both the complete and the incomplete information scenarios) is much lower than the benchmark optimum. Here we ask whether the manager can do something to make the system robust against strategic users, filling, at least partially, the gap between the benchmark optimum and the manager's expected utility in strategic settings.

[19] The expressions of b, $\mathbf{A}$ and $\mathbf{A}^{-1}$ can be found in Appendix B.3

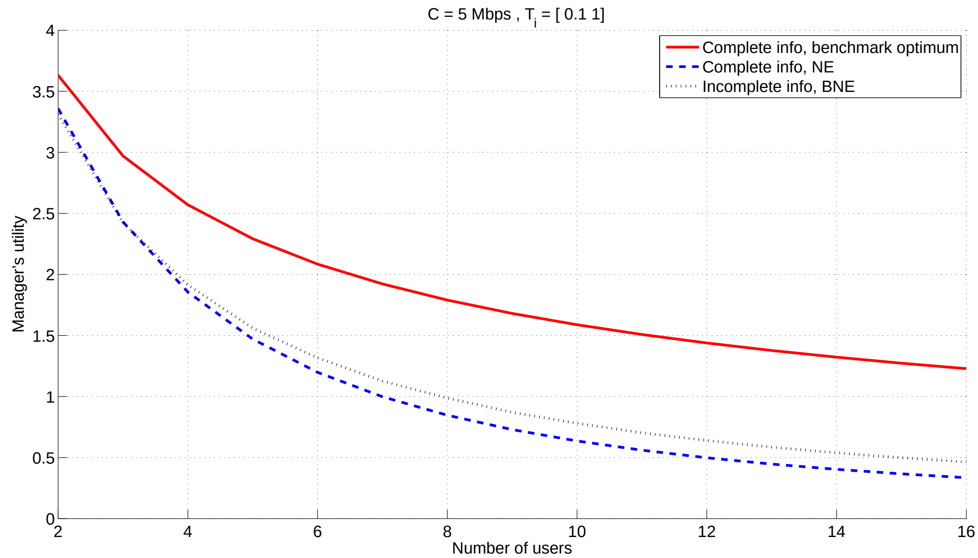


Figure 7.3. *Intervention device utility with respect to the number of users*

We assume that the manager can choose and deploy a *device* in the system that (1) can receive communications from users, (2) can send communications to users, (3) can monitor the actions of users, and (4) can transmit a stream of packet $x \in X = [0, \, \overline{x}]$ to the server, which we interpret as an *intervention*. The intervention action increases the incoming traffic of the server $\lambda = \sum_{i=1}^{n} a_i + x$, and the users' and the manager's utilities change accordingly:

$$U_i^I(a, t_i, x) = a_i^{t_i} \, (C - \lambda) = a_i^{t_i} \left(C - \sum_{i=1}^{n} a_i - x \right) \tag{7.6}$$

$$U^I(a, t, x) = \sqrt[n]{\prod_{i=1}^{n} U_i^{I+}(a, t_i, x)} = (C - \lambda)^+ \prod_{i=1}^{n} a_i^{\frac{t_i}{n}}$$

It is straightforward to check that the users' and the manager's utilities satisfy assumptions **A1-A6**. In particular, the manager's preferred action is $x^* = 0$ (i.e., no intervention), and the game Γ_t^0 defined in Subsection 7.8.2 coincides with the game Γ_t^0 defined in Subsection 7.4.1.

In the complete information scenario, the intervention device is a tool the manager employs to instruct the users on how to behave, giving them the incentive to adopt efficient actions by *threatening punishments* which are *not executed* if users follow the recommendations. In addition, in the incomplete information scenario, the device is also used to retrieve the relevant information from the users, i.e., their types. First, we formalize a device in the more general scenario of incomplete information [20], and we will then discuss the natural simplifications for the complete information scenario.

[20] The formalization will be similar to the one introduced in Section 7.2, but here we assume there are no errors, reports

A device is a tuple $D = \langle \mu, \overline{x}, \Phi \rangle$ where

- $\mu : T \to A$ is the *message rule*, which specifies the recommendations to be sent to the users as a function of the reported types. If $r \in T$ are the reports we write $m = \mu(r)$ for the recommended actions, $m = (m_1, \ldots, m_n) \in A$;
- $\overline{x}$ represents the maximum rate the intervention device is able to transmit;
- $\Phi : T \times A \times A \to X$ is the *intervention rule*, which specifies the transmission rate the device will adopt given the received reports, the transmitted messages and the users' adopted actions. If r are the reports, m the transmitted messages and a the users' adopted actions, we write $\Phi(r, m, a)$ for the adopted intervention action.

In the incomplete information scenario, a strategic user i selects its report r_i and its action a_i in order to maximize its expected utility given the information and the beliefs it has. Specifically, a strategy for user i consists of a pair of functions (f_i, g_i), in which $f_i : T_1 \to T_1$ specifies the report of user i based on its type, and $g : T_1 \times A_i \to A_i$ specifies the action of user i based on its type and on the recommendation received. As usual, we denote by f and g the profiles of the two strategies.

In the following, we summarize the different stages of the interaction between the users and the intervention device in the incomplete information scenario.

Stage 1: each user i sends the report $r_i = f_i(t_i)$ to the intervention device

Stage 2: the intervention device sends the recommended action $m_i = \mu(r)$ to each user i

Stage 3: each user i takes the action $a_i = g_i(t_i, m_i)$

Stage 4: the intervention device monitors the users' action profile[21] a and adopts the intervention action $x = \Phi(r, m, a)$

Here we restrict the attention to the class of *affine intervention* devices $\mathcal{D}$, in which the intervention level increases linearly with the users' actions. It may seem restrictive to only consider such a

and messages are costless and we do not consider *randomized* rules because, as we will see, *pure* rules are sufficient to obtain optimal results in the complete information case and to satisfy the conditions we need to use the algorithm in the incomplete information case.

[21]The device can estimate the users' rates by counting in real time the number of packets that each user has sent since the beginning of the communication session. Such estimates may be inaccurate, in particular in the first phases of the session. Here we neglect this issue, implicitly assuming that the session is long enough (with respect to the users' rates) to converge very soon to accurate estimations. We will take into consideration an extension of this work in which we analyze in more detail the impact of imperfect monitoring.

simple class of devices. However, $\mathcal{D}$ will turn out to be optimal, i.e., it is not possible to increase the manager's expected utility by considering more complex devices.

$D = \langle \mu, \overline{x}, \Phi \rangle$ is an affine intervention device if the intervention rule Φ is of the form

$$\Phi(r, m, a) = \left[\sum_{i=1}^{n} c_i(r, m) \left(a_i - \tilde{a}_i(r, m) \right) \right]_0^{\overline{x}}$$

where $\tilde{a}_i(r, m) \geq 0$ represents a target action for user i, $c_i(r, m) \geq 0$ is the rate of increase of the intervention level due to an increase of i's action, and $[\cdot]_a^b = \min\{\max\{a, \cdot\}, b\}$. Though in this abstract definition $\tilde{a}_i(r, m)$ might be different from the recommended action $m_i = \mu(r)$, in the schemes we will propose in the following we will have $\tilde{a}_i(r, m) = m_i$, $\forall\, r$, m_{-i}, i.e., $\tilde{a}(r, m)$ will represent the recommended action profile.

Fig. 7.4 shows how the intervention rule Φ changes the relation between i's utility and i's action, for given type, report and message profiles and assuming that the other users adopt the target action profile $\tilde{a}_{-i}(r, m)$. The utility of user i is plotted for three different values of the parameter $c_i(r, m)$ ($c_i(r, m) = 0$ means that the intervention device never intervenes). For an action a_i lower than the target action $\tilde{a}_i$, i's utility is as if the device did not exist. However, for an action a_i higher than the target action $\tilde{a}_i(r, m)$, i's utility is lower compared to the utility it would have obtained without the device, and the gap increases as c_i increases. In fact, if the users adopt the target action profile $\tilde{a}(r, m)$ the intervention level is 0, but if a single user i deviates from the recommendation adopting an action $a_i > \tilde{a}_i(r, m)$, the intervention device reacts transmitting a flow of packets with a positive rate $x = \Phi(r, m, a)$, that is increasing in $c_i(r, m)$ and affects the utility of every user. This agrees with our view of intervention as a threat of punishments which are not executed if all users follow the recommendations.

As noted before, in the complete information scenario the interaction between the users and the device can be simplified, because the type profile t is known by everybody. In particular, since the device already knows t, the reports do not play any role and we can consider $f_i(t_i) = t_i$, $\forall\, i$ (or, alternatively, we can skip **Stage 1**). Moreover, the users know in advance the messages they will receive because messages are a deterministic function of the type profile (hence, also **Stage 2** can be skipped). Finally, since reports and messages are given, the intervention rule can simply be written as a function of the users' actions: $\overline{\Phi}(a) = \Phi(t, \mu(t), a)$. In particular, for an affine device the parameters $c_i = c_i(t, \mu(t))$ and $\tilde{a}_i = \tilde{a}_i(t, \mu(t))$ are constant. Thus, in the complete information scenario a device is simply described by $\overline{x}$ and $\overline{\Phi}$. In this context, each user i has to select an action a_i to maximize its utility.

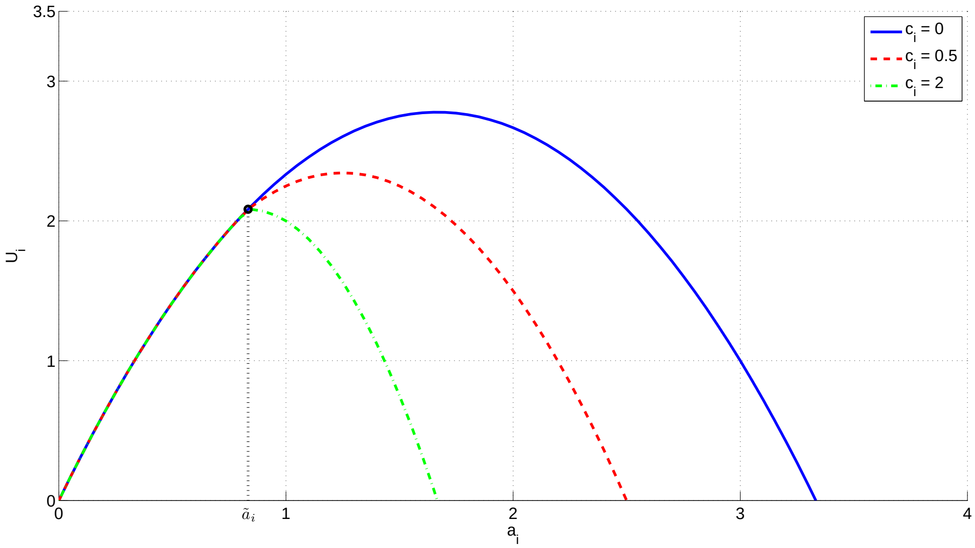


Figure 7.4. *Manager's expected utility vs. number of users for the complete and incomplete information scenarios*

Given a device D, in both the complete and the incomplete information cases, the interaction among users can be modeled as a game. In the following we provide the tools for the manager to choose a device in the class $\mathcal{D}$, for both the complete and the incomplete information scenarios.

7.9.1 Complete information scenario

In the complete information scenario, given a device $D = \langle \overline{x}, \overline{\Phi} \rangle$, the interaction among users is modeled with the game

$$\Gamma_t = \left(N, A, D, \{U_i^I\}_{i=1}^n\right)$$

in which each user i strategically selects the action $g_i(t)$ (the dependence on t shows that if the type profile t changes, the game Γ_t changes as well and the users may decide to take different actions) to maximize its utility U_i^I, see Eq. (7.6).

The outcome of such interaction is represented by the NE. The manager faces the problem of choosing a device D so that there exists a NE of the game Γ_t that gives it the highest utility U^I among what is achievable with all possible NEs.

Lemma 7. *Consider the affine device D such that, $\forall\, i \in N$,*

$$c_i \geq \frac{t_i\left(C - \sum_{k=1}^n \tilde{a}_k\right) - \tilde{a}_i}{\tilde{a}_i} \quad , \quad \overline{x} \geq \frac{c_i\left[t_i\left(C - \sum_{k=1}^n \tilde{a}_k\right) - \tilde{a}_i\right]}{1 + t_i(1 + c_i)} \tag{7.7}$$

If $\tilde{a} \leq g^{NE^0}$, then $\tilde{a}$ is a NE of Γ_t.

Proof. See Appendix B.4 □

Interpretation: If a c_i high enough is selected, and if the device is able to transmit with a large enough transmission rate, the threat of punishment discourages the users from adopting actions higher than the target. This situation is shown in Fig. 7.4 for $c_i = 2$. Hence, if the utility of user i is increasing before the target action $\tilde{a}_i$ (in particular, this is valid if $\tilde{a}_i \leq g_i^{NE^0}$), as in Fig. 7.4, the target action $\tilde{a}_i$ becomes the best response action for user i.

Proposition 17. *$\forall\, t \in T$, the optimal profile $g^M(t)$ is sustainable without intervention in Γ_t, adopting the device $\overline{x} \geq \dfrac{C}{1+\tau_1}$, $\tilde{a} = g^M(t)$ and $c_i \geq n-1$, $i \in N$.*

$\forall\, t \in T$, every strategy profile $a \leq g^{NE^0}(t)$ is sustainable without intervention in Γ_t, adopting the device $\overline{x} \geq C$, $\tilde{a} = a$ and c_i high enough (i.e., $c_i \geq \frac{\tau_v (C - \sum_{k=1}^n \tilde{a}_k) - \tilde{a}_i}{\tilde{a}_i}$), $i \in N$.

