PHYSICAL REVIEW A **91**, 011604(R) (2015)

Composite bosons in the two-dimensional BCS-BEC crossover from Gaussian fluctuations

L. Salasnich and F. Toigo
Dipartimento di Fisica e Astronomia "Galileo Galilei" and CNISM, Università di Padova, Via Marzolo 8, 35131 Padova, Italy
(Received 6 October 2014; published 26 January 2015)

We study Gaussian fluctuations of the zero-temperature attractive Fermi gas in the two-dimensional (2D) BCS-BEC crossover showing that they are crucial to get a reliable equation of state in the Bose-Einstein condensation (BEC) regime of composite bosons, bound states of fermionic pairs. A low-momentum expansion up to the fourth order of the quadratic action of the fluctuating pairing field gives an ultraviolent divergent contribution of the Gaussian fluctuations to the grand potential. Performing dimensional regularization we evaluate the effective coupling constant in the beyond-mean-field grand potential. Remarkably, in the BEC regime our grand potential gives exactly the Popov's equation of state of 2D interacting bosons, and allows us to identify the scattering length a_B of the interaction between composite bosons as $a_B = a_F/(2^{1/2}e^{1/4}) = 0.551\ldots a_F$, with a_F is the scattering length of fermions. Remarkably, the value from our analytical relationship between the two scattering lengths is in full agreement with that obtained by recent Monte Carlo calculations.

DOI: 10.1103/PhysRevA.91.011604 PACS number(s): 03.75.Ss, 03.70.+k, 05.70.Fh, 03.65.Yz

Thermal and quantum fluctuations play a relevant role in any generic two-dimensional (2D) superfluid system [1–4]. Triggered by the experimental realization of the BCS-BEC crossover with three-dimensional ultracold atoms [5–7], in past years several theoretical papers [8–14] have been devoted to the study of thermal fluctuations in the 2D BCS-BEC crossover, i.e., in the crossover of a 2D fermionic superfluid from weakly bound BCS-like Cooper pairs to the Bose-Einstein condensation (BEC) of strongly bound molecules. Recently, zero-temperature quantum effects beyond the old mean-field predictions of Randeria *et al.* [15] have been investigated by Bertaina and Giorgini [16]. By using the fixed-node diffusion Monte Carlo (MC) numerical method they have found that in the BEC regime the zero-temperature MC equation of state shows dimer-dimer and atom-dimer interaction effects that are completely neglected in the mean-field picture [16].

In this Rapid Communication we study quantum fluctuations of the zero-temperature attractive Fermi gas in the 2D BCS-BEC crossover by using a path-integral approach. Our theoretical analysis is the 2D counterpart of similar beyond-mean-field investigations performed in the three-dimensional (3D) crossover [17–19]. However, the calculations presented in the above-mentioned references are by no means readily extended from 3D to 2D. In fact, to our knowledge, no extension has been made to 2D of the regularization approach of Refs. [17–19] based on convergence factors. For this reason we adopt a completely different method, i.e., the dimensional regularization plus flow equation quantum-field-theory technique, which is widely used in high-energy particle physics, but rarely encountered in condensed matter theory.

We show that, contrary to the 3D case, in our 2D fermionic system the interaction between composite bosons is fully induced by quantum fluctuations. To obtain this intriguing result we investigate the zero-point energy of collective bosonic excitations obtained from the the quadratic Gaussian action of the fluctuating pairing field. This divergent zero-point energy can be set to zero on the basis of dimensional regularization [20] only if one considers a low-momentum expansion of the Gaussian action up to second order [21]. Here we perform a low-momentum expansion up to the fourth order and using dimensional regularization we find a running coupling constant from which we derive an effective beyond-mean-field grand potential. Remarkably, from this effective grand potential, which includes quantum fluctuations, we find exactly the recursive Popov's equation of state [22] of 2D interacting bosons with its beyond-mean-field logarithmic correction, which reduces to the Schick's equation of state [23] at the leading order [24]. In particular, we find that the scattering length a_B of composite bosons is given by $a_B = a_F/(2^{1/2}e^{1/4}) = 0.551\ldots\ a_F$, with a_F the scattering length of fermions. This fully analytical result is in very good agreement with recent Monte Carlo calculations of Bertaina and Giorgini [16] and previous four-body scattering calculations of Petrov *et al.* [25].

