Electoral Competition with Primaries and Quality Asymmetries

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We introduce primaries—both closed and open—into a Downsian model of two-party electoral competition allowing the two candidates in each party's primary to differ in valence as well as in policy platform. The good news is that the introduction of either type of primary acts as a stabilizing force since equilibriums exist quite generally, serves as an arena for policy debates since all candidates propose differentiated platforms, and guarantees that each party's nominee is of higher quality than its primary opponent. Moreover, primaries tend to benefit the party whose median voter is closer to the overall median. The bad news is that the winner of the general election need not be the candidate with the highest overall quality since primaries that are too competitive can prove harmful. Given the differences between open and closed primaries, we show that the choice of primary type is particularly important and may determine the winner of the general election.

Party primaries have become an increasingly common method of nominating candidates for a general election. In the United States (after World War II), primaries are by and large conducted in the same manner as a general election and run by the same electoral authorities. In Europe and Latin America, primary elections are a more recent phenomenon, and primaries are generally run by the parties themselves. Several questions of interest naturally arise. For example, how does the introduction of primaries influence candidates' policy proposals? How does the introduction of primaries affect the outcome of the general election? If we allow candidates to vary both in policy positions and on a valence or quality dimension, how does the differentiation of candidates in the party primary on these two dimensions affect the party's performance in the general election? Also, how does the choice between a closed and an open primary type matter?

We answer such questions in the framework of a standard Downsian model of electoral competition by considering plurality two-party competition in a two-stage election (primary and general) in which candidates choose and commit to a given platform prior to the primary election.[1] We first assume that both parties run closed primaries with two candidates competing in each primary. At the time of closed primaries, parties are treated as exogenous and differ in the preferences of their primary electorates. Candidates may differ both in policy platforms and in terms of a commonly valued and commonly known nonpolicy characteristic, which we label a *valence* or quality dimension. By introducing this quality heterogeneity among candidates, voters who participate in the primaries and in the general election sincerely vote not only on the basis of policy proposals but also on the basis of which candidate is considered to be "better" (e.g., in terms of charisma, corruption allegations, and expe-

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1. Standard assumptions of a Downsian model are office-motivated candidates, full commitment, and sincere voting among others. See Grofman (2004) and Osborne (1993) for a list of assumptions that are generally perceived to best justify the description of a model as a Downsian one.

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rience).[2] Regarding candidates' behavior, we present all of our formal results by positing that all candidates aim at maximizing their general election vote share in the absence of any kind of uncertainty.[3]

In our setup, we characterize the unique equilibrium of the game with several interesting properties developing. In equilibrium, each party's low-valence candidate proposes a platform that coincides with the ideal policy of her party's median. Each party's high-valence candidate is relatively more moderate than the low-valence primary candidate, and targeting the best electoral outcome in the general election locates the closest possible to the society's median. The valence asymmetry between the two primary candidates ultimately determines how differentiated the two platforms are. The larger the advantage of one candidate is, the more she is capable of moving toward moderate policies and hence becoming more appealing in the general election while guaranteeing a primary victory. As far as valence is concerned, since high-valence candidates always win their primary, our result is in line with recent empirical evidence showing that primaries tend to be effective at selecting high-quality types (Hirano and Snyder 2014). In terms of policy proposals, primaries serve as the arena for meaningful intraparty policy debates since primary candidates propose differentiated platforms. Nevertheless, the intensity of such debates is crucial in determining the winner of the general election, and primaries can prove harmful to the party with the highest valence candidate. Our results show that the low-valence general election candidate may win if she faced a weak party primary opponent, while the highest valence candidate could not propose a moderate enough platform because of tight primaries inside the losing party. Hence, our divergent equilibrium result shows that primaries may prove harmful to a party if they create too much competition during the nomination process. This *within-party competition effect* of primaries on electoral outcomes is the first substantial result of our analysis and relates to the negative aspect of the divisive effect of primaries (Agranov 2016; Key 1953).

Second, our analysis suggests that primaries have a matching effect: candidates nominated by leftist (rightist) parties win more often when the society's median is leftist (rightist). This is a result of primaries making candidates more responsive to the policy preferences of their primary electorate rather than the general electorate. If, for example, the society becomes more leftist, the leftist party will win more often than before, since the high-valence primary candidate of the rightist party cannot react to the median's shift. If the rightist high-valence candidate were to propose leftist policies that would please the new median, this would potentially make her lose the primary. Note that this otherwise intuitive feature of our equilibrium is surprisingly absent from most electoral competition models without primaries: models with office-motivated candidates usually generate equilibriums in which candidates converge (either in deterministic or in probabilistic terms), and models with policy-motivated candidates often predict that candidates will locate equidistantly away from the society's median (and will hence tie) independently of whether the society's median is leftist or rightist (see, e.g., Besley and Coate 1997; Matakos et al. 2016; Ortuño-Ortín 1997; Osborne and Slivinski 1996; Saporiti 2014). Overall, the matching effect may further strengthen the stability of the party system as it allows parties to form a more durable ideological framework. However, the asymmetry of party competition, with the party whose median voter is closer to the overall median being advantaged, allows us to recognize an important stylized fact about much political competition, namely, that there may be (extended) periods during which one party is dominant (Merrill et al. 2008).

Third, the introduction of primaries in this intuitive setup extends our knowledge on equilibrium existence in modifications of the standard Downsian model. Although without primaries a pure-strategy Nash equilibrium in a similar two-party setup does not exist (Aragonès and Palfrey 2002),[4] we show that with closed primaries a unique equilibrium in pure strategies always exists, and this is true for any distribution of voters' preferences. That is, while the standard Downsian model of electoral competition with valence asymmetries predicts that stability may (Ansolabehere and Snyder 2000)

2. While in the general election there are no incentives for strategic voting, this is not the case in the primary. We discuss the effect of introducing a small share of strategic voters in the appendix (available online), and we argue that the main qualitative features of our equilibrium analysis are robust to such extension.

3. Our results, however, are to some extent compatible with the alternative interpretation of candidates having some uncertainty regarding the location of the median voter in the general election when choosing their primary platforms so as to maximize the probability of winning the general election.

4. To be precise, Aragonès and Palfrey (2002) consider that the two heterogeneous candidates are win-motivated and hold imperfect information regarding voters' policy preferences. As is argued by Aragonès and Xefteris (2017a), in these models, win-motivation with imperfect information about voters' preferences is technically equivalent to vote-share maximization and perfect information about voters' preferences. Equilibrium existence with win-motivation and perfect information about voters preferences equilibrium existence is rarely an issue (see, e.g., Ansolabehere and Snyder 2000).

or may not be reached (Aragonès and Palfrey 2002), the introduction of primaries provides a clear *stabilizing effect*.[5]

To the best of our knowledge, this is the first article to point at these three effects of primaries simultaneously. The second and the third effects are obviously positive ones: primaries stabilize the electoral process and generate consistency between the party of the elected candidate and voters' policy preferences, and these in turn promote a sense of trust in the political system. The first effect has, arguably, negative implications: a high-valence candidate who faces hard within-party competition might end up losing the general election to a mediocre candidate who won in her party's primaries against a low-quality opponent.

We propose several modifications to our analysis and find that our results are robust in a number of directions. First, we focus on open primaries. Once all candidates make their policy proposals, active citizens who are willing to participate in the procedure then vote in the primary of the party in which their top-ranked candidate participates. This implies that by proposing moderate platforms, candidates not only increase their general election vote share but also increase the amount of active voters participating in their party's primary. Hence, both parties' size as well as the ideal policy of the parties' medians are now endogenously determined and depend on the quality characteristics of all candidates. This is in contrast to closed primaries, where candidates gain nomination by focusing only on their party's median voter and the quality characteristics of their party's candidates; in open primaries, attention is also paid to the quality characteristics of the other party given the endogenous sorting of voters across primaries. Interestingly, and despite the endogenous party formation described, the stabilizing effect of primaries prevails since for a large class of voters' distributions we still obtain a unique equilibrium in pure strategies such that (*a*), in some instances, the highest valence candidate does not emerge as the winner of the general election (within-party competition effect of primaries), and (*b*) the winner of the leftist (rightist) primary is more likely to be the general election's winner when the society's median is leftist (rightist; *matching effect of primaries*).