Proof. First, consider the second affirmation. The condition of Eq. (7.7) on $\overline{x}$ is automatically satisfied if the right hand side is lower than 0. Moreover, if it is higher than 0, the right hand side is increasing in c_i. In fact, the function $h(c_i) = \frac{ac_i}{b+ac_i}$, with $a, b \geq 0$, is increasing in c_i, because $h'(c_i) = \frac{ab}{(b+ac_i)^2} > 0$. Thus, the condition of Eq. (7.7) on $\overline{x}$ becomes stricter as c_i increases. Taking the limit for $c_i \to +\infty$ we can find the following stricter condition on $\overline{x}$ that does not depend on c_i:

$$\overline{x} \geq \frac{t_i\,(C - \sum_{k=1}^n \tilde{a}_k) - \tilde{a}_i}{t_i} = C - \sum_{k=1}^n \tilde{a}_k - \frac{\tilde{a}_i}{t_i}$$

In order to obtain conditions that are independent of users' types and action profiles to sustain, we can consider the following stricter conditions:

$$\overline{x} \geq C - \sum_{k=1}^n \tilde{a}_k - \frac{\tilde{a}_i}{\tau_v} \quad , \quad \overline{x} \geq C - \sum_{k=1}^n \tilde{a}_k \quad , \quad \overline{x} \geq C \tag{7.8}$$

As for c_i, we can find a stricter condition independent of users' types substituting t_i with τ_v. Thus, once the action profile to sustain is fixed, it is sufficient to select a c_i satisfying

$$c_i \geq \frac{\tau_v\,(C - \sum_{k=1}^n \tilde{a}_k) - \tilde{a}_i}{\tilde{a}_i} \tag{7.9}$$

Now consider the first affirmation. Substituting $\tilde{a} = g^M(t)$ we obtain

$$c_i \geq \frac{t_i\,(C - \sum_{k=1}^n \tilde{a}_k)}{\tilde{a}_i} - 1 = n + \sum_{k=1}^n t_k - \frac{t_i C \sum_{k=1}^n t_k}{t_i C} - 1 = n - 1$$

As to $\overline{x}$, substituting $\tilde{a} = g^M(t)$ into the second condition of Eq. (7.8) we obtain

$$\overline{x} \geq \frac{nC}{n + \sum_{k=1}^n t_k}$$

Finally, since the right hand side is decreasing in $\sum_{k=1}^{n} t_k$, a stricter condition can be obtained substituting $t_k = \tau_1, \forall k \in N$, obtaining

$$\overline{x} \geq \frac{C}{1+\tau_1}$$

□

If the device is able to transmit a stream of packets with a rate higher than a certain threshold (that is upper-bounded by C), if $\tilde{a} = g^M(t)$ and if $c_i \geq n-1$, the threat of punishments is an incentive for the users to adopt the optimal action profile $g^M(t)$. Note that, in this case, the *punishments are not executed*. Thus, the manager can extract the maximum utility from the game Γ_t. The following corollary is an implication of this consideration.

Corollary 5. *The class of affine intervention rules $\mathcal{D}$ is optimal (i.e., it is not possible to gain more by considering more complex devices) in the complete information scenario.*

Finally, the manager's expected utility for the complete information scenario with intervention device is equal to the maximum efficiency utility $EU(g^M(t))$.

7.9.2 Incomplete information scenario

In the incomplete information scenario, given a device $D = \langle \mu, \overline{x}, \Phi \rangle$, the interaction among users is modeled with the Bayesian game

$$\Gamma = \left(N, A, T, \pi, D, \{U_i^I\}_{i=1}^{n} \right)$$

in which each user i strategically adopts the functions $f_i : T_1 \to T_1$ (which specifies the report of user i based on its type) and $g : T_1 \times A_i \to A_i$ (which specifies the action of user i based on its type and on the recommendation received) to maximize its expected utility

$$EU_i^I(f, g, t_i, D) = \sum_{t_{-i} \in T_{-i}} \pi(t_{-i}) U_i^I(a, t, x)$$

where, $\forall\, i \in N$,

$$r_i = f_i(t_i)\ ,\ m = \mu(r)\ ,\ a_i = g_i(t_i, m_i)\ ,\ x = \Phi(r, m, a)$$

The outcome of such interaction is represented by the BNE. The manager faces the problem of choosing a device D so that there exists a BNE of the game Γ that gives it the highest expected utility

$$EU^I(f, g, D) = \sum_{t \in T} \pi(t) U^I(a, t, x)$$

among what is achievable with all possible BNEs.

In the following we apply the results derived in Subsection 7.5.1 to find the conditions for the existence and to compute a maximum efficiency device, that allows the manager to achieve its benchmark optimum. In case such device does not exist, the network cannot operate as efficiently as in the compliant users scenario. Moreover, in this case the optimal device is hard to compute. For this reason, we consider two suboptimal devices which are easier to compute than the optimal device.

7.9.2.1 Existence and calculation of a maximum efficiency device

We wonder if there are some conditions under which the manager can select a device to obtain the same utility it would achieve with compliant users. The following result provides an answer to this question.

Proposition 18. *If* $\forall\, l = \{1, \ldots, v-1\}$ *and* $\forall\, t_{-i} \in T_{-i}$,

$$\left(\frac{n + \sum_{j \neq i} t_j + \tau_{l+1}}{n + \sum_{j \neq i} t_j + \tau_l} \right)^{\tau_l + 1} \left(\frac{\tau_l}{\tau_{l+1}} \right)^{\tau_l} \geq 1 \tag{7.10}$$

then the affine device $\mu(t) = g^M(t)$, $\overline{x} \geq \dfrac{C}{1+\tau_1}$, $\tilde{a}_i(r, m) = m$ *and* $c_i \geq n-1$, $i \in N$, *is a maximum efficiency incentive compatible device.*

Proof. See Appendix B.5 □

Notice that all maximum efficiency incentive compatible devices must be of the form $\mu(t) = g^M(t)$, $\overline{x} \geq \dfrac{C}{1+\tau_1}$, $\tilde{a}_i(r, m) = m$ and $c_i \geq n - 1$, $i \in N$. However, if condition (7.10) is not satisfied, the device might not be able to give to the users the incentive to report truthfully.

7.9.2.2 Algorithm that converges to an incentive compatible device

Here we specialize, for the flow control application, the general algorithm proposed in Subsection 7.6 that converges to an incentive compatible device. Prop. 17 guarantees that, at each step of the algorithm, the considered device sustains without intervention the suggested action profile $\mu(r)$ in Γ_r (note that the suggested action profile will never be higher than $g^{NE^0}(t)$).

The algorithm has been designed with the idea of minimizing the distance between the optimal action profile $g^M(t)$ and the suggested action profile $\mu(t)$, for each possible type profile t. If a maximum efficiency device exists, the initialization of the algorithm corresponds to a maximum efficiency incentive compatible device and the algorithm stops after the first iteration. If a user i having type τ_s

can benefit by pretending to be of type τ_l , for each type profile $\tau = (\tau_l, t_{-i})$ the algorithm increases the recommended action for user i – if it is lower than $g_i^{NE^0}(\tau)$ – or the recommended actions for the other users. In both cases, the new device is selected such that the new suggested action profile $\mu(\tau)$ is sustained without intervention in Γ_τ. Proceeding in this way, the algorithm will converge to a device in which no user can benefit by pretending to be of another type.

Algorithm 2 Flow control algorithm.

1: **Initialization**: $\forall\, r \in T, \overline{x} \geq C, \mu(r) = g^M(r), \tilde{a}(r, \mu(r)) = \mu(r), c_i(r, \mu(r)) \geq n - 1$
2: **For** $s = 1 : v$ and $l = 1 : v$
3: **If** $W_i(\tau_s, \tau_s) < W_i(\tau_s, \tau_l)$
4: **For** $t_{-i} \in T_{-i}$
5: $\tau \leftarrow (\tau_l, t_{-i})$
6: **If** $\mu_i(\tau) < g_i^{NE^0}(\tau)$
7: $\mu_i(\tau) \leftarrow \min\left\{\mu_i(\tau) + \epsilon_i,\ g_i^{NE^0}(\tau)\right\}$, $\tilde{a}(\tau, \mu(\tau)) = \mu(\tau)$, $c_i(\tau, \mu(\tau))$ satisfying (7.9)
8: **Else for** $k = 1 : m, k \neq i$
9: $\mu_k(\tau) \leftarrow \min\left\{\mu_k(\tau) + \epsilon_k,\ g_k^{NE^0}(\tau)\right\}$, $\tilde{a}(\tau, \mu(\tau)) = \mu(\tau)$, $c_k(\tau, \mu(\tau))$ satisfying (7.9)
10: Repeat from 2 until 3 is unsatisfied $\forall\, s, l$

7.9.2.3 Communication-free device

In this Subsection we define a new type of device, called *communication-free device*, in which reports do not play any role for the final outcome, i.e., the message and intervention rules do not depend on reports. This is particularly useful in situations where it is not possible for the users to communicate with the device, or where communication is very expensive. However, also for scenarios where users can send reports, a communication-free device might represent a good sub-optimal device that is efficient and easy to compute.

Consider the communication-free device D that, independently of users' types, suggests action profile $\overline{a}$,

$$\begin{aligned} &\overline{a} = \operatorname*{argmin}_a \left[-\ln\left(C - \sum_{i=1}^n a_i \right) \mathbb{E}_t \left[\prod_{i=1}^n a_i^{\frac{t_i}{n}} \right] \right] \\ &a_i \geq 0 \ \ , \ \ a_i \leq C \ \ , \ \ \forall i \in N \end{aligned} \qquad (7.11)$$

Proposition 19. *Eq. (7.11) defines a convex problem if* $\tau_v \leq n$. *Moreover, if the device* D *sustains* $\overline{a}$

without intervention in Γ*, then* D *is an optimal communication-free incentive compatible device and the manager's expected utility is* $EU(\overline{a})$*.*

Proof. See Appendix B.6 □

Corollary 6. *Consider the communication-free device* D *such that,* $\forall\, r \in T$ *and* $\forall\, i \in N$*,*

$$\mu(r) = \overline{a} \;\;,\;\; \overline{\Phi}(r, \overline{a}, a) = \left[\sum_{i=1}^{n} c_i(\overline{a})(a_i - \overline{a}_i)\right]_0^{\overline{x}} \;\;,\;\; c_i(\overline{a}) \geq \frac{\tau_v\left(C - \sum_{k=1}^{n} \overline{a}_k\right) - \overline{a}_i}{\overline{a}_i} \;\;,\;\; \overline{x} \geq C \tag{7.12}$$

If $\overline{a} \leq g^{NE^0}(t)$*,* $\forall\, t \in T$*, then* D *is an optimal communication-free incentive compatible device and the manager's expected utility is* $EU(\overline{a})$*.*

Proof. It is sufficient to show that D sustains $\overline{a}$ without intervention in Γ. Notice that $\overline{\Phi}(r, \overline{a}, \overline{a}) = 0$, so it is sufficient to show that $\overline{a}$ is an equilibrium in Γ. Notice that D satisfies the conditions of Lemma 7, $\forall\, t \in T$, therefore D sustains $\overline{a}$ in Γ_t, i.e., $\forall\, t \in T \;,\; \forall\, i \in N \;,\; \forall\, t_i \in T_1 \;,\; \forall\, \hat{a}_i \in A_i$,

$$U_i\left(\overline{f}, \overline{a}, t\right) \geq U_i\left(\overline{f}, \hat{a}_i, \overline{a}_{-i}, t\right)$$

As a consequence, $\forall\, i \in N \;,\; \forall\, t_i \in T_1 \;,\; \forall\, \hat{a}_i \in A_i$,

$$\sum_{t_{-i} \in T_{-i}} \pi(t_{-i}) U_i\left(f, a, t\right) \geq \sum_{t_{-i} \in T_{-i}} \pi(t_{-i}) U_i\left(f, \hat{a}_i, a_{-i}, t\right)$$

Hence, $\overline{a}$ in an equilibrium in Γ. □

Notice that $\overline{a} \leq g^{NE^0}(t)$, $\forall\, t \in T$, is a sufficient condition such that D is an optimal communication-free incentive compatible mechanism, but it is not necessary. In fact, D might sustain $\overline{a}$ without intervention in Γ even if $\overline{a} \nleq g^{NE^0}(t)$ for some $t \in T$.

7.9.3 Illustrative results

In the following we are going to quantify the manager's expected utility and the expected throughput and delay for each type of user in different scenarios. We consider $C = 5\, Mbps$ and a common type set $T_1 = \{0.1, 1\}$. Except for Fig. 7.6, we assume that the types are uniformly distributed, i.e., $P(0.1) = P(1) = 0.5$, and we plot the results varying the number of users from 2 to 16.

We first look at how the manager's expected utility varies increasing the number of users, in the complete and incomplete information scenarios. The left side of Fig. 7.5 refers to the complete information scenario. The overlapped upper lines represent the manager's expected utility when users are compliant and when they are strategic with the device derived in Subsection 7.9.1. The manager's expected utility is decreasing in the number of users because, as the number of users increases, the total congestion experienced by every user increases as well. However, it is remarkable that with the intervention scheme the manager can completely fill the gap between the benchmark optimum and its expected utility when the users are strategic but no incentive scheme is adopted (dotted line). The right side of Fig. 7.5 refers to the incomplete information scenario. In this scenario the manager is guaranteed to achieve the benchmark optimum using the device derived from the algorithm (dashed line) if the number of users is sufficiently small. In fact, for a number of users less than or equal to 3, it is straightforward to check that the sufficient condition (7.10) is satisfied, hence, a maximum efficiency device exists and the algorithm converges to it. For a larger number of users, there is no guarantee of optimality, and in fact the results of Fig. 7.5 show that in this case the manager's expected utility is lower than what could be obtained with compliant users. However, the manager can still considerably increase its expected utility compared to the case of strategic users and no incentive scheme (dotted line), by adopting the device derived from the algorithm for a number of users lower than 8 and the communication-free device (dash-dot line) for a number of users greater than or equal to 8 ($\overline{f}$ defined in (7.12) turns out to sustain the solution of (7.11) without intervention in Γ). It is not surprising that the communication-free device is able to obtain good performance for a large number of users, in fact in this situation the manager is able to foresee more accurately the fraction of users of a certain type, hence the information about users' types becomes less important.

