Direct Reconstruction of the Quantum Density Matrix by Strong Measurements

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Abstract: We propose a quantum state reconstruction method based on ancilla-assisted strong measurements. It is simpler than standard tomography and more accurate than methods that require weak measurements. We validate it with an optical experiment.

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Introduction

Quantum state reconstruction is a fundamental task for many modern quantum devices such as computers or simulators. We propose a new strategy to tackle this problem which can be preferable to current techniques, especially for systems of large dimension [1]. The traditional way of finding the density operator of a d-dimensional state is known as quantum state tomography (QST), which in general requires $O(d^2)$ different operations. Recently, Lundeen *et al.* proposed a *direct reconstruction* method based on ancilla-assisted weak measurements which can reduce such number to $O(d)$ [2]. However, weak measurements need many statistical repetitions of the experiment to extract enough information, can be more vulnerable to experimental systematic errors and use approximate relations.

We extended one such method in a way that eliminates the need for these approximations, allowing the use of more practical strong measurements. Our results are backed by exact mathematical expressions and are less demanding in terms of statistical repetitions. We verified experimentally the feasibility of our measurement protocol applying it to the polarization of light in single-photon regime.

Theoretical model

In the weak measurement framework, the object of the observation is coupled to a quantum measuring device, usually called ancilla. Our protocol and the weak counterpart that it extends need two independent bidimensional ancillae A and B which are both prepared in $|0\rangle$, the $+1$ eigenstate of the Pauli operator σ_z. The initial general system is $\varrho_{in} = \varrho \otimes |0\rangle_A \langle 0| \otimes |0\rangle_B \langle 0|$ where ϱ is the unknown density operator that we wish to reconstruct. To find its jkth element in the basis $\{|a_j\rangle \,|\, j = 1, \dots, d\}$ we apply two unitary operations $U_{A,j} = e^{-i\theta_A \Pi_{a_j} \otimes \sigma_{yA}} \otimes \mathbb{1}_B$ and $U_B = e^{-i\theta_B \Pi_{b_0} \otimes \sigma_{yB}} \otimes \mathbb{1}_A$ in this order, where $\theta_{A,B} \in [0, \pi/2]$ represents the strength of the coupling, $|b_0\rangle = \frac{1}{\sqrt{d}} \sum_j |a_j\rangle$, while symbol Π labels the one-dimensional projector on the subscripted state. Then, two appropriate observables are measured on the ancillae, jointly with Π_{a_k} on the object system. Lundeen *et al.* found that in the weak regime the initial density operator ϱ is approximated by ϱ^W:

$$\begin{aligned}
\Re(\varrho^W_{jk}) &= \mathcal{N}_{AB} \left(\langle \sigma_{xA} \sigma_{xB} \rangle_{j,k} - \langle \sigma_{yA} \sigma_{yB} \rangle_{j,k} \right) \\
\Im(\varrho^W_{jk}) &= \mathcal{N}_{AB} \left(\langle \sigma_{yA} \sigma_{xB} \rangle_{j,k} + \langle \sigma_{xA} \sigma_{yB} \rangle_{j,k} \right) ,
\end{aligned} \tag{1}$$

in which $\mathcal{N}_{AB} = \frac{d}{4 \sin\theta_A \sin\theta_B}$. However, our main result is that this expression can be made exact, finding $\varrho = \varrho^S$:

$$\begin{aligned}
\varrho^S_{jj} &= 16 \mathcal{N}^2_{AB} \langle \Pi_{1A} \Pi_{1B} \rangle_{j,k} && \forall k \\
\Re(\varrho^S_{jk}) &= -2 \mathcal{N}_{AB} \langle \sigma_{yA} \sigma_{yB} \rangle_{j,k} && j \neq k \\
\Im(\varrho^S_{jk}) &= 2 \mathcal{N}_{AB} \langle \sigma_{xA} \sigma_{yB} \rangle_{j,k} && j \neq k
\end{aligned} \tag{2}$$