The model. We consider a two-dimensional attractive Fermi gas of ultracold and dilute two-spin-component neutral atoms. We adopt the path integral formalism, where the atomic fermions are described by the complex Grassmann fields $\psi_\sigma(\mathbf{r},\tau)$, $\bar{\psi}_\sigma(\mathbf{r},\tau)$ with spin $\sigma = (\uparrow,\downarrow)$ [4]. The Euclidean Lagrangian density of the uniform system in a two-dimensional box of area L^2 and with chemical potential μ is given by

$$\mathscr{L} = \bar{\psi}_\sigma\left[\hbar\partial_\tau - \frac{\hbar^2}{2m}\nabla^2 - \mu\right]\psi_\sigma + g\bar{\psi}_\uparrow\bar{\psi}_\downarrow\psi_\downarrow\psi_\uparrow, \tag{1}$$

where $g < 0$ is the strength of the s-wave interatomic coupling [4]. Summation over the repeated index σ in the Lagrangian is meant. The interaction strength g of s-wave pairing is related to the binding energy ϵ_b of a fermion pair in vacuum by the expression [15,26]

$$-\frac{1}{g} = \frac{1}{2L^2}\sum_{\mathbf{k}} \frac{1}{\epsilon_k + \frac{1}{2}\epsilon_b}. \tag{2}$$

Note that, contrary to the 3D case, in 2D realistic interatomic potentials always have a bound state [16,26]. In addition, according to Mora and Castin [27] the binding energy ϵ_b of two fermions can be written in terms of the 2D fermionic scattering length a_F as

$$\epsilon_b = \frac{4}{e^{2\gamma}}\frac{\hbar^2}{ma_F^2}, \tag{3}$$

where $\gamma = 0.577\ldots$ is the Euler-Mascheroni constant.

Through the usual Hubbard-Stratonovich transformation [4] the Lagrangian density $\mathscr{L}$ of Eq. (1), quartic in the fermionic fields, can be rewritten as a quadratic form by introducing the auxiliary complex scalar field $\Delta(\mathbf{r},\tau)$ so that

$$\mathscr{L}_e = \bar{\psi}_\sigma\left[\hbar\partial_\tau - \frac{\hbar^2}{2m}\nabla^2 - \mu\right]\psi_\sigma + \bar{\Delta}\,\psi_\downarrow\,\psi_\uparrow + \Delta\bar{\psi}_\uparrow\,\bar{\psi}_\downarrow - \frac{|\Delta|^2}{g}. \quad (4)$$

The partition function $\mathcal{Z}$ of the system at temperature T can then be written as

$$\mathcal{Z} = \int \mathcal{D}[\psi_\sigma,\bar{\psi}_\sigma]\,\mathcal{D}[\Delta,\bar{\Delta}]\,\exp\left\{-\frac{S_e(\psi_\sigma,\bar{\psi}_\sigma,\Delta,\bar{\Delta})}{\hbar}\right\}, \quad (5)$$

where

$$S_e(\psi_\sigma,\bar{\psi}_\sigma,\Delta,\bar{\Delta}) = \int_0^{\hbar\beta} d\tau \int_{L^2} d^2\mathbf{r}\,\mathscr{L}_e(\psi_\sigma,\bar{\psi}_\sigma,\Delta,\bar{\Delta}) \quad (6)$$

is the effective action and $\beta \equiv 1/(k_B T)$ with k_B Boltzmann's constant.

Review of mean-field results. We shall investigate the effect of fluctuations of the gap field $\Delta(\mathbf{r},t)$ around its mean-field value Δ_0 which may be taken to be real. For this reason we set

$$\Delta(\mathbf{r},\tau) = \Delta_0 + \eta(\mathbf{r},\tau), \quad (7)$$

where $\eta(\mathbf{r},\tau)$ is the complex paring field of bosonic fluctuations [4].