Now we investigate how the results depend on the type probability distribution for the incomplete information scenario. In Fig. 7.6 we fix the number of users to 4 and we vary the probability of the low type, $P(0.1)$, from 0 to 1, which is equivalent to varying $P(1)$ from 1 to 0. We can see that the gap between the benchmark optimum and the manager's expected utility achievable with the device derived from the algorithm is not strongly dependent on the type probability distribution. In fact, such a mechanism provides incentives for each type of user to be honest and obedient, even though some user types rarely occur. Notice that in the algorithm the recommended action profile for a certain type profile is increased by a finite amount ϵ if the users do not have an incentive to report truthfully, which has the effect to produce the little step visible in Fig. 7.6 (the lower ϵ, the smoother the step). On the contrary, the communication-free device is strongly dependent on the probability distribution of user types. In fact, the recommended and enforced action profile depends exclusively on the type probability distribution. As an example, if the low type occurs rarely, the device will suggest to the users to adopt an action profile that is close to the objective of the users with high type, that will probably be the majority of the users in the network. In the extreme case, if low type users are for sure

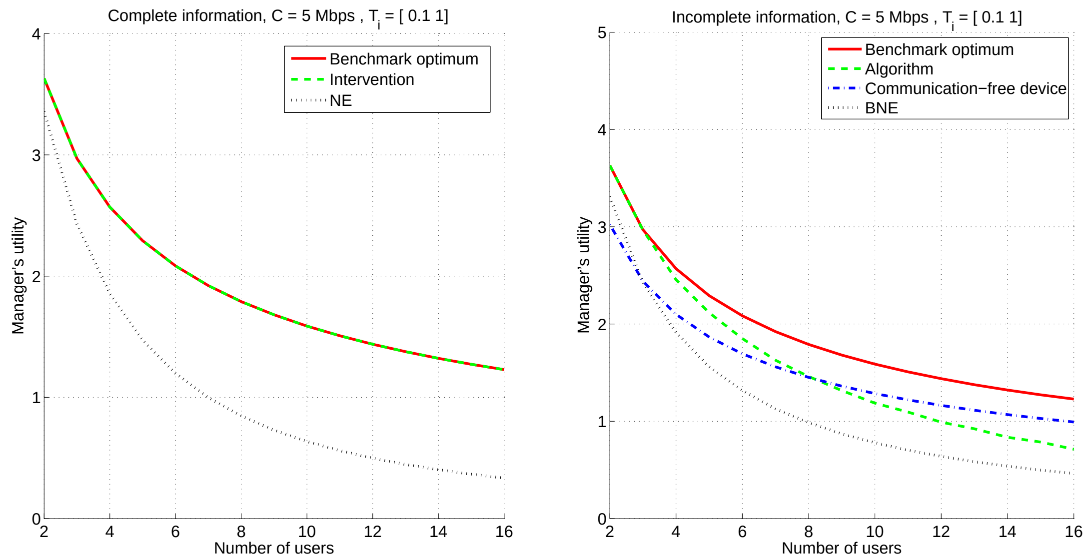


Figure 7.5. *Manager's expected utility vs. number of users for the complete and incomplete information scenarios*

not present in the network (i.e., P(0.1) = 0), than the adopted action profile will maximize the interests of the users having high type and the communication-free device is able to achieve the benchmark optimum. Notice that in this situation the manager has no uncertainty about the types of the users in the network, which is the reason why it is able to extract the maximum utility. In some sense, the uniform probability distribution represents the worst case for the communication-free device because the manager has the highest uncertainty over the types of the users in the network.

So far we have only considered the utility as performance indicator. However, the utility includes the two real performance metrics, throughput and delay. Now we investigate the expected throughput and delay achievable with the considered schemes in the complete and incomplete information scenarios, for each type of user.[22] Fig. 7.7 shows the expected throughput (left-side) and delay (right-side) for the complete information scenario. Continuous lines refer to the high type users, while dashed lines refer to the low type users. Notice that the high type users obtain a higher expected throughput and a higher expected delay compared to the low type users (this will be true also for the incomplete information scenario), confirming that the higher the type the higher the user's preference for throughput with respect to delay. In both pictures, the upper (continuous and dashed) lines refer

[22]Notice that all users in the network experience the same delay. However, such delay depends on the type profile: the higher the number of high type users with respect to the number of low type users, the higher the delay. Thus, the expected delay for a low type user is lower than the expected delay for a high type user.

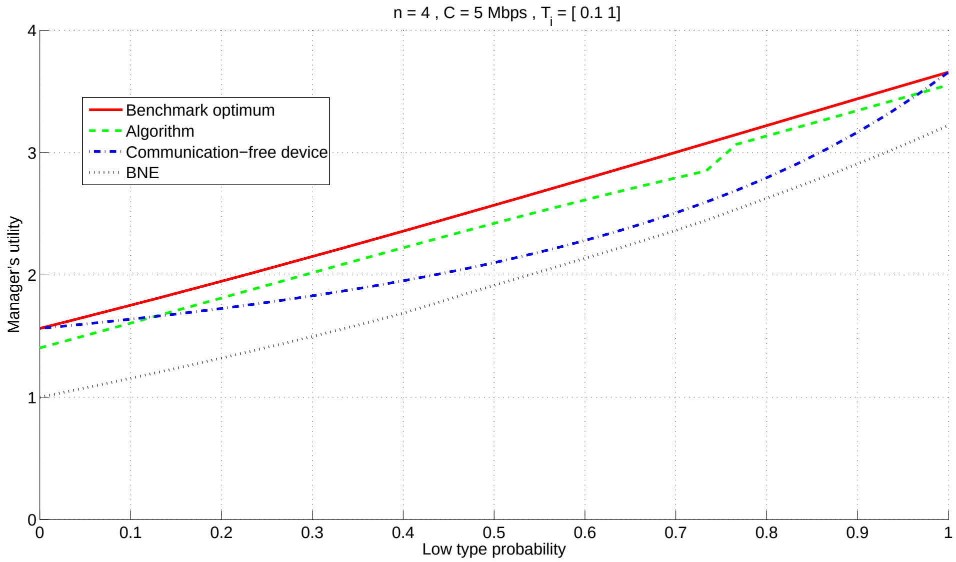


Figure 7.6. *Manager's expected utility vs. low type probability for the incomplete information scenario*

to the strategic scenario without intervention device, in which the users adopt the NE action profile, while the overlapped lower (continuous and dashed) lines represent the optimal action policy, obtainable with compliant users or with strategic users with the device derived in Subsection 7.9.1. With no incentive scheme, strategic users tend to overuse the resources of the network, transmitting with higher rates compared to the optimal ones. This translates into much higher delays, that increase quickly as the number of users increases. Conversely, the optimal transmission policy is such that the expected delay is almost constant with respect to the number of users. This means that also the aggregate throughput is almost constant, and the rate of each user scales as $\frac{1}{n}$.

Fig. 7.8 shows the expected throughput (left-side) and delay (right-side) for the incomplete information scenario. Continuous lines refer to the high type users, while dashed lines refer to the low type users, with the exception of the performance obtainable adopting the communication-free device, represented by the dash-dot line, in which different types of users adopt the same action and experience the same throughput and delay. In both pictures, the upper (continuous and dashed) lines refer to the strategic scenario without intervention device, in which the users adopt the BNE action profile, while the lower (continuous and dashed) lines represent the optimal action policy. The performance obtainable adopting the device derived from the algorithm lies in between. The lines that represent the expected delay for the BNE action profile are truncated for a number of users equal to 3 and 5 because for more users the system might become unstable. In fact, in the BNE the expected utility of a user is maximized, given that the other users adopt the BNE. However, for some type profile instances, the

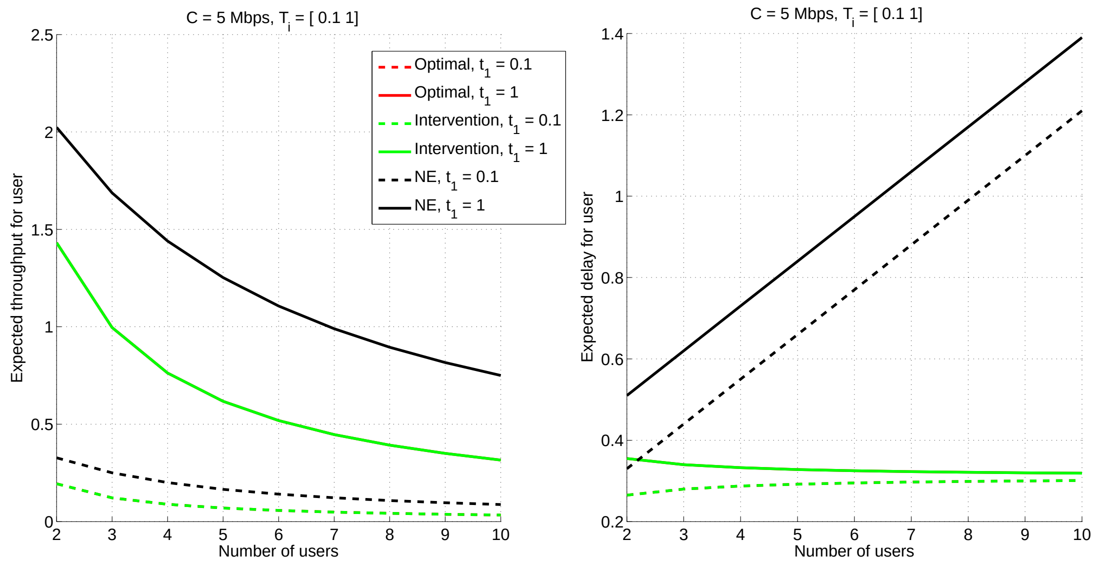


Figure 7.7. *Total expected throughput and delay vs. number of users for the complete information scenario*

utility might be equal to 0, i.e., the delay might diverge. Thus, the expected delay diverges as well. In words, there is a positive probability that the network becomes congested. The device derived from the algorithm allows to improve this situation, limiting the delay experienced by each user. However, such a delay increases almost linearly as the number of users increases. This is the reason why the communication-free device, at a certain point, even though it is not able to differentiate the service given to different classes of traffic, is able to obtain a better performance (from the manager's utility point of view) than the mechanism derived from the algorithm. In the communication-free device each user, independently of its type, adopts a rate which is between the optimal rates adopted by the low type users and the high type users, and this situation reflects in the expected delay. This allows to keep a very low and constant delay with respect to the number of users.

7.10 Conclusion

In this chapter we extend the intervention framework introduced by [66] to take into account situations of private information, imperfect monitoring and costly communication – in addition to intervention. We allow the designer to use a device that can communicate with users and intervene in the system. The goal of the designer is to choose the device that allows him to obtain the highest possible utility in the considered scenario. For a class of environments that includes many engineering scenarios of interest (e.g., power control [15, 16], medium access control (MAC) [12, 25], and

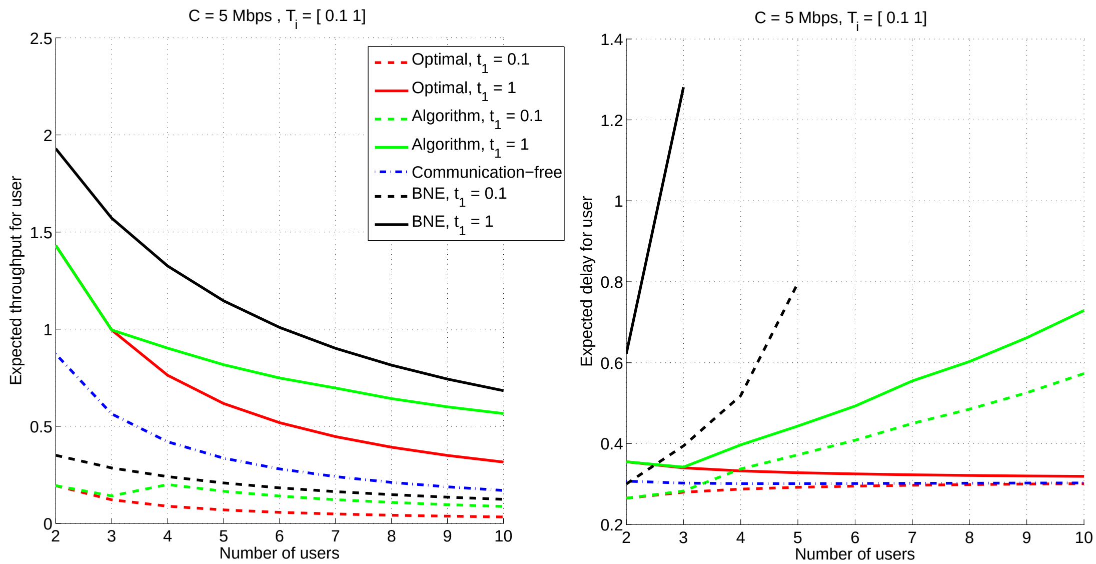


Figure 7.8. *Expected throughput and delay per user vs. number of users for the incomplete information scenario*

flow control [24–28]) we find conditions under which there exist devices that achieve the benchmark optimum and conditions under which such devices do not exist. In case they do not exist, we find conditions such that the problem of finding an optimal device can be decoupled. Because the optimal device may still be difficult to compute, we also provide a simple algorithm that converges to a device that, although perhaps not optimal, still yields a ‘good’ outcome for the designer.

Then we consider the design of a flow control management system, in both the complete and the incomplete information scenarios. We quantify the inefficiency of the NE of the complete information game and the BNE of the incomplete information game. We design an intervention scheme for the complete information scenario able to provide the incentive for the users to adopt the optimal transmission rate, by threatening punishments if they deviate. Such scheme is able to obtain the optimal performance achievable when the users act cooperatively. For the incomplete information scenario, we designs two devices: the first one is able to retrieve the private information from the users giving them the incentive to report it truthfully; the second one is based only on the a priori information that the device has about the users. Illustrative results show that these devices can considerably increase the efficiency of the network in the incomplete information scenario as well.

Chapter 8

Conclusions

This thesis discusses the application of game theory to the design of wireless network protocols which are robust against self-interested and strategic users. To reach this goal, the designer has to provide to the users an incentive to follow the protocol rules. On one hand, this constrains the choice of the protocol and results in schemes which are, in general, less efficient than optimal centralized schemes. On the other hand, this allows to obtain more stable protocols, not vulnerable to strategic users. As networks become more decentralized, users' terminals become more autonomous, programmable and (computationally) powerful, this design approach is fundamental to avoid high unforeseen inefficiencies.

This dissertation presents the following contributions to the design of efficient game-theoretic schemes in wireless networks. In Chapter 3 a virtual game among the radio resources allocator and the scheduler is proposed to manage the resources in an LTE system, trading off fairness and throughput, while ensuring the modularity of the overall system.

In Chapter 4 we address the problem of promoting cooperative relaying in a wireless network. This objective is reached with a dynamic scheduling rule which increases the access opportunities of cooperative users. We model this access scheme as a Stackelberg game, where a network unit plays the role of access coordinator, and we prove the existence of a Stackelberg equilibrium. A careful analysis of the numerical results justifies our scheme as a valid solution to increase the network performance in a viable manner from an implementation standpoint.

In Chapter 5 we develop a framework which can be used to select some nodes to be shared between two coexisting wireless networks. We consider a wireless network simulator that evaluates the network behavior at the physical, MAC and network layers. A Bayesian network approach is used by the two networks to evaluate their performance based on observable topological parameters. The interaction among the two networks is then modeled as a repeated game, and a trigger strategy is used to promote cooperation. Numerical results show that, even when only a small fraction of the nodes is shared, our scheme approaches the performance of a full cooperation scheme, in which the networks are assumed to share all their nodes.