Finally, we investigate situations in which a primary is held only in one of the two parties (either closed or open). This is of interest because incumbents often run for reelection without going through a nomination process and face a challenger who emerged from a primary. In our model, we assume that the position of the incumbent is fixed and that primary candidates in the opposition strategically choose their platforms (typically the incumbent has less flexibility than the challenger in credibly promising something different from the implemented policies). We find that when the incumbent implements socially detrimental (appealing) policies, the highest valence challenger is elected less (more) often in closed primaries than in open primaries. The reason why bad incumbents are less threatened by challengers that emerge from closed primaries than from open primaries is that closed primaries hold candidates close to their party's median, which might be quite far from society's median. Open primaries pose no such restriction and allow candidates to expand their primary electorate by moving toward the center. In other words, open primaries give incentives to high-valence candidates of initially less moderate parties to move toward the center and thus to (*a*) win more often and (perhaps more importantly) (*b*) make their parties more moderate by moving toward the center and hence attracting new moderate voters for their primaries. These results point to an interesting effect of the organization of the party in opposition on the incumbent's decisions when the latter cares about reelection: incumbents have stronger incentives to implement moderate policies when the challenger's party holds open primaries than when it holds closed primaries.

Our article complements the existing literature on primaries with valence asymmetries by adding several insightful new results.[6] Our work closely relates to Adams and Merrill (2008), since, to the best of our knowledge, this is the only other setup where all four primary candidates may differ in valence. Nevertheless, while Adams and Merrill (2008) focus on a probabilistic voting model, we focus on a deterministic one.[7] As is well known, probabilistic and deterministic voting models with valence asymmetries deliver very diverse pre-

5. The search for stabilizing forces in multidimensional competition models has attracted previous attention and several proposals. Among others, Lin et al. (1999) consider probabilistic voting; Aragonès and Xefteris (2017b), Dziubiński and Roy (2011), and Krasa and Polborn (2012) allow for differentiated candidates; and Bräuninger (2007) allows for costly voting.

6. Research on primaries without valence issues was presented in Aranson and Ordeshook (1972), Coleman (1971), Meirowitz (2005), and Owen and Grofman (2006), among others. For articles interested in the noncommitment of primary winners and flip-flopping between primary and general elections, see Agranov (2016) and Hummel (2010). For the effect of sequential primaries on electoral competition, see Callander (2007) and Deltas et al. (2016). For work on different ways of candidates' nomination including primaries, see Amorós et al. (2016), Benoit, Crutzen, and Sahuguet (2018), Buisseret and Van Weelden (2017), Crutzen et al. (2010), Hortala-Vallve and Mueller (2015), Jackson et al. (2007), Kselman (2015), and Winer et al. (2014), among others.

7. In a probabilistic voting model one votes for a certain candidate with a probability that is increasing in the utility that one derives from the election of this candidate; i.e., one need not vote for the top-ranked candidate, although this is one's most probable action. In a deterministic voting model one always votes for the candidate offering the highest utility.

dictions on candidates' equilibrium behavior, and we show this is also true in the context of primaries.[8] The presence of a random element eventually leads to primary candidates proposing identical platforms, while the point of convergence might differ across parties (Adams and Merrill 2008). However, we show that primary candidates run on different platforms giving back to primaries the element of an internal battlefield.

Hummel (2013) also employs a nonprobabilistic valence model, but unlike us, he considers that (*a*) the higher (lower) valence candidate of the leftist party has precisely the same valence as the higher (lower) valence of the rightist party and (*b*) voters may strategically decide not to support their top-ranked candidates. Similar to Hummel (2013) we show that high-valence candidates propose more moderate policies than low-valence candidates. Nevertheless, by allowing all four candidates to differ in valence, we provide new results on the effect of primaries on the winner of the general election and demonstrate why and when the highest valence candidate may not win the general election. Takayama (2014) uses similar assumptions to those of Hummel (2013) and models only the primaries in the challenger's party with three candidates of different valence overall. She shows that as the incumbent's valence increases, the qualifying challenger becomes more moderate. Our results under the presence of an incumbent are different and depend on the primary type. We show that if the party runs closed primaries, the policy proposed by the high-valence challenger is not affected by the incumbent's characteristics. However, when the party organizes an open primary, the challenger becomes more moderate as the incumbent's valence decreases.

In Hummel (2013), as in Kartik and McAfee (2007), there are two levels of valence; although they are different from the aforementioned papers, high-valence types are committed to an exogenous platform. In Serra (2011) and Snyder and Ting (2011), both primaries and the general election function as a valence revelation mechanism, and their focus is more on the adoption of primaries rather than on primary candidates' platforms proposals. In Andreottola (2016), only primaries serve as a valence revelation mechanism; in contrast to our findings, his results show that the high-valence primary candidate proposes more extreme platforms than the low-valence candidate. (For models of primary elections with endogenous valence, see Casas [2013] and Serra [2010].)

The remainder of the article is structured as follows: in the next section we present the model; after that, we present our results for closed, open, and one-party primaries; finally, we conclude. In the appendix we discuss possible extensions and justifications of some of our main modeling elements, and we present all of the proofs.

THE MODEL

The policy space is the [0, 1] interval. We have a unit mass of general-election voters whose ideal policies are distributed according to an absolutely continuous, strictly increasing, and twice differentiable distribution function $\Phi : [0,\ 1] \to [0,\ 1]$ with a unique median, $m \in (0,\ 1)$, defined by $\Phi(m) = 1/2$. Two positive-measure subsets of these voters form the two exogenously given parties and participate in a closed primary election where no other voters can participate. Let the median of the leftist party be the primary voter with ideal policy l and the median of the rightist party be the primary voter with ideal policy r, with all l, r, and m known and $0 < l < m < r < 1$.[9] Candidates A and B compete in the primary of the leftist party, and candidates C and D compete in the primary of the rightist party. Each candidate $J \in \{A,\ B,\ C,\ D\}$ is characterized by a valence parameter $v_J \geq 0$ and strategically chooses and commits to an electoral platform $x_J \in S_J$, where $S_J = [0,\ m]$ if $J \in \{A,\ B\}$ and $S_J = [m,\ 1]$ if $J \in \{C,\ D\}$. We assume that $v_B > v_A$, $v_C > v_D$, and $v_B > v_C$. This ordering of valences assumes that B and C are the high-valence candidates in each of the parties, places the highest valence candidate B in the leftist party, and provides equilibrium locations in order with candidates' "names."[10] We focus on the interesting case when valence differences are not very large (the exact formal constraints are presented in the statements of our propositions).[11]

The game has three stages. In stage 1, all four candidates choose and announce their policy platforms simultaneously. In stage 2, closed primary elections take place in each of the two parties.[12] In stage 3, the general election takes place, and each voter votes for one of the two primaries' winners.

8. For example, in standard two-party competition models, while probabilistic voting models do not rule out convergent equilibriums when valence asymmetries are not very large (see, e.g., Schofield 2007), this never occurs in deterministic voting models (see, e.g., Aragonès and Palfrey 2002).

9. Note that we impose very little structure on the precise kind of closed primaries that each party holds. While one party may run a (primary) election in which only "core" party members are eligible, another party may run a primary open to all party members.

10. One can show that our equilibrium results hold when allowing one or more equalities but at a considerable cost in the proof length.

11. Indeed, most of the literature focuses in characterizing equilibriums for this scenario (e.g., Aragonès and Palfrey 2002; Groseclose 2001), as when valence differences are very large electoral competition might become trivial.

12. Our analysis would carry through if closed primaries in each party took place sequentially.

All ties, either in primaries or in the general election, are broken with equiprobable draws.

The utility of a voter with ideal policy $i \in [0,\ 1]$ when candidate $J \in \{A,\ B,\ C,\ D\}$ is elected into office (or else, wins in the general election) is given by

$$u_i(x_J, v_J) = -|i - x_J| + v_J,$$

in line with literature on electoral competition among candidates of unequal valence (see, e.g., Aragonès and Palfrey 2002; Groseclose 2001). Voters are sincere in both primaries and the general election and vote for the candidate who offers them the highest utility. We assume that when some voters are indifferent among a number of candidates, they evenly split among them.