Mean-field results are obtained neglecting bosonic fluctuations, i.e., setting $\eta(\mathbf{r},t) = 0$. Integrating over the fermionic fields $\psi_s(\mathbf{r},t)$ and $\bar{\psi}_s(\mathbf{r},t)$ in Eq. (5) one finds immediately the mean-field partition function [4,8–10,14]

$$\mathcal{Z}_{\rm mf} = \exp\left\{-\frac{S_{\rm mf}}{\hbar}\right\} = \exp\{-\beta\,\Omega_{\rm mf}\}, \quad (8)$$

where

$$\begin{aligned}\frac{S_{\rm mf}}{\hbar} &= -\mathrm{Tr}\left[\ln\left(G_0^{-1}\right)\right] - \beta L^2\frac{\Delta_0^2}{g}\\ &= -\sum_{\mathbf{k}}(2\ln\{2\cosh[\beta E_{sp}(k)/2]\} - \beta(\epsilon_k-\mu))\\ &\quad - \beta L^2\frac{\Delta_0^2}{g},\end{aligned} \quad (9)$$

with $\epsilon_k = \hbar^2k^2/(2m)$,

$$G_0^{-1} = \begin{pmatrix} \hbar\partial_\tau - \frac{\hbar^2}{2m}\nabla^2 - \mu & \Delta_0 \\ \Delta_0 & \hbar\partial_\tau + \frac{\hbar^2}{2m}\nabla^2 + \mu \end{pmatrix} \quad (10)$$

the inverse mean-field Green function, and

$$E_{\rm sp}(k) = \sqrt{(\epsilon_k-\mu)^2 + \Delta_0^2} \quad (11)$$

the energy of the fermionic single-particle elementary excitations.

At zero temperature ($T = 0$, i.e., $\beta \to +\infty$) the mean-field grand potential $\Omega_{\rm mf}$ becomes

$$\Omega_{\rm mf} = -\sum_{\mathbf{k}}[E_{\rm sp}(k) - \epsilon_k + \mu] - L^2\frac{\Delta_0^2}{g}. \quad (12)$$

In the continuum limit $\sum_{\mathbf{k}} \to L^2\int d^2\mathbf{k}/(2\pi)^2$ the logarithmic divergence of the grand potential $\Omega_{\rm mf}$ is removed by using Eq. (2), which gives the interaction strength g in terms of the binding energy ϵ_b of pairs. In this way one obtains

$$\Omega_{\rm mf} = -\frac{mL^2}{4\pi\hbar^2}\left[\mu^2 + \mu\sqrt{\mu^2+\Delta_0^2} + \frac{1}{2}\Delta_0^2 - \Delta_0^2\ln\left(\frac{-\mu+\sqrt{\mu^2+\Delta_0^2}}{\epsilon_b}\right)\right]. \quad (13)$$

The constant, uniform and real gap parameter Δ_0 is obtained by minimizing $\Omega_{\rm mf}$ with respect to Δ_0, namely,

$$\left(\frac{\partial\Omega_{\rm mf}}{\partial\Delta_0}\right)_{\mu,L^2} = 0, \quad (14)$$

from which one finds the gap equation

$$\Delta_0 = \sqrt{2\epsilon_b\left(\mu + \tfrac{1}{2}\epsilon_b\right)}, \quad (15)$$

which gives the energy gap Δ_0 as a function of the chemical potential μ and the binding energy ϵ_b. Inserting this formula into Eq. (13) we find

$$\Omega_{\rm mf} = -\frac{mL^2}{2\pi\hbar^2}\left(\mu + \frac{1}{2}\epsilon_b\right)^2. \quad (16)$$

The total number density $n = N/L^2$ of fermions is obtained from the familiar zero-temperature thermodynamic relation

$$n = -\frac{1}{L^2}\frac{\partial\Omega_{\rm mf}}{\partial\mu} \quad (17)$$

which immediately gives the chemical potential μ as a function of the number density $n = N/L^2$, i.e.,

$$\mu = \frac{\pi\hbar^2}{m}n - \frac{1}{2}\epsilon_b. \quad (18)$$

This is the mean-field equation of state of the 2D superfluid Fermi gas in the BCS-BEC crossover obtained by Randeria *et al.* [15]. In the BCS regime, where $\epsilon_b \ll \epsilon_F$ with $\epsilon_F = \pi\hbar^2 n/m$ the Fermi energy of the 2D ideal Fermi gas, one finds $\mu \simeq \epsilon_F > 0$ while in the BEC regime, where $\epsilon_b \gg \epsilon_F$ one has $\mu \simeq -\epsilon_b/2 < 0$.

Introducing $\mu_B = 2(\mu + \epsilon_b/2)$ as the chemical potential of composite bosons (made of bound fermionic pairs) with mass $m_B = 2m$ and density $n_B = n/2$, we may rewrite the above equation of state, Eq. (18), in terms of bosonic quantities as

$$\mu_B = \frac{8\pi\hbar^2}{m_B}n_B. \quad (19)$$

Clearly, this mean-field equation of state showing a bosonic chemical potential μ_B independent of the interaction between bosons is lacking important information which must be encoded in quantum fluctuations. As previously explained, the main goal of this Rapid Communication is to take into account these quantum fluctuations, which are crucial in reduced dimensionalities [1–4].