In Chapter 6 we use two incentive schemes, pricing and intervention, to design a random access protocol robust against strategic users. We compare the two schemes in terms of the network environment, the knowledge of the designer and the knowledge of the users. Our results show that the intervention scheme, differently from the pricing scheme, is able to achieve the optimal performance if the user actions are perfectly observable. On the other hand, if they are not, the intervention scheme may punish the users even if they follow the recommendations, resulting in a degradation of the system performance. Nevertheless, we notice that intervention outperforms pricing if the users are not aware that their actions are imperfectly observed. While if they are aware of it, as a rough general principle, intervention achieves greater efficiency than pricing when the number of users is small and the opposite is true when the number of users is large.

In Chapter 7 we extend the intervention framework to take into account situations of private information, imperfect monitoring and costly communication, in which a device is adopted to provide to the users an incentive to report truthfully their information and to follow the instructions. For a class of environments that include many resource allocation games in communication networks, we provide tools for the designer to design an efficient system. In an abstracted environment, we find conditions under which the designer can achieve the same outcome it could if users were compliant, and conditions under which it can not. We also provide a simple algorithm that converges to a scheme that, although perhaps not optimal, still yields a good outcome for the designer. Then we consider the design of a flow control management system, in both the complete and the incomplete information scenarios. In the former we design a scheme which is able to obtain the optimal performance achievable when the users act cooperatively; in the latter we propose two mechanisms that, though not optimal, can considerably increase the efficiency of the network.

8.1 Future Directions

The application of game theory to design robust protocols in wireless networks is a challenging topic. Game theory can be applied to a variety of wireless networking problems at different levels: power control, interference avoidance, resource allocation, relaying, flow/congestion control, network routing, network formation, content distribution, security, etc. Here we want to focus on the adopted methodologies, rather than on a particular application.

To derive analytical results, game theory is usually applied to simple models, which only partially capture the real problems. This analysis may provide useful insights about the inefficiencies that may occur in the presence of strategic users and about possible solutions. However, such solutions cannot be directly applied to the real problems, because small perturbations (derived from unrealistic assumptions) may significantly change the equilibrium. We believe that game theoretic schemes must get rid of such simplified assumptions to be adopted in real systems. In particular, we tried to highlight the role of *information* in game theoretic approaches. The last three chapters of this dissertation are focused on this aspect.

Specifically, in Chapter 5 we used a Bayesian network approach to *estimate* some performance parameters, which are then used by the two networks – in place of the *real non observable values* – to make the decisions. In Chapter 6 we showed that the equilibrium efficiency strongly depends on the *observability* of the user actions and on the *information heterogeneity*. In Chapter 7 we developed a framework and derived some results to *retrieve the unknown information* from the users.

We believe that these types of approaches must be explored more deeply in game theoretic studies. In fact, in real wireless systems users do not usually have access to all the information. They may not (perfectly) know the other users' objectives. They may not (perfectly) observe the other users' actions. They may not even (perfectly) know the number of users in the system. In these cases, the users must act based on the *belief* they have about the missing information, and mechanism design or learning based schemes – to elicit or estimate the missing information – can be used to form and update the beliefs. Learning based techniques can be particularly useful when the number of users is large and it is expensive to keep track of each single user in the network and to exchange information. In this context, users are assumed to be aware of the environment and to dynamically adapt to it, learning from outcomes of past decisions. Finally, if the number of users is very large, *mean-field learning techniques* [116] may provide a suitable framework to model and analyze the interaction among them.

Appendix A

Proofs Chapter 6

A.1 Proof of Proposition 8

Proof. The unique NE of the game (6.9) is $a_i = \frac{\theta_i}{c_i}$, $i \in N$. Hence, the expected social welfare is

$$U(a) = \sum_{i=1}^{n} \mathbb{E}\left[\theta_i \ln\left[a_i \prod_{j=1, j\neq i}^{n} (1-a_j)\right] - c_i\left[a_i + n_i\right]_0^1\right] = \sum_{i=1}^{n} \theta_i \ln\left[a_i \prod_{j=1, j\neq i}^{n} (1-a_j)\right] - \frac{\theta_i}{a_i}\mathbb{E}\left[\left[a_i + n_i\right]_0^1\right]$$

where, considering $n_i \sim \mathcal{U}\left[-\epsilon_i\, \epsilon_i\right]$,

$$\mathbb{E}\left[\left[a_i + n_i\right]_0^1\right] = \begin{cases} \frac{(a_i+\epsilon_i)^2}{4\epsilon_i} & \text{if } a_i < \epsilon_i \\ a_i & \text{if } \epsilon_i \leq a_i \leq 1-\epsilon_i \\ \frac{-a_i^2 + 2(\epsilon_i+1)a_i + 2\epsilon_i - \epsilon_i^2 - 1}{4\epsilon_i} & \text{if } a_i > 1-\epsilon_i \end{cases}$$

Therefore

$$\frac{\partial U(a)}{\partial a_k} = \begin{cases} \frac{\theta_k}{a_k} - \frac{\sum_{i\neq k}\theta_i}{1-a_k} - \frac{\theta_k a_k^2 - \epsilon_k^2\theta_k}{4\epsilon_k a_k^2} & \text{if } a_k < \epsilon_k \\ \frac{\theta_k}{a_k} - \frac{\sum_{i\neq k}\theta_i}{1-a_k} & \text{if } \epsilon_k \leq a_k \leq 1-\epsilon_k \\ \frac{\theta_k}{a_k} - \frac{\sum_{i\neq k}\theta_i}{1-a_k} + \theta_k\frac{a_k^2 - (1-\epsilon_k)^2}{4\epsilon_k a_k^2} & \text{if } a_k > 1-\epsilon_k \end{cases}$$

and

$$\frac{\partial^2 U(a)}{\partial a_k^2} = \begin{cases} -\frac{\theta_k}{a_k^2} - \frac{\sum_{i\neq k}\theta_i}{(1-a_k)^2} - \frac{\epsilon_k\theta_k}{2a_k^3} & \text{if } a_k < \epsilon_k \\ -\frac{\theta_k}{a_k^2} - \frac{\sum_{i\neq k}\theta_i}{(1-a_k)^2} & \text{if } \epsilon_k \leq a_k \leq 1-\epsilon_k \\ \frac{\theta_k}{a_k} - \frac{\sum_{i\neq k}\theta_i}{1-a_k} + \theta_k\frac{(1-\epsilon_k)^2}{2\epsilon_k a_k^3} & \text{if } a_k > 1-\epsilon_k \end{cases}$$

$$\frac{\partial^2 U(a)}{\partial a_k \partial a_j} = 0 \ \ , \ \ \forall j \neq k$$

$\dfrac{\partial^2 U(a)}{\partial a_k^2}$ is negative. For $a_k < \epsilon_k$ and $\epsilon_k \leq a_k \leq 1-\epsilon_k$ this is trivial, for $a_k > 1-\epsilon_k$ we have

$$\frac{\partial^2 U(a)}{\partial a_k^2} = \frac{-2\epsilon_k \theta_k a_k \left(1-a_k\right)^2 - 2\epsilon_k \sum_{i\neq k} \theta_i a_k^3 + \left(1-\epsilon_k\right)^2 \theta_k \left(1-a_k\right)}{2\epsilon_k a_k^3 \left(1-a_k\right)^2} <$$
$$< \frac{-2\epsilon_k \sum_{i\neq k} \theta_i a_k^3 + \left(1-\epsilon_k\right)^2 \theta_k \left(1-a_k\right)}{2\epsilon_k a_k^3 \left(1-a_k\right)^2} < \frac{-2\left(1-\epsilon_k\right)^3 \sum_{i\neq k} \theta_i + \epsilon_k \left(1-\epsilon_k\right)^2 \theta_k \left(1-a_k\right)}{2 a_k^3 \left(1-a_k\right)^2} < 0$$

where the second and third inequalities are valid because $\dfrac{\theta_k}{\sum_{i=1}^n \theta_i} > 1-\epsilon_k$ (as we will see, the optimal transmission probability a_k is higher than $1-\epsilon_k$ if and only if $\dfrac{\theta_k}{\sum_{i=1}^n \theta_i}$ is higher than $1-\epsilon_k$) and $\epsilon_k \ll 1$ respectively. Hence, the Hessian of $U(a)$ is negative definite (it is a diagonal matrix with strictly negative diagonal entries), so $U(a)$ is concave. The global maximizer of $U(a)$ can be obtained with the first order condition.

For $a_k < \epsilon_k$ we obtain the condition

$$\theta_k a_k^3 - \left(\theta_k + 4\epsilon_k \sum_{i=1}^n \theta_i\right) a_k^2 + \left(4\epsilon_k\theta_k - \epsilon_k^2\theta_k\right) a_k + \epsilon_k^2 \theta_k = 0 \qquad \text{(A.1)}$$

The solution of Eq. (A.1) exists and is unique assuming $\dfrac{\theta_k}{\sum_{i=1}^n \theta_i} < \epsilon_k$. In fact the left hand side is a continuous function, decreasing in a_k (its derivative with respect to a_k corresponds to the second derivative of $U(a)$ with respect to a_k), equal to $\dfrac{\epsilon_k^2\theta_k}{0^+} > 0$ for $a_k \to 0^+$ and to $\dfrac{\epsilon_k \sum_i \theta_i - \theta_k}{\epsilon_k \left(1-\epsilon_k\right)} < 0$ for $a_k \to \epsilon_k^-$.

For $\epsilon_k \leq a_k \leq 1-\epsilon_k$ we obtain the condition

$$a_k = \frac{\theta_k}{\sum_{i=1}^n \theta_i} \rightarrow c_k = \sum_{i=1}^n \theta_i$$

For $a_k > 1-\epsilon_k$ we obtain the condition

$$-\theta_k a_k^3 + \left(\theta_k - 4\epsilon_k \sum_{i=1}^n \theta_i\right) a_k^2 + \left(4\epsilon_k\theta_k + \left(1-\epsilon_k\right)^2 \theta_k\right) a_k - \left(1-\epsilon_k\right)^2 \theta_k = 0 \qquad \text{(A.2)}$$

The solution of Eq. (A.2) exists and is unique assuming $\dfrac{\theta_k}{\sum_{i=1}^n \theta_i} > 1-\epsilon_k$. In fact the left hand side is a continuous function, decreasing in a_k (its derivative with respect to a_k corresponds to the second derivative of $U(a)$ with respect to a_k), equal to $\dfrac{\theta_k - \left(1-\epsilon_k\right)\sum_{i=1}^n \theta_i}{2\epsilon_k + \left(1-\epsilon_k\right)} > 0$ for $a_k \to \left(1-\epsilon_k\right)^+$ and to $\dfrac{-\sum_{i\neq k} \theta_i}{0^+} < 0$ for $a_k \to 1^-$.

Finally, notice that the solutions found are consistent with the case considered and

$$\begin{aligned} a_k < \epsilon_k \quad &\Leftrightarrow \quad \frac{\theta_k}{\sum_{i=1}^n \theta_i} < \epsilon_k \\ \epsilon_k \le a_k \le 1-\epsilon_k \quad &\Leftrightarrow \quad \epsilon_k \le \frac{\theta_k}{\sum_{i=1}^n \theta_i} \le 1-\epsilon_k \\ a_k > 1-\epsilon_k \quad &\Leftrightarrow \quad \frac{\theta_k}{\sum_{i=1}^n \theta_i} > 1-\epsilon_k \end{aligned}$$

□

A.2 Proof of Proposition 9

Proof. Given the intervention rule $\tilde{a}_k$ and $r_k = \frac{1}{\tilde{a}_k}$, $\forall\, k$, and the NE action profile $a = \tilde{a}$, the intervention level for user i is equal to

$$f_i^I(\tilde{a}_i) = \left[\frac{1}{\tilde{a}_i}([\tilde{a}_i + n_i]_0^1 - \tilde{a}_i)\right]_0^1$$

Consequently, the expected throughput of a generic user i and the social welfare are

$$T_i^I(\tilde{a}) = \mathbb{E}\left[\tilde{a}_i\left(1 - f_i^I(\tilde{a}_i)\right)\prod_{j=1,j\neq i}^{n}(1-\tilde{a}_j)\right] = \tilde{a}_i\left(1 - \mathbb{E}\left[f_i^I(\tilde{a}_i)\right]\right)\prod_{j=1,j\neq i}^{n}(1-\tilde{a}_j)$$

$$U(\tilde{a}) = \sum_{i=1}^{n}\theta_i \ln T_i(\tilde{a}) = \sum_{i=1}^{n}\theta_i \ln\left(\tilde{a}_i - \tilde{a}_i\mathbb{E}\left[f_i^I(\tilde{a}_i)\right]\right) + \sum_{i=1}^{n}\left(\sum_{j=1,j\neq i}^{n}\theta_j\right)\ln\left(1-\tilde{a}_i\right)$$

Now we want check if $U(\tilde{a})$ is concave analyzing its Hessian. To do so, we first compute the average intervention level $\mathbb{E}\left[f_i^I(\tilde{a}_i)\right]$, then we calculate $\frac{\partial U(\tilde{a})}{\partial \tilde{a}_i}$ and finally we compute $\frac{\partial^2 U(\tilde{a})}{\partial \tilde{a}_i^2}$ and $\frac{\partial^2 U(\tilde{a})}{\partial \tilde{a}_i \partial \tilde{a}_j}$, $i \neq j$. Notice that, to do so, we should calculate each function in three different cases: for $\tilde{a}_i < \epsilon_i$, for $\epsilon_i \le \tilde{a}_i \le 1-\epsilon_i$, and for $\tilde{a}_i > 1-\epsilon_i$. However, to avoid a heavy notation, we do not take into consideration the case $\tilde{a}_i > 1-\epsilon_i$. In fact this case is not interesting because, since $\epsilon_i \ll 1$, the best target action for user i is close to 1 if and only if there are few users in the network and the conditions are strongly asymmetric (i.e., $\theta_i \gg \theta_j$, $\forall\, j \neq i$). On the contrary, we are interested in the case $\tilde{a}_i < \epsilon_i$ because the best $\tilde{a}_i$ scales with the number of users. Thus, if the network is crowded, $\tilde{a}_i$ may become close to 0.