Since voters' behavior in stages 2 and 3 is essentially mechanic, one may define the expected vote share of candidate J in the general election as

$$P_J(x_J,\ x_{-J} : v,\ \Phi,\ l,\ r),$$

where x_{-J} is the vector of platforms of the other candidates and $v = (v_A,\ v_B,\ v_C,\ v_D)$. Candidates are Downsian; that is, they maximize expected vote shares in the general election. The equilibrium concept we employ is Nash equilibrium in pure strategies, which in this setup is a vector $\hat{x} = (\hat{x}_A,\ \hat{x}_B,\ \hat{x}_C,\ \hat{x}_D)$, such that for every $J \in \{A,\ B,\ C,\ D\}$ it is true that $P_J(\hat{x}_J,\ \hat{x}_{-J} : v,\ \Phi,\ l,\ r) \geq P_J(\acute{x}_J,\ \hat{x}_{-J} : v,\ \Phi,\ l,\ r)$ for any $\acute{x}_J \in S_J$.[13]

RESULTS

Before presenting our main results, let us define two concepts of crucial relevance. In equilibrium, each primary is won by the high-valence candidate (B wins the leftist primary, and C wins the rightist primary). We refer to the valence difference in the general election as the "toughness" faced by B (defined as $T_G = -(v_B - v_C)$). Similarly, we refer to the valence difference in each party as the toughness candidates B and C face in their primary elections respectively (defined as $T_L = -(v_B - v_A)$ and $T_R = -(v_C - v_D)$). Note that given our restrictions on candidates' valence characteristics, toughness takes negative values, approaching zero when both candidates are of almost equal valence representing the "toughest" of all cases.

Proposition 1. If valence differences are not very large, there exists a unique Nash equilibrium; it is such that in each party (a) the low-valence candidate proposes the platform preferred by the party's median voter, (b) the high-valence candidate proposes a more moderate platform than the one preferred by the party's median voter, and (c) the high-valence candidate (i.e., B in the leftist party and C in the rightist party) wins the primary. Formally, if $l < m - v_B < m + v_B < r$, then there exists a unique Nash equilibrium $\hat{x}$ such that $\hat{x}_A = l$, $\hat{x}_B = l - T_L$, $\hat{x}_C = r + T_R$, and $\hat{x}_D = r$.

All proofs can be found in the appendix.

The existence of a unique Nash equilibrium points at the stabilizing effect of primaries on electoral competition. That is, while in the absence of primaries a clear prediction is hard to be derived—either the model does not admit an equilibrium in pure strategies (Aragonès and Palfrey 2002) or it admits a continuum of equilibriums (Ansolabehere and Snyder 2000)—this is no longer true when the two candidates have been selected through a primary race. Moreover, valence asymmetries in primaries create interesting electoral dynamics and in equilibrium lead to the divergence of proposed platforms in the primary race (in line with Hummel [2013] and in contrast to Adams and Merrill [2008]) with general election candidates locating somewhere between their parties' and the general election median (a platform ordering supported in the literature; Adams et al. 2005; Aranson and Ordeshook 1972; Burden 2001; Coleman 1971). Specifically, the low-valence candidate locates exactly at the party's median, while the high-valence candidate is more moderate and locates closer to the society's median.[14] How far toward the society's median the high-valence candidate is able to move depends on the toughness of the primary. The higher the

13. Maximization of the general election vote share is one of the possibilities when introducing office motives, and since we focus on pure strategies, it is a refinement of the following ordered pair of objectives: (a) a candidate prefers all outcomes of the game in which she is the winner of the general election to any other outcome, and (b) among all outcomes in which a candidate does not win the general election, this candidate prefers the outcomes in which she wins her party's primaries. Recall also the alternative interpretation of our model hinted in n. 3 according to which Φ may be viewed as the candidates' beliefs regarding the location of m and candidates' objective as maximizing the probability of winning the election (we elaborate on this in the appendix).

14. Recall that candidates in the leftist (rightist) party are constrained to propose platforms to the left (right) of the median. Parties, however, may impose different constraints on candidates' platforms (e.g., through a prerequisite of a minimum number of party officials' endorsements or resolutions of party summits regarding the party's flexibility on certain policy issues). Fortunately, all of the arguments supporting the existence of the equilibrium, $\hat{x} = (l,\ l + v_B - v_A, r - v_C + v_D,\ r)$, continue to hold for all alternative strategy sets $\{\bar{S}_A,\ \bar{S}_B,\ \bar{S}_C,\ \bar{S}_D\}$, as long as $\hat{x}_J \in \bar{S}_J$ for each $J \in \{A,\ B,\ C,\ D\}$. That is, as long as the constraints on candidates' platforms set by the party allow a candidate to locate sufficiently close to the party's median, our equilibrium continues to exist (even in the extreme case in which parties do not constrain candidates at all). Of course, the uniqueness arguments that we develop may not extend to all conceivable alternative strategy sets.

valence asymmetry inside a party, the more the winning candidate can converge toward the society's median, thus improving her future performance in the general election. However, if both candidates are of almost equal valence, then the high-valence candidate is not able to differentiate much; this may have a negative impact in her performance in the general election. In the characterized equilibrium, in each primary, all of the party members more extreme than the party's median are indifferent between the two candidates and therefore split between the two primary candidates. All of the party members more moderate than the party's median, however, strictly prefer the high-valence candidate over the low-valence candidate and vote for her. Hence, the high-valence candidate wins the primary for sure, obtaining the support of three-quarters of party members. Regarding the winner of the general election, the following corollary indicates that any of the qualifying candidates may win.

Corollary 1. Candidate B's prospects in the general election benefit from a moderate leftist party (i.e., high l), an extreme rightist party (i.e., high r), a leftist median voter (i.e., low m), and a "tough" primary in the rightist party (i.e., high T_R), while harmed by a "tough" leftist primary and general election (i.e., high T_L and T_G). The reverse holds for candidate C. Formally, candidate B wins the general election if $T_G + T_L < l + r - 2m + T_R$, candidate C wins the general election if $T_G + T_L > l + r - 2m + T_R$, and candidates B and C win with equal probability if $T_G + T_L = l + r - 2m + T_R$.

The winner of the general election ultimately depends on both the parties' and society's medians, as well as the valence asymmetries that determine the toughness of both primaries and the general election. We further explain our results by focusing on the winning prospects of the highest valence candidate B with symmetric arguments holding for the prospects of candidate C. Our results indicate that candidate B wins the general election when the aggregate toughness she faces in the primary and general election is low "enough" (i.e., $T_G + T_L < l + r - 2m + T_R$). This condition illustrates that B is favored by a moderate leftist party (large l), an extreme rightist party (large r), and a tough primary in the rightist party (large T_R). This condition also points at the matching effect of primaries since the leftist party wins more often as the society becomes more leftist (i.e., small m). However, candidate B's election prospects are harmed by a tough general election (large T_G) since voters compare the valence characteristics of the two general election candidates and candidate B loses more often as her advantage compared to candidate C gets smaller. Finally, a tough primary (large T_L) is also harmful for candidate B since the presence of an almost equally competent primary opponent obliges her to remain close to the party median so as to guarantee nomination, thus harming her general election performance (within-party competition effect). This last effect also points to the fact that parties may not benefit when choosing both their primary candidates from a pool of highly competent members and some heterogeneity proves desirable. Ideally, when the time of primaries arrives, parties would opt for a competition between their most competent member and a low-valence internal opponent so as to have good chances in the general election.

Open primaries

After obtaining a very general result for closed primaries, a natural question is what occurs when parties hold open primaries. Open primaries are an increasingly popular method used by several parties to select their nominees.[15] We therefore extend our setup by allowing voters to decide in which primary to participate once all four candidates announce their platforms.[16]

For the analysis of open primaries, some further assumptions are necessary. Again, candidates A and B constitute the candidates of the leftist party, and candidates C and D constitute the candidates of the rightist party; we assume that only a subset of voters are "active" and participate in the primaries. Let active voters have ideal policies distributed according to any continuous log-concave distribution F with a unique median m^a.[17] These active voters participate in the primary of the party in which the candidate who gives them the highest utility is competing and vote for that candidate.

15. In the United States, around one-third of the states hold an open primary. In Europe, the socialist parties of France, Greece, and Italy run open primaries for their leaders, as is the case for the conservatives in the United Kingdom for some parliamentary candidates. The European Green Party ran a pan-European open primary for the 2014 European Union election. In Latin America, open primaries take place in Argentina. For mixed empirical evidence on the effect of primaries on political competition, see Gerber and Morton (1998), Kanthak and Morton (2001), and Kaufmann et al. (2003).