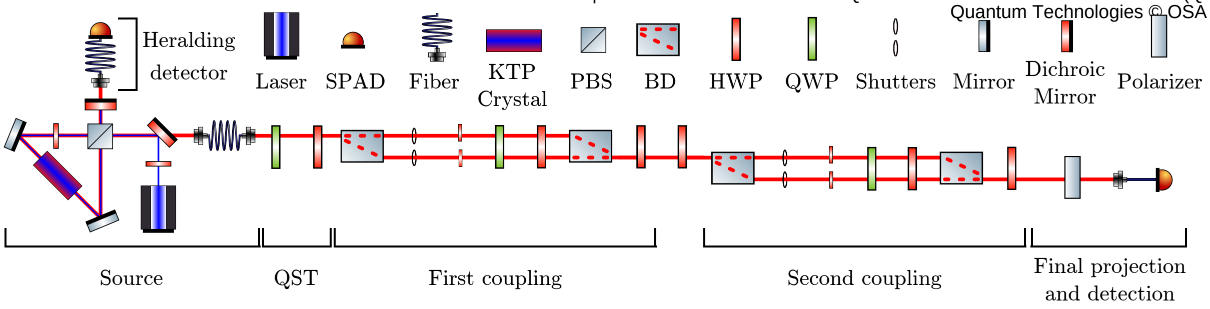

Fig. 1: Scheme of the experimental setup.

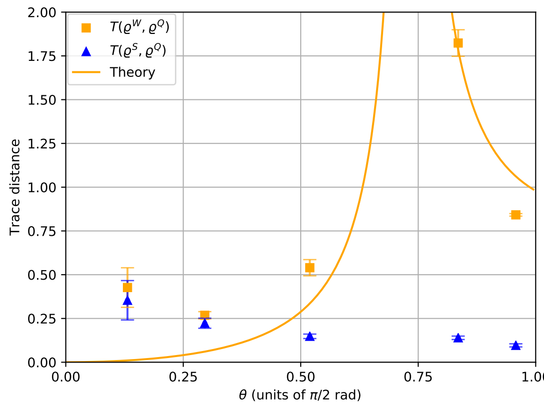

Fig. 2: Trace distance for a pure state.

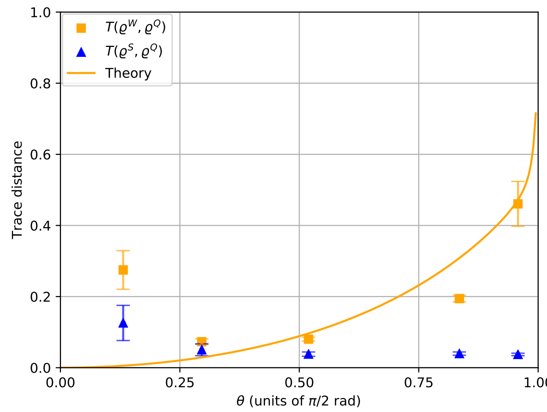

Fig. 3: Trace distance for a mixed state.

Experimental method

We use a custom-built source of polarization entangled photon pairs at 808 nm to produce polarization states of different degrees of purity, that we reconstruct with the protocols of Eqs. (1) and (2). One photon of each pair is sent directly to a single photon avalanche detector and used as a herald, while the other reaches our measurement setup. This consists of two Mach-Zehnder interferometers in a sequence, one for $U_{A,j}$ and one for U_B, with the path representing the ancilla. Internal waveplates can tune the coupling strength and perform the ancilla measurement, while a final polarizer enacts the projection onto the $\{|a_k\rangle\}$ basis. Two plates at the beginning of the setup can perform standard QST to enrich our comparison. A complete scheme of the experiment is shown in Fig. 1.

Results

We use the trace distance between the QST state ϱ^Q and the results produced by the two protocols as a figure of merit. We present it here as a function of the coupling strength $\theta = \theta_A = \theta_B$, for two different input states, one that is almost pure (Fig. 2) and one that is almost maximally mixed (Fig. 3). The solid line shows the expected trace distance of the protocol of Eq. (1). We can see that its results are inaccurate not only for high θ, but also in the weak regime because of the increased vulnerability to systematic errors. Our method instead shows a low trace distance for $\theta \to \pi/2$, thus proving its feasibility and the superiority of strong measurements.

Discussion

Because of being based on exact relations, our method allows the use of strong measurements, which are easier to implement and more precise. It can also be preferable to standard QST when the dimension d of the system is large. This is because it needs only $d+1$ unitary operations $U_{A,j}$ and U_B, one d-outcome projection in the $\{|a_k\rangle\}$ basis and a small finite number of pointer measurements (three: $\Pi_{1A}\Pi_{1B}$, $\sigma_{yA}\sigma_{yB}$ and $\sigma_{xA}\sigma_{yB}$). With the exception of U_B, all the operations on the object system involve states in the measurement basis, which is often experimentally more accessible.

References

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