If $\tilde{a}_i + n_i < 0$ then $f_i^I(\tilde{a}_i) = 0$. If $\tilde{a}_i + n_i \ge 0$ then $f_i^I(\tilde{a}_i) = \frac{1}{\tilde{a}_i}\left[n_i\right]_0^{\tilde{a}_i}$. Hence, we obtain

$$\mathbb{E}\left[f_i^I(\tilde{a}_i)\right] = \begin{cases} \frac{1}{2\epsilon_i\tilde{a}_i}\int_0^{\tilde{a}_i} x\partial x + \frac{1}{2\epsilon_i}\int_{\tilde{a}_i}^{\epsilon_i}\partial x = \frac{2\epsilon_i - \tilde{a}_i}{4\epsilon_i} & \text{if } \tilde{a}_i < \epsilon_i \\ \frac{1}{2\epsilon_i\tilde{a}_i}\int_0^{\epsilon_i} x\partial x = \frac{\epsilon_i}{4\tilde{a}_i} & \text{if } \tilde{a}_i \ge \epsilon_i \end{cases}$$

Therefore

$$\frac{\partial U(\tilde{a})}{\partial \tilde{a}_i} = \begin{cases} \frac{2\theta_i\left(\epsilon_i+\tilde{a}_i\right)}{2\epsilon_i\tilde{a}_i+\tilde{a}_i^2} - \frac{\sum_{j\neq i}\theta_j}{1-\tilde{a}_i} & \text{if } \tilde{a}_i < \epsilon_i \\ \frac{\theta_i}{\tilde{a}_i-\frac{\epsilon_i}{4}} - \frac{\sum_{j\neq i}\theta_j}{1-\tilde{a}_i} & \text{if } \tilde{a}_i \geq \epsilon_i \end{cases}$$

$$\frac{\partial^2 U(\tilde{a})}{\partial \tilde{a}_i^2} = \begin{cases} \frac{2\theta_i\left(-2\epsilon_i^2-2\epsilon_i\tilde{a}_i-\tilde{a}_i^2\right)}{\left(2\epsilon_i\tilde{a}_i+\tilde{a}_i^2\right)^2} - \frac{\sum_{j\neq i}\theta_j}{\left(1-a_i\right)^2} & \text{if } \tilde{a}_i < \epsilon_i \\ \frac{-\theta_i}{\left(\tilde{a}_i-\frac{\epsilon_i}{4}\right)^2} - \frac{\sum_{j\neq i}\theta_j}{\left(1-\tilde{a}_i\right)^2} & \text{if } \tilde{a}_i \geq \epsilon_i \end{cases}$$

$$\frac{\partial^2 U}{\partial a_i \partial a_j} = 0 \ , \ \forall\, i \neq j$$

$\frac{\partial^2 U(\tilde{a})}{\partial \tilde{a}_i^2} < 0$. Hence, the Hessian of $U(\tilde{a})$ is negative definite (it is a diagonal matrix with strictly negative diagonal entries), so $U(\tilde{a})$ is concave. The global maximizer of $U(\tilde{a})$ can be obtained with the first order condition, i.e., imposing $\frac{\partial U(\tilde{a})}{\partial \tilde{a}_i} = 0$. Notice that $\frac{\partial U(\tilde{a})}{\partial \tilde{a}_i}$ is continuous, decreasing (because $\frac{\partial^2 U(\tilde{a})}{\partial \tilde{a}_i^2} < 0$), and tends to $+\infty$ for $\tilde{a}_i \to 0^+$ and to $-\infty$ for $\tilde{a}_i \to 1^-$. Thus, there exists one and only one $\tilde{a}_i$ such that $\frac{\partial U(\tilde{a})}{\partial \tilde{a}_i} = 0$.

Imposing $\frac{\partial U(\tilde{a})}{\partial \tilde{a}_i} = 0$ for $\tilde{a}_i < \epsilon_i$, we obtain

$$\left(-\theta_i - \sum_{j=1}^{n}\theta_j\right)\tilde{a}_i^2 + \left(2\theta_i - 2\epsilon_i\sum_{j=1}^{n}\theta_j\right)\tilde{a}_i + 2\epsilon_i\theta_i = 0$$

Imposing $\frac{\partial U(\tilde{a})}{\partial \tilde{a}_i} = 0$ for $\tilde{a}_i \geq \epsilon_i$, we obtain

$$\tilde{a}_i = \frac{4\theta_i + \epsilon_i\sum_{j=1,j\neq i}^{n}\theta_j}{4\sum_{j=1}^{n}\theta_j}$$

This results is compatible with the condition $\tilde{a}_i \geq \epsilon_i$ if and only if

$$\epsilon_i \leq \frac{4\theta_i}{4\sum_{j=1}^{n}\theta_j - \sum_{j=1,j\neq i}^{n}\theta_j}$$

□

A.3 Proof of Lemma 3

Proof.

$$\mathbb{E}\left[\left[a_i+n_i\right]_0^1\right] = \begin{cases} \frac{\left(a_i+\epsilon_i\right)^2}{4\epsilon_i} & \text{if } a_i \leq \epsilon_i \\ a_i & \text{if } \epsilon_i < a_i \leq 1-\epsilon_i \\ \frac{-a_i^2+2(\epsilon_i+1)a_i+2\epsilon_i-\epsilon_i^2-1}{4\epsilon_i} & \text{if } a_i > 1-\epsilon_i \end{cases}$$

$$U_i(a) = \begin{cases} \theta_i \ln\left[a_i \prod_{j\neq i}(1-a_j)\right] - c_i\dfrac{(a_i+\epsilon_i)^2}{4\epsilon_i} & \text{if } \ a_i < \epsilon_i \\ \theta_i \ln\left[a_i \prod_{j\neq i}(1-a_j)\right] - c_i a_i & \text{if } \ \epsilon_i \le a_i \le 1-\epsilon_i \\ \theta_i \ln\left[a_i \prod_{j\neq i}(1-a_j)\right] - c_i\dfrac{-a_i^2+2(\epsilon_i+1)a_i+2\epsilon_i-\epsilon_i^2-1}{4\epsilon_i} & \text{if } \ a_i > 1-\epsilon_i \end{cases}$$

$$\frac{\partial U_i(a)}{\partial a_i} = \begin{cases} \dfrac{\theta_i}{a_i} - 2c_i\dfrac{a_i+\epsilon_i}{4\epsilon_i} & \text{if } \ a_i < \epsilon_i \\ \dfrac{\theta_i}{a_i} - c_i & \text{if } \ \epsilon_i \le a_i \le 1-\epsilon_i \\ \dfrac{\theta_i}{a_i} - c_i\dfrac{-a_i+\epsilon_i+1}{2\epsilon_i} & \text{if } \ a_i > 1-\epsilon_i \end{cases}$$

To compute the best response function of user i, we impose the first derivative of $U(a)$ equal to 0 and we analyse the concavity of $U(a)$, with respect to a_i.

$$\frac{\partial U_i(a)}{\partial a_i} = 0 \longrightarrow a_i = \begin{cases} \dfrac{-\epsilon_i}{2} + \dfrac{1}{2}\sqrt{\epsilon_i^2 + \dfrac{8\epsilon_i\theta_i}{c_i}} & \text{if } \ \dfrac{\theta_i}{c_i} < \epsilon_i \\ \dfrac{\theta_i}{c_i} & \text{if } \ \epsilon_i \le \dfrac{\theta_i}{c_i} \le 1-\epsilon_i \\ \dfrac{\epsilon_i+1}{1} + \dfrac{1}{2}\sqrt{(\epsilon_i+1)^2 - \dfrac{8\epsilon_i\theta_i}{c_i}} & \text{if } \ \dfrac{1}{2} < \dfrac{\theta_i}{c_i} < 1-\epsilon_i \end{cases}$$

$$\frac{\partial^2 U_i(a)}{\partial a_i^2} = \begin{cases} -\dfrac{\theta_i}{a_i^2} - \dfrac{c_i}{2\epsilon_i} & \text{if } \ a_i < \epsilon_i \\ \dfrac{\theta_i}{a_i^2} & \text{if } \ \epsilon_i \le a_i \le 1-\epsilon_i \\ -\dfrac{\theta_i}{a_i^2} + \dfrac{c_i}{2\epsilon_i} & \text{if } \ a_i > 1-\epsilon_i \end{cases}$$

$\dfrac{\partial^2 U_i(a)}{\partial a_i^2} < 0$ for $a_i \in \left[0, \max\left(\sqrt{\dfrac{2\epsilon_i\theta_i}{c_i}}, 1-\epsilon_i\right)\right]$ and in $\max\left(\sqrt{\dfrac{2\epsilon_i\theta_i}{c_i}}, 1-\epsilon_i\right)$ there is a change in the concavity. If $\dfrac{1}{2} < \dfrac{\theta_i}{c_i} \le 1-\epsilon_i$ then, after the change of concavity, the function reaches a local minimum in $a_i = \dfrac{\epsilon_i+1}{2} + \dfrac{1}{2}\sqrt{(\epsilon_i+1)^2 - \dfrac{8\epsilon_i\theta_i}{c_i}}$ and then restarts to increase. Hence, in this case there are 2 local maxima: $a_i = \dfrac{\theta_i}{c_i}$ and $a_i = 1$. Comparing the 2 maxima we obtain $U_i(\dfrac{\theta_i}{c_i}, a_{-i}) \ge U_i(1, a_{-1}) \Longleftrightarrow \dfrac{\theta_i}{c_i}\ln\dfrac{\theta_i}{c_i} - \dfrac{\theta_i}{c_i} \ge \dfrac{\epsilon_i}{4} - 1$.

Summarizing:

Case 1) if $\dfrac{\theta_i}{c_i} < \epsilon_i$ then there is one local maximum which is the global maximum: $a_i = \dfrac{-\epsilon_i}{2} + \dfrac{1}{2}\sqrt{\epsilon_i^2 + \dfrac{8\epsilon_i\theta_i}{c_i}}$

Case 2) If $\epsilon_i \le \dfrac{\theta_i}{c_i} \le \dfrac{1}{2}$ then there is one local maximum which is the global maximum: $a_i = \dfrac{\theta_i}{c_i}$

Case 3) If $\frac{1}{2} < \frac{\theta_i}{c_i} \leq 1 - \epsilon_i$ and $\frac{\theta_i}{c_i} \ln \frac{\theta_i}{c_i} - \frac{\theta_i}{c_i} \geq \frac{\epsilon_i}{4} - 1$ then there are two local maxima and the global one is $a_i = \frac{\theta_i}{c_i}$

Case 4) If $\frac{1}{2} < \frac{\theta_i}{c_i} \leq 1 - \epsilon_i$ and $\frac{\theta_i}{c_i} \ln \frac{\theta_i}{c_i} - \frac{\theta_i}{c_i} < \frac{\epsilon_i}{4} - 1$ then there are two local maxima and the global one is $a_i = 1$

Case 5) if $\frac{\theta_i}{c_i} > 1 - \epsilon_i$ then the function is increasing and the maximum is obtained for $a_i = 1$

□

A.4 Proof of Proposition 10

Proof. Considering that users adopt the NE action profile (6.13), we want to maximize $U(a)$ with respect to a_k, $\forall k \in N$. The optimal a_k must be lower than 1, therefore we can consider only the first three cases listed at the end of Appendix A.3.

We obtain:

$$\frac{\partial U(a)}{\partial a_k} = \begin{cases} \frac{\theta_k}{a_k} - \frac{\sum_{i \neq k} \theta_i}{1 - a_k} + \frac{\theta_k \epsilon_k}{2a_k^2} & \text{if } \ a_k < \epsilon_k \\ \frac{\theta_k}{a_k} - \frac{\sum_{i \neq k} \theta_i}{1 - a_k} & \text{if } \ \epsilon_k \leq a_k \leq a_{k,5} \end{cases}$$

$$\frac{\partial U(a)}{\partial a_i} = 0 \longrightarrow a_i = \begin{cases} a_{k,4} & \text{if } \ a_k < \epsilon_k \\ \frac{\theta_k}{\sum_{i \neq k} \theta_i} & \text{if } \ \epsilon_k \leq a_k \leq a_{k,5} \end{cases}$$

$$\frac{\partial^2 U(a)}{\partial a_k^2} = \begin{cases} \frac{-\theta_k}{a_k^2} - \frac{\sum_{i \neq k} \theta_i}{(1 - a_k)^2} - \frac{\theta_k \epsilon_k}{a_k^3} & \text{if } \ a_k < \epsilon_k \\ \frac{-\theta_k}{a_k^2} - \frac{\sum_{i \neq k} \theta_i}{(1 - a_k)^2} & \text{if } \ \epsilon_k \leq a_k \leq a_{k,5} \end{cases}$$

$$\frac{d^2 G(a)}{d a_k d p_i} = 0 \ , \ i \neq k$$

The Hessian of $U(a)$ is negative definite in $[0,\, a_{k,5}]$. $U(a)$ is a continuous and concave function in $[0,\, a_{k,5}]$, increasing in $a_k = 0$. However, its first partial derivative is not continuous in $a_k = \epsilon_k$. In particular, if there exists a user k such that $\frac{\partial G(a)}{\partial a_k} \neq 0$ in $[0,\, a_{k,5}]$, then either (1) $\frac{\partial U(a)}{\partial a_k} > 0$ for $a_k < \epsilon_k$ and $\frac{\partial U(a)}{\partial a_k} < 0$ for $a_k > \epsilon_k$, or (2) $U(a)$ increases in a_k until reaching a maximum in $a_k = a_{k,5}$. Finally, the global maximum is located where partial derivatives are equal to 0 or, in case this condition is not satisfied for some users k, in $a_k = \epsilon_k$ if $\frac{\partial G(a)}{\partial a_k} > 0$ for $a_k < \epsilon_k$ and $\frac{\partial G(a)}{\partial a_k} < 0$ for $a_k > \epsilon_k$, or in $a_{k,5}$ otherwise; i.e.,

$$a_k = \begin{cases} a_{k,4} & \text{if } \ a_{k,4} < \epsilon_k \\ \epsilon_k & \text{if } \ a_{k,4} \geq \epsilon_k \ \text{ and } \ \dfrac{\theta_k}{\sum_i \theta_i} \leq \epsilon_k \\ \dfrac{\theta_k}{\sum_i \theta_i} & \text{if } \ \epsilon_k \leq \dfrac{\theta_k}{\sum_i \theta_i} \leq \dfrac{1}{2} \ \text{ or } \ \dfrac{\theta_k}{\sum_i \theta_i} \in \mathcal{C}\left(\epsilon_k\right) \\ a_{k,5} & \text{otherwise} \end{cases}$$

which is equivalent to Eq. (6.14).

□

A.5 Proof of Lemma 4

Proof. We study i's utility, $U_i^I(a)$, varying i's action, a_i. To do so, we first analyze the average intervention level $E := \mathbb{E}\left[\left[r_i\left([a_i + n_i]_0^1 - \overline{a}_i - \epsilon_i\right)\right]_0^1\right]$, for $r_i \to +\infty$.