16. Our open primaries model has a spirit similar with the literature modeling endogenous parties (e.g., Baron 1993; Eguia 2011a, 2011b, 2012; Gomberg et al. 2004, 2016).

17. We consider that a continuous distribution function F is log-concave if $\partial^2 \ln F(x)/\partial x^2 < 0$ and $\partial^2 \ln[1 - F(x)]/\partial x^2 < 0$ for every $x \in (0, 1)$. That is, the notion of log-concavity that we employ implies that F is strictly increasing and twice differentiable in its support too. While log-concavity of the distribution of voters' ideal policies is a general assumption and widely used in the political economy literature, our definition is weaker compared to the "standard" one assuming a log-concave density function (see Bagnoli and Bergstrom [2005] for further properties of log-concave distributions).

Hence, the distribution of active voters across parties and therefore the location of the primary median voters are endogenously determined and depend on candidates' policy proposals and valence characteristics. As far as candidates are concerned, we assume that each candidate $J \in \{A, B, C, D\}$ strategically chooses and commits to an electoral platform $x_J \in S_J$, where $S_J = [0, m^a]$ if $J \in \{A, B\}$ and $S_J = [m^a, 1]$ if $J \in \{C, D\}$, while valence differences are again not very large. Finally, if no voters participate in one of the two primaries (i.e., when all active voters prefer the candidate[s] of one party compared to those of the other), we assume that each of the candidates qualifies for the general election with equal probability. The following proposition characterizes the equilibrium:

Proposition 2. If valence differences are not very large, there exists a unique Nash equilibrium, and it is such that parties and their medians are uniquely defined and in each party (a) the low-valence candidate proposes the platform preferred by the party's median voter, (b) the high-valence candidate proposes a more moderate platform than the one preferred by the party's median voter, and (c) the high-valence candidate (i.e., B in the leftist party and C in the rightist one) wins the primary. Formally, for every F there exists $\tilde{v}_B > 0$ such that for every $v_B \in (0, \tilde{v}_B)$ (a) the endogenous party medians $(l^\star, r^\star) \in (0, 1)^2$ are the unique values that solve

$$2F(l^\star) = F\left(\frac{l^\star + v_B - v_A + r^\star - v_C + v_D}{2} + \frac{v_B - v_C}{2}\right)$$

and

$$2[1 - F(r^\star)] = 1 - F\left(\frac{l^\star + v_B - v_A + r^\star - v_C + v_D}{2} + \frac{v_B - v_C}{2}\right),$$

and (b) there exists a unique Nash equilibrium $\hat{x}$ such that $\hat{x}_A = l^\star$, $\hat{x}_B = l^\star - T_L$, $\hat{x}_C = r^\star + T_R$, and $\hat{x}_D = r^\star$.

The equilibrium structure is similar to the one in closed primaries with primary losers locating on parties' medians and primary winners diverging from the party median toward the median of the society. What is different compared to closed primaries is that now party medians (i.e., $l^\star$ and $r^\star$) are endogenously determined and depend on all four values of valence characteristics (see ex. 1) as well as the distribution of active voters. Again, candidates B and C propose the two most moderate platforms, but now the voter indifferent between the two determines not only their vote shares in the general election but also the distribution of active voters across the two primaries. Since active voters decide to participate in the primary in which they can identify the candidate who gives them the highest utility, all active voters on the left (right) of the indifferent voter in the general election participate in the primary of the leftist (rightist) party.

Notice that an equilibrium exists and is unique, guaranteeing the stabilizing effect of primaries even if voters can freely choose in which primary to participate. Unlike the case of closed primaries in which the fixed party structure guarantees equilibrium existence for any distribution of voters, in open primaries existence is obtained by some mild restriction on such distribution (F being log-concave).[18] All of this suggests that indeed the stabilizing effect of primaries holds even with open primaries for a very general class of preference profiles, but, as expected, it is weaker compared to the closed primaries case. We note though that the fact that open primaries might stabilize electoral competition for such a general class of preference profiles is more surprising, at least to us, than the fact that they are less prone to lead to stability compared to closed primaries. At first sight, one could expect that the dynamics that lead to the existence of an equilibrium when primary electorates are fixed would disappear once we considered that parties are endogenous.

As in closed primaries and corollary 1, any of the two general election candidates may win the election, and the condition such that one or the other candidate wins is similar. The highest valence candidate B wins the general election as long as the aggregate toughness she faces is lower than a given threshold (i.e., $T_G + T_L < l^\star + r^\star - 2m + T_R$). Such a threshold now clearly depends on the endogenous location of the primary median voters (i.e., $l^\star$ and $r^\star$). Candidate B benefits if the endogenously formed leftist party is relatively moderate while the endogenously formed rightist party is relatively extreme. Similar to closed primaries, the highest valence candidate B is harmed if the primaries in the leftist party are much tougher in terms of valence than the ones of the rightist party (within-party competition effect). Moreover, the matching effect of primaries in which the leftist candidate benefits from a leftist electorate is still present since candidate B wins more often as the society becomes more leftist (i.e., small m).[19]

18. This restriction is necessary because an equilibrium may not exist when F is too convex around the location of the indifferent voter in the general election $(l^\star + v_B - v_A + r^\star - v_C + v_D)/2 + (v_B - v_C)/2$. If, e.g., F is log-convex about this point (where the notion of log-convexity is symmetric to the one of log-concavity), a slight transition of B from her equilibrium platform $l^\star + v_B - v_A$ to $l^\star + v_B - v_A + \varepsilon$ brings into the leftist primary many new supporters of B. Hence, B still wins in the primary and improves her performance in the general election.

19. To see that this is not an artifact of having distinct primary and general election electorates, let, e.g., $\Phi = F = (2 - a + ax)/2x$, where $a \in [-2, 2]$. This is a simple class of distributions with linear densities where $a = -2$ corresponds to the triangular distribution with a peak at zero, $a = 0$ corresponds to the uniform distribution as in ex. 1, and $a = 2$ corresponds to the triangular distribution with a peak at one. One can show that the matching effect is strongly present: when the society is left

Remember that the main difference between open and closed primaries is that while in closed primaries candidates propose platforms that depend exclusively on the exogenous location of the party's median voter and the valence characteristics of the party's candidates (proposition 1), in open primaries proposed platforms depend on the endogenous location of the party's median voter and therefore the valence characteristics of all four candidates (proposition 2). In other words, in open primaries valence characteristics in each party also affect nomination in the other party, a feature absent in closed primaries. The following example illustrates such interaction in open primaries.

Example 1. From proposition 2 we know that parties' median voters ($l^\star$, $r^\star$) are the unique values that solve

$$2F(l^\star) = F\left(\frac{l^\star + v_B - v_A + r^\star - v_C + v_D}{2} + \frac{v_B - v_C}{2}\right)$$

and

$$2[1 - F(r^\star)] = 1 - F\left(\frac{l^\star + v_B - v_A + r^\star - v_C + v_D}{2} + \frac{v_B - v_C}{2}\right).$$

Let active voters be uniformly distributed (i.e., $F(x) = x$). Then these two equations simplify to

$$2l^\star = \frac{l^\star + v_B - v_A + r^\star - v_C + v_D}{2} + \frac{v_B - v_C}{2},$$
$$2(1 - r^\star) = 1 - \frac{l^\star + v_B - v_A + r^\star - v_C + v_D}{2} + \frac{v_B - v_C}{2}.$$

By solving these two equations with respect to $l^\star$ and $r^\star$, we identify the location of the primary median voter in each party as $l^\star = (1/4) - (1/2)v_A + v_B - v_C + (1/2)v_D$ and $r^\star = (3/4) - (1/2)v_A + v_B - v_C + (1/2)v_D$. The unique equilibrium platforms are hence

$$\hat{x}_A = l^\star = \frac{1}{4} - \frac{1}{2}v_A + v_B - v_C + \frac{1}{2}v_D,$$
$$\hat{x}_B = l^\star - T_L = \frac{1}{4} - \frac{3}{2}v_A + 2v_B - v_C + \frac{1}{2}v_D,$$
$$\hat{x}_C = r^\star + T_R = \frac{3}{4} - \frac{1}{2}v_A + v_B - 2v_C + \frac{3}{2}v_D,$$
$$\hat{x}_D = r^\star = \frac{3}{4} - \frac{1}{2}v_A + v_B - v_C + \frac{1}{2}v_D.$$

As this example shows, all equilibrium platforms depend on all valence characteristics and illustrate forces that push parties to the extremes or the center. Ceteris paribus, an increase in the valence of either B or D moves platforms to the right. However, an increase in the valence of either A or C moves platforms to the left. In other words, as a nominee's valence increases (i.e., B's or C's), not only does she move to moderate ground but also makes the other party propose extreme platforms. However, an increase in the valence of a losing primary contender (i.e., A or D) not only ties the whole party to the extremes but also permits the opposing party to propose moderate platforms.