Gaussian quantum fluctuations. We now consider the effect of quantum fluctuations, i.e., in Eq. (7) we allow $\eta(\mathbf{r},t) \neq 0$. Expanding the effective action $S_e(\psi_s,\bar{\psi}_s,\Delta,\bar{\Delta})$ of Eq. (6)

around Δ_0 up to the quadratic (Gaussian) order in $\eta(\mathbf{r},t)$ and $\bar{\eta}(\mathbf{r},t)$ one finds

$$Z = Z_{\mathrm{mf}} \int \mathcal{D}[\eta,\bar{\eta}] \exp\left\{ -\frac{S_g(\eta,\bar{\eta})}{\hbar} \right\}, \tag{20}$$

where

$$S_g(\eta,\bar{\eta}) = \frac{1}{2}\sum_Q (\bar{\eta}(Q),\eta(-Q))\,\mathbf{M}(Q)\begin{pmatrix}\eta(Q)\\ \bar{\eta}(-Q)\end{pmatrix} \tag{21}$$

is the Gaussian action of fluctuations in the reciprocal space with $Q = (\mathbf{q}, i\nu_m)$ the 3-vector denoting the momenta $\mathbf{q}$ and Matsubara frequencies $\nu_m = 2\pi m/\beta$. Integrating over the bosonic fields $\eta(Q)$ and $\bar{\eta}(Q)$ in Eq. (20) one finds the Gaussian grand potential [13,19,28,29]

$$\Omega_g = \frac{1}{2\beta}\sum_Q \ln \mathrm{Det}(\mathbf{M}(Q)). \tag{22}$$

The 2×2 matrix $\mathbf{M}(Q)$ is the inverse fluctuation propagator, whose nontrivial dependence on Q can be found in Refs. [13,29]. $\mathrm{Det}(\mathbf{M}(Q)) = \mathrm{Det}(\mathbf{M}(\mathbf{q},z))$ has zero on the real axis of the z complex plane at $z = \pm\omega_0(\mathbf{q})$ which correspond to the poles of the fluctuation propagator, and describe the spectrum $E_{\mathrm{col}}(q) = \hbar\omega_0(q)$ of the bosonic collective excitations [13,19,28,29]. These excitations can be extracted from $\mathbf{M}(Q)$ with a low-energy and low-momentum expansion [13,29] to give

$$E_{\mathrm{col}}(q) = \sqrt{\epsilon_q\left(\lambda\epsilon_q + 2\,m\,c_s^2\right)}, \tag{23}$$

where $\epsilon_q = \hbar^2 q^2/(2m)$ is the free-particle energy, λ takes into account the first correction to the familiar low-momentum phonon dispersion $E_{\mathrm{col}}(q) \simeq c_s\hbar q$, with c_s the sound velocity. Both λ and c_s depend on the chemical potential μ and the energy gap Δ_0, which is itself a function of μ and ϵ_b on the basis of the gap equation (15). In particular, one finds [26,29]

$$\lambda = \frac{4x_0^2 + 1 - 8x_0\sqrt{x_0^2+1}}{24\sqrt{x_0^2+1}\left(\sqrt{x_0^2+1} - x_0\right)} \tag{24}$$

with $x_0 = \mu/\Delta_0 = [(\mu+\epsilon_b/2) - \epsilon_b/2]/\sqrt{2\epsilon_b(\mu+\epsilon_b/2)}$ and

$$mc_s^2 = \frac{\Delta_0}{2}\left(x_0 + \sqrt{x_0^2+1}\right) = \mu + \frac{1}{2}\epsilon_b, \tag{25}$$

where the last equality is obtained using Eq. (15). An inspection of Eq. (24) shows that λ is positive if $x_0 < 0.132$ and it goes quickly to $\lambda = 1/4$ in the BEC regime, where x_0 is large and negative, i.e., for $-x_0 \gg 1$. Thus in the BEC regime the spectrum (23) of collective bosonic excitations reduces to the familiar Bogoliubov spectrum [4,28] of bosonic excitations with mass $m_B = 2m$. Instead, λ is negative for $x_0 > 0.132$ and it goes to $\lambda = -x_0^2/3$ for $x_0 \gg 1$ that is the BCS regime. At zero temperature, the total grand potential finally reads