If $a_i < \overline{a}_i$, the term that multiplies r_i is always negative (notice that $[a_i + n_i]_0^1 \leq a_i + \epsilon_i$) and, consequently, the intervention level is always equal to 0 and $E = 0$.

If $a_i > \overline{a}_i + 2\epsilon_i$, the term that multiplies r_i is always positive (notice that $[a_i + n_i]_0^1 \geq a_i - \epsilon_i$) and, consequently, the intervention level is always equal to 1 and $E = 1$.

If $\overline{a}_i \leq a_i \leq \overline{a}_i + 2\epsilon_i$, the intervention might be 0 or 1, depending on the value of the estimation error n_i. Notice that, in this case, $a_i + n_i \geq 0$. Thus, whenever n_i is higher than $\overline{a}_i + \epsilon_i - a_i$, the intervention is 1, and the average intervention level is equal to

$$E = \frac{1}{2\epsilon_i} \int_{\overline{a}_i + \epsilon_i - a_i}^{\epsilon_i} \partial x = \frac{1}{2\epsilon_i}\left(a_i - \overline{a}_i\right)$$

Hence, we obtain

$$U_i(a) = \begin{cases} \theta_i \ln a_i + \theta_i \ln\left[\prod_{j\neq i}\left(1 - a_j\right)\right] & \text{if } \ a_i < \overline{a}_i \\ \theta_i \ln\left[a_i\left(1 - \dfrac{1}{2\epsilon_i}\left(a_i - \overline{a}_i\right)\right)\right] + \theta_i \ln\left[\prod_{j\neq i}\left(1 - a_j\right)\right] & \text{if } \ \overline{a}_i \leq a_i \leq \overline{a}_i + 2\epsilon_i \\ -\infty & \text{if } \ a_i > \overline{a}_i + 2\epsilon_i \end{cases}$$

To predict the best action for user i, we study the trend of $U_i(a)$ varying a_i in the interval $[0, \overline{a}_i + 2\epsilon_i)$. To do so, we calculate $\dfrac{\partial U_i(a)}{\partial a_i}$ and $\dfrac{\partial^2 U_i(a)}{\partial a_i^2}$. and we study their sign.

$$\frac{\partial U_i(a)}{\partial a_i} = \begin{cases} \dfrac{\theta_i}{a_i} & \text{if } \ a_i < \overline{a}_i \\ \theta_i \dfrac{1 + \frac{1}{2\epsilon_i}\overline{a}_i - \frac{1}{\epsilon_i}a_i}{\left(1 + \frac{1}{2\epsilon_i}\overline{a}_i\right)a_i - \frac{1}{2\epsilon_i}a_i^2} & \text{if } \ \overline{a}_i \leq a_i \leq \overline{a}_i + 2\epsilon_i \end{cases}$$

$$\frac{\partial^2 U_i(a)}{\partial a_i^2} = \begin{cases} -\frac{\theta_i}{a_i^2} & \text{if } a_i < \overline{a}_i \\ \theta_i \frac{\frac{1}{\epsilon_i}(a_i - \overline{a}_i) - 1 + \frac{a_i}{2\epsilon_i^2}(\overline{a}_i - a_i) - \frac{a_i^2}{2\epsilon_i^2} - \frac{\overline{a}_i^2}{4\epsilon_i^2}}{\left[\left(1 + \frac{1}{2\epsilon_i}\overline{a}_i\right) a_i - \frac{1}{2\epsilon_i} a_i^2\right]^2} & \text{if } \overline{a}_i \le a_i \le \overline{a}_i + 2\epsilon_i \end{cases}$$

$\frac{\partial^2 U_i(a)}{\partial a_i^2} < 0$ for $\overline{a}_i \le a_i \le \overline{a}_i + 2\epsilon_i$. In fact, for $\overline{a}_i \le a_i \le \overline{a}_i + \epsilon_i$

$$\frac{1}{\epsilon_i}(a_i - \overline{a}_i) - 1 + \frac{a_i}{2\epsilon_i^2}(\overline{a}_i - a_i) - \frac{a_i^2}{2\epsilon_i^2} - \frac{\overline{a}_i^2}{4\epsilon_i^2} \le 1 - 1 + 0\frac{a_i^2}{2\epsilon_i^2} - \frac{\overline{a}_i^2}{4\epsilon_i^2} \le 0$$

For $\overline{a}_i + \epsilon_i \le a_i \le \overline{a}_i + 2\epsilon_i$

$$\frac{1}{\epsilon_i}(a_i - \overline{a}_i) - 1 + \frac{a_i}{2\epsilon_i^2}(\overline{a}_i - a_i) - \frac{a_i^2}{2\epsilon_i^2} - \frac{\overline{a}_i^2}{4\epsilon_i^2} \le 2 - 1 - \frac{a_i}{2\epsilon_i} - \frac{a_i^2}{2\epsilon_i^2} \le 1 - \frac{\overline{a}_i + \epsilon_i}{2\epsilon_i} - \frac{(\overline{a}_i + \epsilon_i)^2}{2\epsilon_i^2} =$$
$$= 1 - \frac{1}{2} - \frac{\overline{a}_i}{2\epsilon_i} - \frac{\overline{a}_i^2 + 2\overline{a}_i\epsilon_i + \epsilon_i^2}{2\epsilon_i^2} = -\frac{\overline{a}_i}{2\epsilon_i} - \frac{\overline{a}_i^2}{2\epsilon_i^2} - \frac{\overline{a}_i}{\epsilon_i} \le 0$$

Thus, $\frac{\partial U_i(a)}{\partial a_i}$ is decreasing in $[\overline{a}_i, \overline{a}_i + 2\epsilon_i]$. Since $\frac{\partial U_i(a)}{\partial a_i} > 0$ in $[0, \overline{a}_i,)$, a necessary and sufficient condition such that $\overline{a}_i$ is a global maximum is that $\frac{\partial U_i(a)}{\partial a_i} \le 0$ for $a_i \to \overline{a}_i^+$. Imposing such a condition we obtain $\overline{a}_i \ge 2\epsilon_i$, which concludes the proof.

□

Appendix B

Proofs Chapter 7

B.1 Proof of Proposition 13

Proof. We prove $\Rightarrow$ by contradiction.

Let $D = \langle (T_i), (A_i), \mu, X, \Phi \rangle$ be a maximum efficiency device (remember that we focus only on incentive compatible devices). Suppose **3** is not valid, i.e., there exists a type profile $\tilde{t}$ such that the non-optimal action profile $z \neq g^M(\tilde{t})$ is suggested with positive probability $\mu_{\tilde{t}}(z) > 0$. Then

$$EU(f, g, D) = \sum_{t \in T} \pi(t) \sum_{a \in A} \mu_t(a) \sum_{x \in X} \Phi_{t,a,a}(x)\, U(a, t, x) = V + W +$$
$$+ \mu_{\tilde{t}}(z) \sum_{x \in X} \Phi_{hatt,z,z}(x)\, U(z, \tilde{t}, x) < V + W + \mu_{\tilde{t}}(z) U(g^M(\tilde{t}), \tilde{t}, x^*) \leq EU^{ben}$$

where

$$V = \sum_{t \in T, t \neq \tilde{t}} \pi(t) \sum_{a \in A} \mu_t(a) \sum_{x \in X} \Phi_{t,a,a}(x)\, U(a, t, x)$$
$$W = \pi(\tilde{t}) \sum_{a \in A, a \neq z} \mu_t(a) \sum_{x \in X} \Phi_{t,a,a}(x)\, U(a, \tilde{t}, x)$$

which contradicts the fact that D is a maximum efficiency device.

Now suppose **4** is not valid, i.e., that $\Phi'_a = \Phi_{t, g^M(t), a}$ does not sustain without intervention $g^M(t)$ in Γ_t. If Φ'_a does not sustain $g^M(t)$ in Γ_t, then there exists a user i and an action $a_i \neq g_i^M(t)$ such that user i prefers to adopt a_i when told to use $g_i^M(t)$, i.e., the strategy $g_i(t_i, g_i^M(t)) = a_i$ allows user i to obtain a higher utility with respect to the obedient strategy g_i^*; this contradicts the fact that the device is incentive compatible. If Φ'_a sustains $g^M(t)$ in Γ_t "with intervention", then there exists $\tilde{t}$ and

$\tilde{x} \neq x^*$ such that $\Phi_{\tilde{t},g^M(\tilde{t}),g^M(\tilde{t})}(\tilde{x}) > 0$. Then

$$EU(f,g,D) = \sum_{t\in T} \pi(t) \sum_{x\in X} \Phi_{t,g^M(t),g^M(t)}(x)\, U(g^M(t),t,x) = V + W +$$
$$+\, \pi(\tilde{t})\Phi_{t,g^M(t),g^M(t)}(\tilde{x})\, U(g^M(t),t,\tilde{x}) < V + W + \pi(\tilde{t})\Phi_{t,g^M(t),g^M(t)}(\tilde{x})\, U(g^M(t),t,x^*) \leq EU^{ben}$$

where

$$V = \sum_{t\in T, t\neq \tilde{t}} \pi(t) \sum_{x\in X} \Phi_{t,g^M(t),g^M(t)}(x)\, U(g^M(t),t,x)$$
$$W = \pi(\tilde{t}) \sum_{x\in X, x\neq \tilde{x}} \Phi_{t,g^M(t),g^M(t)}(x)\, U(g^M(t),t,x)$$

which contradicts the fact that D is a maximum efficiency device.

Finally, if **1** is not satisfied then **4** can not be satisfied either (because $g^M(t)$ is not sustainable without intervention), thus we obtain a contradiction. If **2** is not satisfied then either **3** is not satisfied or the device is not incentive compatible (because, given **3**, **2** is a particular case of the incentive-compatibility constraints), thus, in both cases, we obtain a contradiction.

$\Leftarrow$ It is straightforward to verify that if **1** $-$ **4** are satisfied the resulting mechanism is incentive compatible and the utility of the designer is equal to the benchmark optimum (7.1). □

B.2 Proof of Lemma 6

Proof. Let $D = \langle (T_i), (A_i), \mu, X, \Phi\rangle$ be an optimal device (remember that we focus only on incentive compatible devices). The expected utility of user i having type t_i can be written as

$$EU_i(f,g,t_i,D) = \sum_{t_{-i}\in T_{-i}} \pi[t \mid t_i] V_i(t) \quad , \quad V_i(t) = \sum_{a\in A} \mu_t(a) \sum_{x\in X} \Phi_{t,a,a}(x)\, U_i(a,t,x)$$

Denote by $a_i^{min}(t)$ the minimum possible action suggested to user i when the type profile is t, i.e., $a_i^{min}(t) = \min\{a_i \in A_i : \mu_t(a_i, a_{-i}) > 0, a_{-i} \in A_{-i}\}$. We define the following intervals

$$I_i(t) = \left[a_i^{min},\ \min\left\{a_i^{min}(t)\,,\ g_i^{NE^0}(t)\right\}\right] \quad , \quad i = \{1,\ldots,n\}$$

and we use the notation $I(t)$ and $I_{-i}(t)$ in the usual way.

We define the function $\ell_i(a_{-i})$ in the domain $I_{-i}(t)$ as follows:

$$\ell_i(a_{-i}) = \{a_i \in I_i(t) \text{ such that } U_i\,(a,t,x^*) = V_i(t)\}$$

The function ℓ_i is a non-empty set-valued function from $I_{-i}(t)$ to the power set of $I_i(t)$. In fact, $\forall\, a'_{-i}(t) \in I_{-i}(t)$,

$$U_i\left(a_i^{min}, a'_{-i}, t, x^*\right) = 0 \le V_i(t) \le \sum_{a\in A} \mu_t(a)\, U_i(a, t, x^*) \le \sum_{a\in A} \mu_t(a)\, U_i(a_i, a'_{-i}(t), t, x^*) \quad \text{(B.1)}$$

The second inequality of Eq. (B.1) is valid because i's utility is non increasing with respect to the intervention level, i.e., $U_i\,(a, t, x) \le U_i\,(a, t, x^*)$, $\forall\, a, t, x$. The last inequality of Eq. (B.1) is valid because i's utility is non increasing in the actions of the other users and, from the definition of the set $I_{-i}(t)$, $a'_{-i}(t) \le a_{-i}$, $\forall\, a'_{-i} \in I_{-i}(t)$. Eq. (B.1) and the continuity of i's utility imply that an action $\tilde{a}_i(t) \in I_i(t)$ satisfying $U_i\,(a, t, x^*) = V_i(t)$ exists, $\forall\, a_{-i} \in I_{-i}(t)$. Moreover, by definition $\ell_i(a_{-i})$ has a closed graph (i.e., the graph of $\ell_i(a_{-i})$ is a closed subset of $I(t)$) and, since i's utility is non decreasing in $\left[a_i^{min}\; g_i^{NE^0}(t)\right]$, $\ell_i(a_{-i})$ is convex, $\forall\, a_{-i} \in I_{-i}(t)$.

We define the function $\ell(a) = (\ell_1(a_{-1}), \cdots, \ell_n(a_{-n}))$, $\forall\, a \in I(t)$. The function ℓ is defined from the non-empty, compact and convex set $I(t)$ to the power set of $I(t)$. Thanks to the properties of ℓ_i, ℓ has a closed graph and $\ell(a)$ is non-empty and convex. Therefore we can apply Kakutani fixed-point theorem [51] to affirm that a fixed point exists, i.e., there exists an action profile $\tilde{a}(t) \in I(t)$ such that $U_i\,(\tilde{a}, t, x^*) = V_i(t)$, $\forall\, i \in N$. Notice that $\tilde{a}(t) < g^{NE^0}(t)$, therefore $\tilde{a}(t)$ is sustainable without intervention in Γ_t, and we denote by Φ'_a the intervention rule that sustains without intervention $\tilde{a}(t)$ in Γ_t.