Given the discussed differences in platform proposals across primary types, one may wonder which primary type delivers higher social welfare (defined as the sum of individual utilities). Our analysis indicates that in the most symmetric scenario ($\Phi(x) = F(x)$ symmetric about $1/2$, and $F(l) = 1 - F(r) = 1/4$), the society is better off when parties hold open primaries. This is true because while in this symmetric case the winner (and hence her valence) is not affected by the primary type, the winner's platform is closer to the median's preferred policy when primaries are open.[20] Of course, since l and r need not coincide with the first and third quartile of the active voters' distribution in the open primary, one can think of cases in which closed primaries are better for the society: these would involve exclusion of extreme voters from the open primary (i.e., l and r are close to m^a). When extremist voter participation in closed primaries is significant (this is most often the case), open primaries seem to better serve the centrist voters' interests as they allow the potential winner to move closer to their preferences.

One-party primaries

In reality, both parties need not hold a primary before the general election. A typical situation of interest for the absence of primaries in one party is when an incumbent runs

leaning, the nominee of the leftist party enjoys an electoral advantage and vice versa. Formally, when most voters are leftist ($a < 0$ or $m < 1/2$), then $\lim_{v_B \to 0}\{[(l^\star + v_B - v_A + r^\star - v_C + v_D)/2] + (v_B - v_C)/2\} \in (m, 1/2)$ and when most voters are rightist ($a > 0$ or $m > 1/2$), then $\lim_{v_B \to 0}\{[(l^\star + v_B - v_A + r^\star - v_C + v_D)/2] + (v_B - v_C)/2\} \in (1/2, m)$

20. Given that voters' utilities are linear in policy and the winner's valence is not affected by the primary type, the primary type that delivers higher welfare is the one that brings the winner's platform closer to the median's preferred policy. To see why in this symmetric scenario the winner is not affected by the primary type, one can refer to ex. 1 (i.e., F is uniform, and hence $l = 1/4$ and $r = 3/4$) while also assuming that the general electorate is uniformly distributed. Computing candidates' vote shares it turns out that under both open and closed primaries B wins the election if $T_G + T_L < T_R$ (i.e., B faces relatively less aggregate toughness than C does). Now comparing the winner's proposed platform in open primaries (ex. 1, $\hat{x}_B = (1/4) - (3/2)v_A + 2v_B - v_C + (1/2)v_D$) or in closed primaries (proposition 1, $\hat{x}_B = (1/4) + v_B - v_A$), indeed, the one proposed in open primaries is also more moderate whenever $T_G + T_L < T_R$. Therefore, while the winner is not affected by the primary type, open primaries indeed deliver higher welfare than closed ones.

for reelection. Let the incumbent be candidate C with valence v_C and an ideal policy x_C that is known and fixed.[21] The leftist candidates A and B may run in a closed or open primary, and while we still assume that $v_B > v_A$, we do not require that $v_B > v_C$, permitting the incumbent to be the highest valence candidate. When both parties hold a primary, we assume that each candidate $J \in A, B$ strategically chooses and commits to an electoral platform $x_J \in S_J$ (where $S_J = [0, m]$ in closed primaries and $S_J = [0, m^a]$ in open primaries), and a Nash equilibrium in pure strategies is a vector $\hat{x} = (\hat{x}_A, \hat{x}_B)$ such that neither of the two candidates has incentive to deviate. Let us start by presenting the results when the leftist party runs a closed primary.

Proposition 3. If the leftist party runs a closed primary to challenge an incumbent and valence differences are not very large, there exists a unique Nash equilibrium, and it is such that in the challenger's party (a) the low-valence candidate proposes the platform preferred by the party's median voter, (b) the high-valence candidate proposes a more moderate platform than the one preferred by the party's median voter, and (c) the high-valence candidate (i.e., B) wins the primary. Formally, if $m + \max\{v_B, v_C\} < x_C$, then there exists a unique Nash equilibrium $\hat{x}$ such that $\hat{x}_A = l$ and $\hat{x}_B = l - T_L$.

Proposition 3 suggests that when the incumbent is challenged by a party organizing a closed primary, both primary candidates follow the same strategies as when both parties hold closed primaries (i.e., $\hat{x}_A = l$ and $\hat{x}_B = l - T_L$). Notice that only the toughness of the primary race—and not the incumbent's characteristics—determines how the high-valence candidate is more moderate than the low-valence candidate and the party's median. Of course, the incumbent's characteristics play a crucial role in determining the winner of the general election. The following corollary offers a summary.

Corollary 2. Candidate B's prospects in the general election benefit from a moderate leftist party (i.e., high l), an extreme incumbent (i.e., high x_C), and a leftist median voter (i.e., low m), while harmed by a "tough" leftist primary and general election (i.e., high T_L and T_G). The reverse holds for candidate C. Formally, candidate B wins the general election if $T_G + T_L < l + x_C - 2m$, candidate C wins the general election if $T_G + T_L > l + x_C - 2m$, and candidates B and C win with equal probability if $T_G + T_L = l + x_C - 2m$.

As corollary 2 indicates, whether the leftist challenger succeeds in replacing the incumbent depends on the ideology of the leftist median, the ideal policy of the incumbent, as well as the toughness of the primary and the general election in terms of valence. The aggregate toughness condition such that the challenger B wins (i.e., $T_G + T_L < l + x_C - 2m$) implies that the incumbent is, of course, harmed by her own extreme policies and low quality. Additionally, the challenger increases her chances to successfully replace the incumbent when she emerges from a moderate leftist party with non-competitive primaries and when the median voter in the general election is leftist.

Note that in closed primaries candidates who aim at winning the nomination focus only on their party's median. This explains why the challenger's proposed platform is not affected by the incumbent's platform. As we describe in the following proposition, this is no longer true when the party in opposition holds an open primary. As before, let F indicate the distribution of active voters. The natural way of extending sincere voting in this one-party primary scenario is to let active voters who like either candidate A or B the most participate in the primary of the leftist party supporting their favorite candidate, while active voters that like the incumbent the most do not participate in the primary. Again, if none of the active voters participate in the leftist primary we assume that A and B qualify for the general election with equal probability.

Proposition 4. If the leftist party runs an open primary to challenge an incumbent and valence differences are not very large, then there exists a unique Nash equilibrium; it is such that the challenger's party and its median are uniquely defined and (a) the low-valence candidate proposes the platform preferred by the party's median voter, (b) the high-valence candidate proposes a more moderate platform than the one preferred by the party's median voter, and (c) the high-valence candidate (i.e., B) wins the primary. Formally, for every F there exists $v^{\max} > 0$ such that for every $\max\{v_B, v_C\} \in (0, v^{\max})$ (a) the endogenous party median $l^\star \in (0, m^a)$ is the unique value that solves

$$2F(l^\star) = F\left(\frac{l^\star + v_B - v_A + x_C}{2} + \frac{v_B - v_C}{2}\right),$$

and (b) there exists a unique Nash equilibrium $\hat{x}$ such that $\hat{x}_A = l^\star$, $\hat{x}_B = l^\star - T_L$.

Proposition 4 presents a similar equilibrium structure as in proposition 3 with $l^\star$ denoting the location of the median of the endogenously formed leftist party. The condition

21. Formally, let $x_C > m > l$ for the case of closed primaries and $x_C > m^a$ for the case of open primaries.

providing such a location permits us interesting insights on the effect of the incumbents' characteristics (i.e., x_C and v_C) on the platforms proposed in the leftist primary and participation in the latter. As it turns out, the leftist party tends to be "large" and hence more moderate when the incumbent implements extreme policies or is of low valence (formally l^* is strictly increasing in x_C and strictly decreasing in v_C). Similarly, the leftist party tends to propose moderate platforms when B is of high valence and A is of low valence (formally l^* is strictly increasing in v_B and strictly decreasing in v_A).[22] The aggregate toughness threshold condition such that the winning leftist candidate B also wins the general election follows the above intuition and is similar to when party B runs a closed primary and corollary 2 (i.e., candidate B wins the general election if $T_G + T_L < l^* + x_C - 2m$).