$$\Omega = -\lim_{\beta\to+\infty}\frac{1}{\beta}\ln(Z) = \Omega_{\mathrm{mf}} + \Omega_g, \tag{26}$$

where Ω_{mf} is given by Eq. (16), while Ω_g reads

$$\Omega_g = \frac{1}{2}\sum_{\mathbf{q}} E_{\mathrm{col}}(q). \tag{27}$$

This is the zero-point energy of bosonic collective excitations, i.e., the zero-temperature Gaussian fluctuations. In the continuum limit Eq. (27) is ultraviolet divergent if $\lambda > 0$. Instead, if $\lambda < 0$ the spectrum (23) has a natural ultraviolet cutoff q_c, given by $\hbar^2 q_c^2/(2m) = 2mc_s^2/|\lambda|$, which goes to zero in the deep BCS regime where, consequently, quantum fluctuations are strongly suppressed.

Analysis in the BEC regime. Since we are interested in the BEC regime ($\lambda > 0$ and in particular $\lambda = 1/4$) we must regularize Eq. (27). To this end we use the dimensional regularization [20,28], i.e., we extend the two-dimensional integral to a generic complex $D = 2 - \varepsilon$ dimension, and then take the limit $\varepsilon \to 0$. In this way

$$\begin{aligned}\frac{\Omega_g}{L^D} &= \frac{1}{2}\int \frac{d^D\mathbf{q}}{(2\pi)^D} E_{\mathrm{col}}(q)\\ &= -\frac{A(0)}{2\kappa^{\varepsilon}}\left(mc_s^2\right)^2\,\Gamma\left(-2+\frac{1}{2}\varepsilon\right),\end{aligned} \tag{28}$$

where the regulator κ is an arbitrary scale wave number which enters for dimensional reasons. In Eq. (28) we have defined $A(0) = m/(2\pi\hbar^2\lambda^{3/2})$ and $\Gamma(z)$ is the Euler gamma function, such that $\Gamma(-2+\varepsilon/2) = 1/\varepsilon + O(\varepsilon^0)$ for $\varepsilon \to 0$. Consequently, using Eq. (25), to leading order in $1/\varepsilon$ we get [30]

$$\frac{\Omega_g}{L^D} = -\frac{A(0)}{2\varepsilon\kappa^{\varepsilon}}\left(\mu + \frac{1}{2}\epsilon_b\right)^2. \tag{29}$$

This expression is still divergent. Nevertheless, comparing Ω_g with Ω_{mf} in $D = 2 - \varepsilon$ dimensions [see also Eq. (16)] given by

$$\frac{\Omega_{\mathrm{mf}}}{L^D} = -\frac{1}{2\xi(\varepsilon)}\left(\mu + \frac{1}{2}\epsilon_b\right)^2 \tag{30}$$

with $\xi(\varepsilon) = (4\pi\hbar^2/m)^{1-\varepsilon/2}\epsilon_b^{\varepsilon/2}(1+\varepsilon/2)/[4\Gamma(1+\varepsilon/2)]$ the mean-field coupling constant [31], we conclude that the total grand potential reads

$$\frac{\Omega}{L^2} = \frac{\Omega_{\mathrm{mf}}}{L^2} + \frac{\Omega_g}{L^2} = -\frac{1}{2\xi_r(k,\varepsilon)}\left(\mu + \frac{1}{2}\epsilon_b\right)^2, \tag{31}$$

where it appears the renormalized coupling constant $\xi_r(\kappa,\varepsilon)$ given by

$$\frac{1}{\xi_r(\kappa,\epsilon)} = \kappa^{\varepsilon}\left(\frac{1}{\xi(\varepsilon)} + \frac{A(0)}{\varepsilon\,\kappa^{\varepsilon}}\right). \tag{32}$$

The parameter $\xi_r(\kappa,\epsilon)$ is the "running coupling constant" of our theory which runs by changing κ [28,32]. To extract its dependence on κ we introduce the flow function $\beta(\xi_r) \equiv \kappa d\xi_r/d\kappa$, which encodes the dependence of the renormalized coupling constant ξ_r on the wave-number scale κ [32],

$$\beta(\xi_r) = \kappa\frac{d\xi_r}{d\kappa} = A(0)\,\xi_r^2 + O(\varepsilon). \tag{33}$$