Finally, the original optimal device $D = \langle (T_i), (A_i), \mu, X, \Phi \rangle$ can be substituted with the device $\tilde{D} = \langle (T_i), (A_i), \tilde{\mu}, X, \tilde{\Phi} \rangle$ in which, $\forall\, t$, $\tilde{\mu}(t) = \tilde{a}(t)$ and $\tilde{\Phi}_{t,\tilde{a}(t),a} = \Phi'_a$. With the new device $\tilde{D}$ the users are obedient (because the restriction of the intervention rule, Φ'_a, sustains $\tilde{a}(t)$) and honest (because the utilities they obtain for each combination of reports are the same as in the initial device D that sustains the honest and obedient strategy profile). More specifically, $\tilde{D}$ sustains without intervention the honest and obedient strategy profile. Moreover, in the equilibrium path the users' expected utilities using $\tilde{D}$ coincide with the users' expected utilities using D; thus, also the designer's utility (which is a function of users' utilities) remains the same, and this implies that $\tilde{D}$ is optimal. $\square$

B.3 Proof of Proposition 16

Proof.

$$EU_i(g,t_i) = \mathbb{E}_{t_{-i}}\left[U_i(g(t),t_i)\right] = g_i(t_i)^{t_i}\mathbb{E}_{t_{-i}}\left[(C-\lambda)\right] = g_i(t_i)^{t_i}\left[\left(C - g_i(t_i) - \sum_{j=1,j\neq i}^{n}\mathbb{E}_{t_j}\left[g_j(t_j)\right]\right)\right]$$

$$\frac{\partial \ln EU_i(g,t_i)}{\partial g_i(t_i)} = \frac{t_i}{g_i(t_i)} - \frac{1}{C - g_i(t_i) - \sum_{j=1,j\neq i}^{n}\mathbb{E}_{t_j}\left[g_j(t_j)\right]}$$

$$\frac{\partial^2 \ln EU_i(g,t_i)}{\partial g_i^2(t_i)} = -\frac{t_i}{g_i^2(t_i)} - \frac{1}{\left(C - g_i(t_i) - \sum_{j=1,j\neq i}^{n}\mathbb{E}_{t_j}\left[g_j(t_j)\right]\right)^2} < 0$$

Imposing that the first derivative is equal to 0, we obtain that the Bayesian Nash Equilibrium g^{BNE} must satisfy, $\forall\, i \in N$ and $\forall\, l = 1,\dots,v$,

$$\left(1+\tau_l\right) g_i^{BNE}(\tau_l) + \tau_l \sum_{j=1,j\neq i}^{n}\sum_{k=1}^{v}\pi(\tau_k) g_j^{BNE}(\tau_k) = C\tau_l \tag{B.2}$$

The system of equations defined by (B.2) can be written as a matrix equation of the form

$$\mathbf{A} g^{BNE} = b$$

where

$$g^{BNE} = \begin{bmatrix} g_1^{BNE} \\ \vdots \\ g_n^{BNE} \end{bmatrix}, \quad g_i^{BNE} = \begin{bmatrix} g_i^{BNE}(\tau_1) \\ \vdots \\ g_i^{BNE}(\tau_v) \end{bmatrix}, \quad b = \begin{bmatrix} \hat{b} \\ \vdots \\ \hat{b} \end{bmatrix}, \quad \hat{b} = \begin{bmatrix} C\tau_1 \\ \vdots \\ C\tau_v \end{bmatrix},$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{\Lambda} & \tau\cdot\mathbf{P} & \cdots & \tau\cdot\mathbf{P} \\ \tau\cdot\mathbf{P} & \mathbf{\Lambda} & \cdots & \tau\cdot\mathbf{P} \\ \vdots & \vdots & \ddots & \vdots \\ \tau\cdot\mathbf{P} & \tau\cdot\mathbf{P} & \cdots & \mathbf{\Lambda} \end{bmatrix},$$

$$\mathbf{\Lambda} = \operatorname{diag}\left(1+\tau_1,\dots,1+\tau_v\right), \quad \tau = \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_v \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} \pi(\tau_1) & \dots & \pi(\tau_v) \end{bmatrix}$$

Finally, we want to analytically compute the inverse of the matrix $\mathbf{A}$. We can write $\mathbf{A}$ as

$$\mathbf{A} = \begin{bmatrix} \mathbf{\Lambda}-\tau\cdot\mathbf{P} & & \\ & \ddots & \\ & & \mathbf{\Lambda}-\tau\cdot\mathbf{P} \end{bmatrix} + \begin{bmatrix} \mathbf{I} \\ \vdots \\ \mathbf{I} \end{bmatrix} \cdot \begin{bmatrix} \tau\cdot\mathbf{P} & \dots & \tau\cdot\mathbf{P} \end{bmatrix} \tag{B.3}$$

where $\mathbf{I}$ is the identity matrix in $\mathbb{R}^{m\times m}$.

The matrix inversion Lemma states that

$$(\mathbf{E}+\mathbf{BCD})^{-1} = \mathbf{E}^{-1} - \mathbf{E}^{-1}\mathbf{B}\left(\mathbf{C}^{-1}+\mathbf{DE}^{-1}\mathbf{B}\right)^{-1}\mathbf{DE}^{-1} \tag{B.4}$$

Applying the matrix inversion Lemma to $\mathbf{A}^{-1}$ we obtain

$$\begin{aligned}\mathbf{A}^{-1} = &\begin{bmatrix}\mathbf{\Lambda}-\tau\cdot\mathbf{P} & & \\ & \ddots & \\ & & \mathbf{\Lambda}-\tau\cdot\mathbf{P}\end{bmatrix}^{-1} - \begin{bmatrix}\mathbf{\Lambda}-\tau\cdot\mathbf{P} & & \\ & \ddots & \\ & & \mathbf{\Lambda}-\tau\cdot\mathbf{P}\end{bmatrix}^{-1}\cdot\begin{bmatrix}\mathbf{I}\\ \vdots \\ \mathbf{I}\end{bmatrix}\cdot \\ &\Bigg(\underbrace{\mathbf{I}^{-1}+\begin{bmatrix}\tau\cdot\mathbf{P} & \dots & \tau\cdot\mathbf{P}\end{bmatrix}\cdot\begin{bmatrix}\mathbf{\Lambda}-\tau\cdot\mathbf{P} & & \\ & \ddots & \\ & & \mathbf{\Lambda}-\tau\cdot\mathbf{P}\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{I}\\ \vdots \\ \mathbf{I}\end{bmatrix}}_{\mathbf{Y}}\Bigg)^{-1}\cdot \\ &\begin{bmatrix}\tau\cdot\mathbf{P} & \dots & \tau\cdot\mathbf{P}\end{bmatrix}\cdot\begin{bmatrix}\mathbf{\Lambda}-\tau\cdot\mathbf{P} & & \\ & \ddots & \\ & & \mathbf{\Lambda}-\tau\cdot\mathbf{P}\end{bmatrix}^{-1}\end{aligned} \tag{B.5}$$

First, we calculate

$$\begin{aligned}(\mathbf{\Lambda}-\tau\cdot\mathbf{P})^{-1} &= \mathbf{\Lambda}^{-1}-\mathbf{\Lambda}^{-1}\cdot\tau\cdot\left(-1+\mathbf{P}\cdot\mathbf{\Lambda}^{-1}\cdot\tau\right)^{-1}\cdot\mathbf{P}\cdot\mathbf{\Lambda}^{-1} \\ &= \mathbf{\Lambda}^{-1}-\mathbf{\Lambda}^{-1}\cdot\tau\cdot\frac{1}{-1+\sum_{i=1}^{v}P(\tau_i)\frac{\tau_i}{1+\tau_i}}\cdot\mathbf{P}\cdot\mathbf{\Lambda}^{-1} \\ &= \mathbf{\Lambda}^{-1}-\mathbf{\Lambda}^{-1}\cdot\tau\cdot\beta\cdot\mathbf{P}\cdot\mathbf{\Lambda}^{-1}\end{aligned} \tag{B.6}$$

where $\beta = \dfrac{1}{-1+\sum_{i=1}^{v}P(\tau_i)\frac{\tau_i}{1+\tau_i}}$.

Now we calculate $\mathbf{Y}^{-1}$. We rewrite $\mathbf{Y}$ as

$$\begin{aligned}\mathbf{Y} &= \mathbf{I}+\begin{bmatrix}\tau\cdot\mathbf{P} & \dots & \tau\cdot\mathbf{P}\end{bmatrix}\cdot\begin{bmatrix}\mathbf{\Lambda}-\tau\cdot\mathbf{P} & & \\ & \ddots & \\ & & \mathbf{\Lambda}-\tau\cdot\mathbf{P}\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{I}\\ \vdots \\ \mathbf{I}\end{bmatrix} \\ &= \mathbf{I}+\begin{bmatrix}\tau\cdot\mathbf{P} & \dots & \tau\cdot\mathbf{P}\end{bmatrix}\cdot\begin{bmatrix}\mathbf{\Lambda}^{-1}-\beta\mathbf{\Lambda}^{-1}\tau\mathbf{P}\mathbf{\Lambda}^{-1} & & \\ & \ddots & \\ & & \mathbf{\Lambda}^{-1}-\beta\mathbf{\Lambda}^{-1}\tau\mathbf{P}\mathbf{\Lambda}^{-1}\end{bmatrix}\cdot\begin{bmatrix}\mathbf{I}\\ \vdots \\ \mathbf{I}\end{bmatrix} \\ &= \mathbf{I}+n\cdot\tau\cdot\mathbf{P}\cdot\left(\mathbf{\Lambda}^{-1}-\beta\mathbf{\Lambda}^{-1}\tau\mathbf{P}\mathbf{\Lambda}^{-1}\right) = \mathbf{I}+\tau\cdot\left[n\cdot\left(1-\mathbf{P}\mathbf{\Lambda}^{-1}\tau\beta\right)\right]\cdot\mathbf{P}\mathbf{\Lambda}^{-1} \\ &= \mathbf{I}+\tau\cdot\frac{n}{1-\sum_{i=1}^{v}P(\tau_i)\frac{\tau_i}{1+\tau_i}}\cdot\mathbf{P}\mathbf{\Lambda}^{-1}\end{aligned} \tag{B.7}$$

Applying the matrix inversion Lemma to $\mathbf{Y}^{-1}$ we obtain

$$\begin{aligned}
\mathbf{Y}^{-1} &= \mathbf{I}^{-1} - \mathbf{I}^{-1}\tau \cdot \left(\frac{1 - \sum_{i=1}^{v} P(\tau_i)\frac{\tau_i}{1+\tau_i}}{n} + \mathbf{P} \cdot \mathbf{\Lambda}^{-1} \cdot \mathbf{I}^{-1} \cdot \tau \right)^{-1} \cdot \mathbf{P} \cdot \mathbf{\Lambda}^{-1} \cdot \mathbf{I}^{-1} \\
&= \mathbf{I} - \tau \cdot \left(\frac{1}{\frac{1 - \sum_{i=1}^{v} P(\tau_i)\frac{\tau_i}{1+\tau_i}}{n} + \sum_{i=1}^{v} P(\tau_i)\frac{\tau_i}{1+\tau_i}} \right) \cdot \mathbf{P} \cdot \mathbf{\Lambda}^{-1} \\
&= \mathbf{I} - \frac{n}{1 + (n-1)\sum_{i=1}^{v} P(\tau_i)\frac{\tau_i}{1+\tau_i}} \cdot \tau \mathbf{P} \mathbf{\Lambda}^{-1} \qquad \text{(B.8)}
\end{aligned}$$

Finally, we can calculate $\mathbf{A}^{-1}$ as

$$\begin{aligned}
\mathbf{A}^{-1} &= \begin{bmatrix} \mathbf{\Lambda} - \tau \cdot \mathbf{P} & & \\ & \ddots & \\ & & \mathbf{\Lambda} - \tau \cdot \mathbf{P} \end{bmatrix}^{-1} - \begin{bmatrix} \mathbf{\Lambda} - \tau \cdot \mathbf{P} & & \\ & \ddots & \\ & & \mathbf{\Lambda} - \tau \cdot \mathbf{P} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \mathbf{I} \\ \vdots \\ \mathbf{I} \end{bmatrix} \cdot \\
&= \left(\mathbf{I} - \frac{n}{1 + (n-1)\sum_{i=1}^{v} P(\tau_i)\frac{\tau_i}{1+\tau_i}} \cdot \tau \mathbf{P} \mathbf{\Lambda}^{-1} \right) \cdot \begin{bmatrix} \tau \cdot \mathbf{P} & \ldots & \tau \cdot \mathbf{P} \end{bmatrix} \cdot \\
& \begin{bmatrix} \mathbf{\Lambda} - \tau \cdot \mathbf{P} & & \\ & \ddots & \\ & & \mathbf{\Lambda} - \tau \cdot \mathbf{P} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{B} & & \\ & \ddots & \\ & & \mathbf{B} \end{bmatrix} - \begin{bmatrix} \mathbf{C} & \ldots & \mathbf{C} \\ \vdots & \ddots & \vdots \\ \mathbf{C} & \ldots & \mathbf{C} \end{bmatrix} \qquad \text{(B.9)}
\end{aligned}$$

where

$$\mathbf{B} = \mathbf{\Lambda}^{-1} - \beta \mathbf{\Lambda}^{-1} \tau \mathbf{P} \mathbf{\Lambda}^{-1}$$

$$\mathbf{C} = \left(\mathbf{\Lambda}^{-1} - \beta \mathbf{\Lambda}^{-1} \tau \mathbf{P} \mathbf{\Lambda}^{-1} \right) \cdot \left(\mathbf{I} - \frac{n}{1 + (n-1)\sum_{i=1}^{v} P(\tau_i)\frac{\tau_i}{1+\tau_i}} \cdot \tau \mathbf{P} \mathbf{\Lambda}^{-1} \right) \cdot \tau \mathbf{P} \cdot \left(\mathbf{\Lambda}^{-1} - \beta \mathbf{\Lambda}^{-1} \tau \mathbf{P} \mathbf{\Lambda}^{-1} \right)$$

□

B.4 Proof of Lemma 7

Proof. For a generic user i, we want to prove that $\tilde{a}_i$ is the best action given that the other users adopt $\tilde{a}_{-i}$. We study the sign of the derivative of the logarithm of i's utility with respect to i's action

$$\frac{\partial \ln U_i^I(a_i, \tilde{a}_{-i}, t_i, x)}{\partial a_i} = \begin{cases} \dfrac{t_i}{a_i} - \dfrac{1}{C - \sum_{k \neq i} \tilde{a}_k - a_i} & a_i < \tilde{a}_i \\ \dfrac{t_i}{a_i} - \dfrac{1 + c_i}{C - \sum_{k \neq i} \tilde{a}_k - a_i - c_i(a_i - \tilde{a}_i)} & \tilde{a}_i < a_i < \tilde{a}_i + \dfrac{\overline{x}}{c_i} \\ \dfrac{t_i}{a_i} - \dfrac{1}{C - \sum_{k \neq i} \tilde{a}_k - a_i - \overline{x}} & a_i > \tilde{a}_i + \dfrac{\overline{x}}{c_i} \end{cases}$$

We denote by $a_i^{BR}(a_{-i})$ the best response function of user i, i.e., i's action that maximizes i's utility when the action vector of the other users is a_{-i}. Since the users' utilities satisfy the assumptions **A4-A6** of Subsection 7.4.1, $\dfrac{U_i^I(a_i, \tilde{a}_{-i}, t_i, x)}{\partial a_i} \geq 0$ for $a_i < \tilde{a}_i$. In fact $U_i^I(a, t_i, x)$ is increasing with respect to a_i in $\left[0,\ a_i^{BR}(\tilde{a}_{-i})\right)$ and $\tilde{a}_i \leq a_i^{NE^0} = a_i^{BR}(a_{-i}^{NE^0}) \leq a_i^{BR}(\tilde{a}_{-i})$, where the first inequality is an assumption of the Lemma and the last inequality is valid because of the submodularity of the game.