The following example illustrates equilibrium proposals for the leftist candidates when primary voters are uniformly distributed:

Example 2. Let active voters be uniformly distributed across the policy space. If the leftist party runs an open primary, the median voter in the endogenously formed party is given by l^* such that

$$2l^* = \frac{l^* + v_B - v_A + x_C}{2} + \frac{v_B - v_C}{2},$$

given that F is the uniform distribution. That is, $l^* = (1/3)(x_C - v_A + 2v_B - v_C)$. As proposition 4 indicates, the low-valence candidate A proposes platform $\hat{x}_A = (1/3)(x_C - v_A + 2v_B - v_C)$, while the high-valence candidate B is more moderate by a distance $v_B - v_A$ and equilibrium platform $\hat{x}_B = (1/3)(x_C - 4v_A + 5v_B - v_C)$. In contrast to closed primaries, all platforms depend on the valence characteristics of all candidates including those of the incumbent as well as the implemented policy with $\hat{x}_A$ and $\hat{x}_B$ increasing in x_C and v_B and decreasing in v_A and v_C as commented before. If we also assume that the society is uniformly distributed (i.e., ideologies of the general electorate are evenly distributed across the policy space, or formally stated $\Phi(x) = x$), then the location of the indifferent voter and hence B's vote share under an open primary is given by

$$\frac{\hat{x}_B + x_C}{2} + \frac{v_B - v_C}{2} = \frac{2}{3}\left(2v_B - v_A + x_C - v_C\right).$$

Let us now go back to closed primaries. From proposition 3, we know the proposed platforms are given by $\hat{x}_A = l$ and $\hat{x}_B = l + v_B - v_A$. When the society is uniformly distributed, the location of the indifferent voter and hence B's vote share under a closed primary is given by

$$\frac{l + v_B - v_A + x_C}{2} + \frac{v_B - v_C}{2}.$$

Notice now that the location of the indifferent voter in the general election varies across the two primary types, and hence the selection of one system over the other clearly affects the electoral outcome and possibly the winner of the election. Comparing the location of the indifferent voter, we know that the vote share of the challenger's party is larger under an open primary rather than under a closed primary if and only if

$$\frac{2}{3}\left(2v_B - v_A + x_C - v_C\right) > \frac{l + v_B - v_A + x_C}{2} + \frac{v_B - v_C}{2}.$$

This last condition can be simplified in a more intuitive manner as $T_L + T_G < x_C - 3l$. Simply put, this condition is equivalent to $l^* > l$, meaning that if the challenger were to run an open primary, then the endogenous median would be more moderate than the party's median voter if it were to run a closed primary.

Open or closed primary in the challenger's party?

An interesting question then is, when would the party in opposition increase its vote share by running an open rather than a closed primary? The relevant condition ($T_L + T_G < x_C - 3l$) obtained in example 2 indicates that this occurs when the aggregate toughness the leftist candidate B faces is low enough.[23] This may hold, if for example, candidate B is a candidate of sufficiently high valence. Hence, one would expect that parties for which a highly competent primary candidate competes may prefer open over closed primaries. This is because open primaries permit the highly competent candidate to "open" the party to the society and propose more moderate platforms than if she were to fight for nomination in a closed primary. In a similar spirit, the condition such that open primaries are preferred over closed ones is also

22. Given the equilibrium condition $2F(l^*) = F[(l^* + v_B - v_A + x_C)/2 + (v_B - v_C)/2]$, the log-concavity of F suffices to obtain the aforementioned comparative statics of l^* with respect to all valence characteristics.

23. Remember that $T_L = -(v_B - v_A)$ indicates the toughness in the leftist primary (with $v_A < v_B$). Large values of T_L indicate a very competitive primary that does not allow the primary winning candidate B to be moderate enough and to become attractive to the general electorate. The toughness of the general election $T_G = -(v_B - v_C)$ has a similar effect with larger values of T_G being detrimental for the leftist candidate. Note here that while $T_L < 0$ is still true, for the general election we have that $T_G < 0$ if $v_C < v_B$ and $T_G > 0$ if $v_C > v_B$.

more likely to hold when party members participating in the closed primary are relatively extreme (i.e., low l). This occurs because while under closed primaries the candidates would have to please an extreme primary electorate, open primaries permit them to move to a moderate ground and enrich the primary electorate with some moderate voters. Finally, parties in opposition are more likely to select their nominee through an open primary when the incumbent is implementing a relatively extreme policy (i.e., high x_C). In that instance, given that the implemented policies leave many voters alienated, the opposition has incentive to open the primary to the society, bringing into the party relatively moderate voters who push the endogenously formed median of the party to a moderate policy ground.

Our example so far has illustrated how the challenger's vote share is higher under closed or open primaries, depending on the aggregate toughness the leftist candidate B faces. Table 1 summarizes the winner of the election under all relevant scenarios and illustrates that the choice of primary type may actually determine the winner of the election and a wrong choice may prove detrimental for the challenger's party.

If the aggregate toughness that candidate B faces ($T_L + T_G$) is very low, the challenger always wins.[24] However, if the aggregate toughness is very high, the incumbent always remains in office. Nevertheless, for moderate levels of toughness the type of primary is crucial. Consider, for instance, that the leftist party is relatively extreme (i.e., $l < 0.25$) and that the aggregate toughness is moderately low. If the challenger emerges through a closed primary, then the incumbent remains in office, while if the challenger emerges from an open primary, the incumbent is successfully replaced. The reason why the wrong choice of primary type may be detrimental is that, under a closed primary, candidate B is not able to propose moderate platforms since the party's median voter is well to the left ($l < 0.25$), and a moderate platform would result in a lost closed primary. However, recall that aggregate toughness is moderately low (e.g., because B is quite talented), a fact that can be exploited by candidate B under open primaries. As it turns out, open primaries permit candidate B to move away from l by "opening" the party to society and bringing the whole party to a moderate ground that eventually provides a victory in the general election. For similar reasons, a moderate leftist party facing moderately high aggregate toughness should not opt for an open primary but rather prefer a closed one, given the moderate location of its median voter. To the degree that all parameters of interest (v_A, v_B, v_C, l, and x_C) are measurable, table 1 summarizes the empirically testable predictions of our model when a challenger aims at replacing an incumbent. Overall, our results imply that the strategic choice of a primary type by parties' leaders is of crucial importance since it may shape the nominee's success. In any case, parties seem to benefit by flexibility in their rule and should choose it on a case-by-case basis, taking into consideration the characteristics of all candidates (those of the incumbent and the potential primary candidates).

Table 1. Winner of the General Election for Open and Closed Primaries

Aggregate Toughness ($T_L + T_G$)	Winner with Open	Winner with Closed	
		$l < .25$	$l > .25$
Very low	B	B	B
Moderately low	B	C	B
Moderately high	C	C	B
Very high	C	C	C

Note. Winner of the general election when the general electorate and active voters are uniformly distributed.

When the two parties use different types of primaries

When both parties hold primaries, we have so far focused on situations in which both parties hold the same primary type. Given that we have previously established how open and closed primaries differ when a challenger emerging from primaries faces an incumbent, a natural follow-up question is what occurs when one party holds a closed primary and the other holds an open primary. Our results permit us to discuss such situation. Let S^A indicate the set of all active voters participating in primaries, with a subset of voters S^C being eligible to participate in the closed primary. In the sincere voting setup on which we have been focusing, three natural cases emerge regarding how party members S^C behave with respect to the open primary. Party members eligible to vote in the closed primary still vote in a sincere manner and (*a*) participate only in the closed primary, (*b*) participate in both primaries, or (*c*) participate only in the one primary in which they identify their preferred candidate.

24. Formally, for $l < 0.25$ aggregate toughness is very low if $T_L + T_G < x_C - 1 + l$, moderately low if $x_C - 1 + l < T_L + T_G < x_C - 0.75$, moderately high if $x_C - 0.75 < T_L + T_G < x_C - 3l$, and very high if $T_L + T_G > x_C - 3l$. For $l > 0.25$ aggregate toughness is very low if $T_L + T_G < x_C - 3l$, moderately low if $x_C - 3l < T_L + T_G < x_C - 0.75$, moderately high if $x_C - 0.75 < T_L + T_G < x_C - 1 + l$, and very high if $T_L + T_G > x_C - 1 + l$.

If closed-primary-eligible voters vote only in this primary, the situation is identical to the one we have described under the presence of an incumbent. In the closed primary, the low-valence candidate will be locating at the party's median and the high-valence candidate will be running on a more moderate platform. In the open primary, the two candidates will be focusing on the distribution of the remaining active voters ($S^A \setminus S^C$) and propose platforms as if they were facing an incumbent (i.e., the high-valence candidate winning the closed primary). Hence, all intuition is as previously presented.

If voters who are eligible to vote in the closed primary are permitted to participate also in the open primary and do so, candidates in the closed primary will be behaving exactly as before, focusing only on the party's median and the toughness of the closed primary. Candidates in the open primary will be running as if they were facing an incumbent, but now they will be focusing on the median of the whole set of voters (S^A). Hence, while the closed primary voters (S^C) participating only in the closed primary affect the open primary exclusively through the location of their winning candidate, voting in both primaries also affects the open primary through the distribution of voters participating in the open primary.

Finally, if party members eligible to vote in the closed primary vote only in the primary with the candidate they prefer, the situation is similar to both parties running an open primary. The difference is that while candidates running in open primaries will be focusing on all active voters (S^A), candidates running in the closed primary will be focusing only on a subset of voters (S^C).

CONCLUSION

Neo-Downsian modeling has generated a huge literature, with the initial simplifying assumptions of Downs's classic model of two-party plurality competition over a single policy dimension enriched with more realistic assumptions, including multidimensionality, party primaries (varying from open to closed), and valence as a basis for voter choice, as well as extensions to multiparty competition under electoral rules other than first past the post. Here we have contributed to that tradition by seeking to develop a model of primary competition for two-party plurality elections that matches various stylized facts about the real world (including perhaps most notably a prediction of party differentiation in the general election) and allowing for a party whose support base is closer to the position of the overall median voter to be advantaged, rather than assuming that electoral competition leads to Tweedledum-Tweedledee politics with each party having an equal probability of victory. We have also allowed for differences across primary type and for different results when there is or is not an incumbent. Moreover, from a theoretical perspective, the equilibrium results we have complement the more common nonequilibrium results in multidimensional two-party competition.

We recognize that while we have made advances over previous models of party competition that include both primaries and valence, ours is far from the last word. In particular, models of candidate behavior, our own included, tend to focus on the perspective of a single candidate and impute to that candidate office-seeking or policy-seeking goals. There are two important ways—each of which takes us into areas beyond the scope of the present article—in which that perspective could be modified in future work. First, we might add to prospective candidate's utility function a further consideration, namely, their perception of the consequences of their candidacy on the success of their party in the general election. Second, we could move from the specifics of individual contests to ask about how political parties and interest groups affect the nomination process.

The conflict between what candidates want and what is in the overall interest of their party is especially sharp when it comes to legislative redistricting (Owen and Grofman 1988). Looking at how this conflict is resolved in that domain gives us some ideas about how we might, in the future, model primary elections in a general equilibrium framework recognizing that individual election contests are embedded in a wider institutional setting. In the redistricting context, incumbents and challengers realize that the value of gaining office is enhanced if their party is the majority party. This may make incumbents more willing to "take a hit for the party," that is, accept some loss of certainty about their own reelection in return for increased chances of their party controlling the legislature. Of course, since a fundamental principle of politics is that no incumbent ever regards his district as safe enough, willingness to take a hit for the party may be limited. Relatedly, in the primary context, candidates for office may have an exaggerated sense of their own valence and thus be unwilling to posit that their nomination in the primary will result in a loss for the party in the general election (see, e.g., Uhlaner and Grofman 1986). Moreover, they may not be sophisticated enough to consider that their candidacy may affect the policies proposed by a competing primary contender in a way that harms the party's general election chances even when it is that candidate who wins the primary and not themselves. Thus, while it certainly makes sense to allow for the possibility of candidates caring about overall consequences for their party in modeling platform choices, relying on this kind of altruism to deter primary challengers

that can hurt the party in unintended ways may be unrealistic, especially once we take misperceptions into account.

But there is a second route by which conflicts between party interest and the interests of individual candidates get resolved, and that involves considerations of relative power. In the redistricting example, trade-offs between overall party interests and candidate/officeholder interests is largely resolved by the relative power of incumbents to control the process as opposed to that of other officials, such as governors, who might take a wider party-centric perspective. In the primary context, to really understand the dynamics of candidates' competition, we would need to move beyond our stylized framework and look behind the scenes at the recruitment of candidates (by parties and interest groups) and the nature of campaign support that primary candidates might expect to receive from the party (e.g., access to list of party donors) and from particular interest groups. In particular, there is universal consensus that "money is the mother's milk of politics," as attested by both Democratic liberals (such as the late Jesse Unruh of California) and contemporary Republican conservatives (such as Rush Limbaugh). Thus, as candidates make their strategic decisions, they may be deterred to seek nomination by recognition of the fact that sources of monetary support are already committed to other candidates. The dynamics of party competition we described above show that, even though a primary challenger loses the primary, the mere fact of her candidacy may affect the ability of the party to win the general election because of the impact that the challenge has on the policy position taken by the candidate who does win the primary. However, even if individual candidates may have nonaligned interests with their party, actors such as party officials and interest groups with a longer term and strategic perspective may make decisions about whom to support in light of sophisticated calculations, seeking to deter challengers who might harm the party's general election chances and, if that fails, seeking to reinforce the primary chances of the preferred candidate.

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REFERENCES

Adams, James, and Samuel Merrill III. 2008. "Candidate and Party Strategies in Two-Stage Elections Beginning with a Primary." *American Journal of Political Science* 52 (2): 344–59.

Adams, James, Samuel Merrill III, and Bernard Grofman. 2005. *A Unified Theory of Party Competition: A Cross-National Analysis Integrating Spatial and Behavioral Factors*. New York: Cambridge University Press.

Agranov, Marina. 2016. "Flip-Flopping, Primary Visibility, and Selection of Candidates." *American Economic Journal: Microeconomics* 8 (2): 61–85.

Amorós, Pablo, M. Socorro Puy, and Ricardo Martínez. 2016. "Closed Primaries versus Top-Two Primaries." *Public Choice* 167 (1): 21–35.

Andreottola, Giovanni. 2016. "Signalling Valence in Primary Elections." Working paper no. 2719, presented at the annual EEA-ESEM conference, Geneva.

Ansolabehere, Stephen, and James M. Snyder Jr. 2000. "Valence Politics and Equilibrium in Spatial Election Models." *Public Choice* 103 (3/4): 327–36.

Aragonès, Enriqueta, and Thomas R. Palfrey. 2002. "Mixed Equilibrium in a Downsian Model with a Favored Candidate." *Journal of Economic Theory* 103 (1): 131–61.

Aragonès, Enriqueta, and Dimitrios Xefteris. 2017a. "Imperfectly Informed Voters and Strategic Extremism." *International Economic Review* 58 (2): 439–71.

Aragonès, Enriqueta, and Dimitrios Xefteris. 2017b. "Voters' Private Valuation of Candidates' Quality." *Journal of Public Economics* 156 (December): 121–30.

Aranson, Peter H., and Peter C. Ordeshook. 1972. "Spatial Strategies for Sequential Elections." In Richard G. Niemi and Herbert F. Weisberg, eds., *Probability Models of Collective Decision Making*. Columbus, OH: Merrill, 298–331.

Bagnoli, Mark, and Ted Bergstrom. 2005. "Log-Concave Probability and Its Applications." *Economic Theory* 26 (2): 445–69.

Baron, David P. 1993. "Government Formation and Endogenous Parties." *American Political Science Review* 87 (1): 34–47.

Benoit, S., Y. Crutzen, and Nicolas Sahuguet. 2018. "Uncontested Primaries: Causes and Consequences." *Quarterly Journal of Political Science* 13 (4): 427–62.