Imposing the condition $\dfrac{\partial U_i^I(a_i, \tilde{a}_{-i}, t_i, x)}{\partial a_i} \leq 0$ in $\tilde{a}_i < a_i < \tilde{a}_i + \dfrac{\overline{x}}{c_i}$, we find

$$c_i \geq \frac{t_i \left(C - \sum_{k=1, k\neq i}^{n} \tilde{a}_k - a_i\right) - a_i}{t_i\left(a_i - \tilde{a}_i\right) + a_i} \tag{B.10}$$

The right hand side term of (B.10) is decreasing in a_i, therefore the condition is valid in $\tilde{a}_i < a_i < \tilde{a}_i + \dfrac{\overline{x}}{c_i}$ if and only if it is valid in $\tilde{a}_i$, obtaining

$$c_i \geq \frac{t_i\left(C - \sum_{k=1}^{n} \tilde{a}_k\right) - \tilde{a}_i}{\tilde{a}_i}$$

Notice that the condition on c_i is a necessary condition for $\tilde{a}_i$ to be a NE. In fact if it is not satisfied then $U_i^I(a_i, \tilde{a}_{-i}, t_i, x)$ is strictly increasing in $\tilde{a}_i$ and, for the continuity of $U_i^I(a_i, \tilde{a}_{-i}, t_i, x)$ with respect to a_i, we can find an action $\hat{a}_i > \tilde{a}_i$ such that $U_i^I(\hat{a}_i, \tilde{a}_{-i}, t_i, x) > U_i^I(\tilde{a}_i, \tilde{a}_{-i}, t_i, x)$.

Finally, imposing the condition $\dfrac{\partial U_i^I(a_i, \tilde{a}_{-i}, t_i, x)}{\partial a_i} \leq 0$ in $a_i > \tilde{a}_i + \dfrac{\overline{x}}{c_i}$, we find

$$\overline{x} \geq \frac{c_i\left[t_i\left(C - \sum_{k=1}^{n} \tilde{a}_k\right) - \tilde{a}_i\right]}{1 + t_i(1 + c_i)}$$

Notice that, given the condition on c_i, this last condition is sufficient for $\tilde{a}_i$ to be a global maximizer. In fact in this way $U_i^I(a_i, \tilde{a}_{-i}, t_i, x)$ becomes quasi-concave in a_i: increasing for $a_i < \tilde{a}_i$ and decreasing for $a_i > \tilde{a}_i$.

□

B.5 Proof of Proposition 18

Proof. Conditions **1**, **3** and **4** of Proposition 13 are satisfied. It remains to verify that **2** is satisfied, i.e., $\forall\, t_i, \hat{t}_i \in T_1$,

$$\sum_{t_{-i} \in T_{-i}} \pi(t_{-i}) a_i^{t_i} \left(C - \sum_{k=1}^{n} a_k\right) \geq \sum_{t_{-i} \in T_{-i}} \pi(t_{-i}) \hat{a}_i^{t_i} \left(C - \sum_{k=1}^{n} \hat{a}_k\right) \tag{B.11}$$

where, $\forall j \neq i$,

$$a_i = \frac{t_i C}{n + \sum_{k \neq i} t_k + t_i} \ , \ a_j = \frac{t_j C}{n + \sum_{k \neq i} t_k + t_i} \ , \ \hat{a}_i = \frac{\hat{t}_i C}{n + \sum_{k \neq i} t_k + \hat{t}_i} \ , \ \hat{a}_j = \frac{t_j C}{n + \sum_{k \neq i} t_k + \hat{t}_i} \tag{B.12}$$

In particular, Eq. (B.11) is valid if, $\forall \, t_{-i} \in T_{-i}$,

$$a_i^{t_i} \left(C - \sum_{k=1}^{n} a_k \right) \geq \hat{a}_i^{t_i} \left(C - \sum_{k=1}^{n} \hat{a}_k \right) \tag{B.13}$$

Substituting Eq. (B.12) into Eq. (B.13) we obtain:

$$\left(\frac{n + \sum_{k \neq i} t_k + \hat{t}_i}{n + \sum_{k \neq i} t_k + t_i} \right)^{t_i + 1} \left(\frac{t_i}{\hat{t}_i} \right)^{t_i} \geq 1 \tag{B.14}$$

We use the notation $b = n + \sum_{k \neq i} t_k$, and $y = \frac{\hat{t}_i}{t_i}$. We want to find the condition on t_i and y such that

$$h(y) = \left(\frac{b + t_i y}{b + t_i} \right)^{t_i + 1} y^{-t_i} \geq 1$$

Notice that $h(1) = 1$. We take the derivative of h with respect to y

$$h'(y) = t_i y^{-t_i - 1} \left(\frac{b + t_i y}{b + t_i} \right)^{t_i} \left(\frac{y - b}{b + t_i} \right)$$

$h'(y) \geq 0 \Leftrightarrow y \geq b \Leftrightarrow \frac{\hat{t}_i}{t_i} \geq n + \sum_{k \neq i} t_k$.

$f(\frac{\hat{t}_i}{t_i})$ is decreasing in $\hat{t}_i$ until $\hat{t}_i = t_i \left(n + \sum_{k \neq i} t_k \right)$, then it is increasing. This implies that for $\hat{t}_i < t_i$ Eq. (B.13) is satisfied, i.e., user i has no incentive to report a lower type. However, if $\hat{t}_i \to t_i^+$, since $h'(1) < 0$, then user i has an incentive to communicate a higher type (this result is linked to Proposition 14). In fact Eq. (B.13) is not satisfied $\forall \, t_{-i} \in T_{-i}$, and therefore Eq. (B.12) is unsatisfied. Since the function $h(\frac{\hat{t}_i}{t_i})$ increases for $\hat{t}_i > t_i \left(n + \sum_{k \neq i} t_k \right)$, the only way for Eq. (B.13) to be satisfied is that the function $f(y)$ will eventually reach the value 1 for a value $x^{th} = \frac{\tau^{th}}{t_i}$ and all the types higher than t_i are higher than the threshold value τ^{th}. Notice that it is sufficient that this condition is verified by the type that follows t_i. Substituting t_i with τ_l and $\hat{t}_i$ with τ_{l+1} into Eq. (B.14) we obtain Eq. (7.10).

$\square$

B.6 Proof of Proposition 19

Proof. First, we demonstrate that Eq. (7.11) describes a convex problem if $\tau_v \leq n$. The constraints describe a convex set. We can rewrite the objective function in the following way

$$f(a) = -\ln\left[\left(C - \sum_{i=1}^{n} a_i\right) \sum_{t\in\mathcal{T}} \pi(t) \prod_{i=1}^{n} a_i^{\frac{t_i}{n}}\right] = -\ln\left[\left(C - \sum_{i=1}^{n} a_i\right) \prod_{i=1}^{n} \sum_{l=1}^{v} \pi(\tau_l) a_i^{\frac{\tau_l}{n}}\right] =$$
$$= -\ln\left(C - \sum_{i=1}^{n} a_i\right) - \sum_{i=1}^{n} \ln \sum_{l=1}^{v} \pi(\tau_l) a_i^{\frac{\tau_l}{n}}$$

We calculate the partial derivatives of $f(a)$

$$\frac{\partial f(a)}{\partial a_j} = \frac{1}{C - \sum_{i=1}^{n} a_i} - \frac{\sum_{l=1}^{v} \pi(\tau_l) \frac{\tau_l}{n} a_i^{\frac{\tau_l}{n}-1}}{\sum_{l=1}^{v} \pi(\tau_l) a_i^{\frac{\tau_l}{n}}}$$

$$\frac{\partial^2 f(a)}{\partial a_j^2} = \frac{1}{\left(C - \sum_{i=1}^{n} a_i\right)^2} - \frac{\left(\sum_{l=1}^{v} \pi(\tau_l) \frac{\tau_l}{n} \left(\frac{\tau_l}{n} - 1\right) a_i^{\frac{\tau_l}{n}-2}\right)\left(\sum_{l=1}^{v} \pi(\tau_l) a_i^{\frac{\tau_l}{n}}\right) - \left(\sum_{l=1}^{v} \pi(\tau_l) \frac{\tau_l}{n} a_i^{\frac{\tau_l}{n}-1}\right)^2}{\left(\sum_{l=1}^{v} \pi(\tau_l) a_i^{\frac{\tau_l}{n}}\right)^2}$$

$$\frac{\partial^2 f(a)}{\partial a_j \partial a_k} = \frac{1}{\left(C - \sum_{i=1}^{n} a_i\right)^2}$$

We have $\frac{\partial^2 f(a)}{\partial a_j^2} \geq \frac{\partial^2 f(a)}{\partial a_j \partial a_k} \geq 0$, where the first inequality is valid if $\tau_v \leq n$.

Before concluding, we state and prove the following Lemma.

Lemma 8. *The matrix*

$$H = \begin{bmatrix} \alpha_1 & \beta & \dots & \beta \\ \beta & \alpha_2 & \dots & \beta \\ \vdots & & \ddots & \vdots \\ \beta & \beta & \dots & \alpha_n \end{bmatrix}$$

where $\alpha_i \geq \beta \geq 0$, $\forall\, i = \{1, 2, \cdots, n\}$, *is positive semidefinite. If the first inequality is strict, it is also positive definite.*

Proof.

$$H = \beta \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \vdots & & \ddots & \vdots \\ 1 & 1 & \dots & 1 \end{bmatrix} + \begin{bmatrix} \alpha_1 - \beta & 0 & \dots & 0 \\ 0 & \alpha_2 - \beta & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & \alpha_n - \beta \end{bmatrix}$$

Therefore

$$w^T \cdot H \cdot w = (\alpha_1 - \beta)\, w_1^2 + \cdots + (\alpha_n - \beta)\, w_n^2 + \beta \left(\sum_{i=1}^{n} w_i \right)^2$$

$w^T \cdot H \cdot w \geq 0 \, \forall \, w$ if $\alpha_i \geq \beta \geq 0 \, \forall \, i$. $w^T \cdot H \cdot w > 0 \, \forall \, w \neq 0$ if $\alpha_i > \beta \geq 0 \, \forall \, i$. □

Applying Lemma 8 to the Hessian of the function $f(a)$ we obtain that the Hessian is positive semidefinite, therefore the function $f(a)$ is convex.

As for the optimality of the communication-free incentive compatible device D, we have

$$max_a \mathbb{E}_t \left[\sqrt[n]{\prod_{i=1}^{n} U_i^+(a, t_i, \Phi(r, m, a))} \right] \leq max_a \mathbb{E}_t \left[\sqrt[n]{\prod_{i=1}^{n} U_i^+(a, t_i, x^*)} \right] =$$

$$= \max_a \left(C - \sum_{i=1}^{n} a_i \right)^+ \mathbb{E}_t \left[\sqrt[n]{\prod_{i=1}^{n} a_i^{t_i}} \right] = \max_a \left(C - \sum_{i=1}^{n} a_i \right) \mathbb{E}_t \left[\prod_{i=1}^{n} a_i^{\frac{t_i}{n}} \right]$$

Thus, if D sustains $\overline{a}$, D is an optimal communication-free incentive compatible device.

□

List of Publications

The research activity carried on during my PhD has resulted in the following – accepted or under submission – articles, whose contributions have been presented in this thesis, except for **C2** and **C3** which are isolated works, far from the main research topic followed during my PhD.

Journal papers

[J1] **L. Canzian**, L. Badia, and M. Zorzi, "Promoting Cooperation in Wireless Relay Networks through Stackelberg Dynamic Scheduling," *to appear in IEEE Trans. Commun.*

[J2] G. Quer, F. Librino, **L. Canzian**, L. Badia, and M. Zorzi, "Inter-Network Cooperation exploiting Game Theory and Bayesian Networks," *Submitted to IEEE Trans. Commun.*

[J3] **L. Canzian**, Y. Xiao, M. Zorzi, and M. van der Schaar, "Game Theoretic Design of MAC Protocols: Pricing and Intervention in Slotted-Aloha," *Submitted to IEEE/ACM Trans. Networking*

[J4] **L. Canzian**, Y. Xiao, W. Zame, M. Zorzi, and M. van der Schaar, "Intervention with Private Information, Imperfect Monitoring and Costly Communication: Design Framework," *Submitted to IEEE Trans. Commun.*

[J5] **L. Canzian**, Y. Xiao, W. Zame, M. Zorzi, and M. van der Schaar, "Intervention with Complete and Incomplete Information: Application to Flow Control," *Submitted to IEEE Trans. Commun.*

Conference papers

[C1] L. Anchora, L. Badia, **L. Canzian**, and M. Zorzi, "A Characterization of Resource Allocation in LTE Systems Aimed at Game Theoretical Approaches," in *Proc. IEEE CAMAD*, Miami, FL, USA, Dec. 3-4, 2010

[C2] **L. Canzian**, A. Zanella, and M. Zorzi, "Overlapped NACKs: Improving Multicast Performance in Multi-access Wireless Networks," in *Proc. IEEE PerGroup*, Miami, FL, USA, Dec. 6, 2010

[C3] O. Pozzobon, **L. Canzian**, A. Dalla Chiara, and M. Danieletto, “Anti-Spoofing and open GNSS Signal Authentication with Signal Authentication Sequences,” in *Proc. NAVITEC 2010*, Noordwijk, The Netherlands, Dec. 8-10, 2010

[C4] **L. Canzian**, L. Badia, and M. Zorzi, “Relaying in Wireless Networks Modeled through Cooperative Game Theory,” in *Proc. IEEE CAMAD*, Kyoto, Japan, Jun. 10-11, 2011

[C5] G. Quer, F. Librino, **L. Canzian**, L. Badia, and M. Zorzi, “Using Game Theory and Bayesian Networks to Optimize Cooperation in Ad Hoc Wireless Networks,” in *Proc. IEEE ICC*, Ottawa, Canada, Jun. 10-15, 2012

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