Besley, Timothy, and Stephen Coate. 1997. "An Economic Model of Representative Democracy." *Quarterly Journal of Economics* 112 (1): 85–114.

Bräuninger, Thomas. 2007. "Stability in Spatial Voting Games with Restricted Preference Maximizing." *Journal of Theoretical Politics* 19 (2): 173–91.

Buisseret, Peter, and Richard Van Weelden. 2017. "Crashing the Party? Elites, Outsiders, and Elections." Working paper no. 6327, Department of Economics, University of Pittsburgh.

Burden, Barry C. 2001. "The Polarizing Effects of Congressional Primaries." In Peter F. Galderisi, Marni Ezra, and Michael Lyons, eds., *Congressional Primaries and the Politics of Representation*. Lanham, MD: Rowman & Littlefield, 95–115.

Callander, Steven. 2007. "Bandwagons and Momentum in Sequential Voting." *Review of Economic Studies* 74 (3): 653–84.

Casas, Agustin. 2013. "Partisan Politics: Parties, Primaries and Elections." Working paper no. 13-15, UC3M Economics Series, Universidad Carlos III de Madrid.

Coleman, James S. 1971. "Internal Processes Governing Party Positions in Elections." *Public Choice* 11 (1): 35–60.

Crutzen, Benoît S. Y., Micael Castanheira, and Nicolas Sahuguet. 2010. "Party Organization and Electoral Competition." *Journal of Law, Economics, and Organization* 26 (2): 212–42.

Deltas, George, Helios Herrera, and Mattias K. Polborn. 2016. "Learning and Coordination in the Presidential Primary System." *Review of Economic Studies* 83 (4): 1544–78.

Dziubiński, Marcin, and Jaideep Roy. 2011. "Electoral Competition in 2-Dimensional Ideology Space with Unidimensional Commitment." *Social Choice and Welfare* 36 (1): 1–24.

Eguia, Jon X. 2011a. "Endogenous Parties in an Assembly." *American Journal of Political Science* 55 (1): 16–26.

Eguia, Jon X. 2011b. "Voting Blocs, Party Discipline and Party Formation." *Games and Economic Behavior* 73 (1): 111–35.

Eguia, Jon X. 2012. "A Spatial Theory of Party Formation." *Economic Theory* 49 (3): 549–70.

Gerber, Elisabeth R., and Rebecca B. Morton. 1998. "Primary Election Systems and Representation." *Journal of Law, Economics, and Organization* 14 (2): 304–24.

Gomberg, Andrei, Francisco Marhuenda, and Ignacio Ortuño-Ortín. 2004. "A Model of Endogenous Political Party Platforms." *Economic Theory* 24 (2): 373–94.

Gomberg, Andrei, Francisco Marhuenda, and Ignacio Ortuño-Ortín. 2016. "Endogenous Party Platforms: 'Stochastic' Membership." *Economic Theory* 63 (4): 839–66.

Grofman, Bernard. 2004. "Downs and Two-Party Convergence." *Annual Review of Political Science* 7:25–46.

Groseclose, Tim. 2001. "A Model of Candidate Location When One Candidate Has a Valence Advantage." *American Journal of Political Science* 45 (4): 862–86.

Hirano, Shigeo, and James M. Snyder Jr. 2014. "Primary Elections and the Quality of Elected Officials." *Quarterly Journal of Political Science* 9 (4): 473–500.

Hortala-Vallve, Rafael, and Hannes Mueller. 2015. "Primaries: The Unifying Force." *Public Choice* 163 (3/4): 289–305.

Hummel, Patrick. 2010. "Flip-Flopping from Primaries to General Elections." *Journal of Public Economics* 94 (11): 1020–27.

Hummel, Patrick. 2013. "Candidate Strategies in Primaries and General Elections with Candidates of Heterogeneous Quality." *Games and Economic Behavior* 78 (March): 85–102.

Jackson, Matthew O., Laurent Mathevet, and Kyle Mattes. 2007. "Nomination Processes and Policy Outcomes." *Quarterly Journal of Political Science* 2 (1): 67–92.

Kanthak, Kristin, and Rebecca Morton. 2001. "The Effects of Electoral Rules on Congressional Primaries." In Peter F. Galderisi, Marni Ezra, and Michael Lyons, eds., *Congressional Primaries and the Politics of Representation*. Lanham, MD: Rowman & Littlefield, 116–31.

Kartik, Navin, and R. Preston McAfee. 2007. "Signaling Character in Electoral Competition." *American Economic Review* 97 (3): 852–70.

Kaufmann, Karen M., James G. Gimpel, and Adam H. Hoffman. 2003. "A Promise Fulfilled? Open Primaries and Representation." *Journal of Politics* 65 (2): 457–76.

Key, Valdimer Orlando. 1953. *Politics, Parties, and Pressure Groups*. New York: Crowell.

Krasa, Stefan, and Mattias K. Polborn. 2012. "Political Competition between Differentiated Candidates." *Games and Economic Behavior* 76 (1): 249–71.

Kselman, Daniel M. 2015. "A Median Activist Theorem for Two-Stage Spatial Models." In Norman Schofield and Gonzalo Caballero, eds., *The Political Economy of Governance*. New York: Springer, 193–210.

Lin, Tse-Min, James M. Enelow, and Han Dorussen. 1999. "Equilibrium in Multicandidate Probabilistic Spatial Voting." *Public Choice* 98 (1/2): 59–82.

Matakos, Konstantinos, Orestis Troumpounis, and Dimitrios Xefteris. 2016. "Electoral Rule Disproportionality and Platform Polarization." *American Journal of Political Science* 60 (4): 1026–43.

Meirowitz, Adam. 2005. "Informational Party Primaries and Strategic Ambiguity." *Journal of Theoretical Politics* 17 (1): 107–36.

Merrill, Samuel, Bernard Grofman, and Thomas L. Brunell. 2008. "Cycles in American National Electoral Politics, 1854–2006: Statistical Evidence and an Explanatory Model." *American Political Science Review* 102 (1): 1–17.

Ortuño-Ortín, Ignacio. 1997. "A Spatial Model of Political Competition and Proportional Representation." *Social Choice and Welfare* 14 (3): 427–38.

Osborne, Martin J. 1993. "Candidate Positioning and Entry in a Political Competition." *Games and Economic Behavior* 5 (1): 133–51.

Osborne, Martin J., and Al Slivinski. 1996. "A Model of Political Competition with Citizen-Candidates." *Quarterly Journal of Economics* 111 (1): 65–96.

Owen, Guillermo, and Bernard Grofman. 1988. "Optimal Partisan Gerrymandering." *Political Geography Quarterly* 7 (1): 5–22.

Owen, Guillermo, and Bernard Grofman. 2006. "Two-Stage Electoral Competition in Two-Party Contests: Persistent Divergence of Party Positions." *Social Choice and Welfare* 26 (3): 547–69.

Saporiti, Alejandro. 2014. "Power Sharing and Electoral Equilibrium." *Economic Theory* 55 (3): 705–29.

Schofield, Norman. 2007. "The Mean Voter Theorem: Necessary and Sufficient Conditions for Convergent Equilibrium." *Review of Economic Studies* 74 (3): 965–80.

Serra, Gilles. 2010. "Polarization of What? A Model of Elections with Endogenous Valence." *Journal of Politics* 72 (2): 426–37.

Serra, Gilles. 2011. "Why Primaries? The Party's Trade-off between Policy and Valence." *Journal of Theoretical Politics* 23 (1): 21–51.

Snyder, James M., Jr., and Michael M. Ting. 2011. "Electoral Selection with Parties and Primaries." *American Journal of Political Science* 55 (4): 782–96.

Takayama, Shino. 2014. "A Model of Two-Stage Electoral Competition with Strategic Voters." Working paper no. 525, School of Economics, University of Queensland.

Uhlaner, Carole Jean, and Bernard Grofman. 1986. "The Race May Be Close but My Horse Is Going to Win: Wish Fulfillment in the 1980 Presidential Election." *Political Behavior* 8 (2): 101–29.

Winer, Stanley L., Lawrence W. Kenny, and Bernard Grofman. 2014. "Explaining Variation in the Degree of Electoral Competition in a Mature Democracy: US Senate Elections, 1922–2004." *Public Choice* 161 (3/4): 471–97.