After integration of Eq. (33) in the limit $\varepsilon \to 0$ we get

$$\frac{1}{\xi_r(\kappa',0)} - \frac{1}{\xi_r(\kappa,0)} = -A(0)\ln\left(\frac{\kappa'}{\kappa}\right), \quad (34)$$

where $A(0) = m/(2\pi\hbar^2\lambda^{3/2})$. We set the Landau pole [32] of Eq. (34) at the high-energy scale of the system ϵ_b, i.e., we set $1/\xi_r(\kappa',0) = 0$ at κ' such that $\hbar^2\kappa'^2/(2m) = \epsilon_b/2$. Then, when κ corresponds to the actual energy of our system, i.e., $\hbar^2\kappa^2/(2m) = \mu + \epsilon_b/2$, from Eqs. (31) with $\varepsilon \to 0$ we obtain

$$\Omega = -\frac{mL^2}{8\pi\hbar^2\lambda^{3/2}}\left(\mu + \frac{1}{2}\epsilon_b\right)^2 \ln\left(\frac{\epsilon_b}{2\left(\mu + \frac{1}{2}\epsilon_b\right)}\right). \quad (35)$$

In this BEC limit, where $\lambda = 1/4$, introducing again $\mu_B = 2(\mu + \epsilon_b/2)$ as the chemical potential of composite bosons with mass $m_B = 2m$ and density $n_B = n/2$, the total grand potential can be rewritten as

$$\Omega = -\frac{m_B L^2}{8\pi\hbar^2}\mu_B^2 \ln\left(\frac{\epsilon_b}{\mu_B}\right). \quad (36)$$

As usual, the total density of bosons $n_B = n/2$ is obtained in terms of $\mu_B = 2(\mu + \epsilon_b/2)$ from the zero-temperature thermodynamic formula

$$n = -\frac{1}{L^2}\frac{\partial\Omega}{\partial\mu}, \quad (37)$$

which leads to

$$n_B = \frac{m_B}{4\pi\hbar^2}\mu_B \ln\left(\frac{\epsilon_b}{\mu_B e^{1/2}}\right). \quad (38)$$

Inserting Eq. (3), which gives the binding energy ϵ_b of two fermions in terms of their s-wave scattering length a_F, into Eq. (38) we exactly recover the Popov's 2D equation of state [22] of weakly -interacting bosons with scattering length a_B [33], i.e.,

$$n_B = \frac{m_B}{4\pi\hbar^2}\mu_B \ln\left(\frac{4\hbar^2}{m_B\mu_B a_B^2 e^{2\gamma+1}}\right), \quad (39)$$

provided that we identify the effective bosonic scattering length a_B with

$$a_B = \frac{1}{2^{1/2}e^{1/4}}\, a_F = 0.551\ldots\, a_F. \quad (40)$$

With this choice of the relation between a_B and a_F, the equation of state of low density $[n \ll m/(\pi\hbar^2)\epsilon_b]$ fermions with a strong attractive interaction in 2D indeed coincides with the iterative Popov's 2D equation of state of weakly interacting bosons [22,33].

Remarkably, recent Monte Carlo results of Giorgini and Bertaina [16] give $a_B = 0.55(4)a_F$, in full agreement with our determination, Eq. (40), and also with previous four-body scattering calculations of Petrov *et al.* [25]. Notice that, at the first iteration [24] of Eq. (39), the leading contribution gives

$$\mu_B = \frac{4\pi\hbar^2}{m_B}\frac{n_B}{\ln\left(\frac{1}{n_B a_B^2}\right)}, \quad (41)$$

which is the equation of state found by Schick in 1971 [23].

In conclusion, by using a functional integral approach with dimensional regularization of Gaussian quantum fluctuations we have derived the flow equation of the running coupling constant which appears in the beyond-mean-field grand potential of the 2D attractive fermionic superfluid. We have found that in the BEC regime of the 2D BCS-BEC crossover this running coupling constant has a logarithmic dependence which exactly reproduces the Popov's equation of state of interacting 2D bosons. We have also shown that in the BCS regime quantum fluctuations are not divergent but they are, however, strongly suppressed. As a final comment, we notice that our approach, limited to the quartic term in the low-momentum expansion of bosonic collective excitations, cannot describe the entire 2D BCS-BEC crossover. In fact, in the region where μ changes sign, the coefficient λ is extremely small indicating that further terms must be included in the theory.

Acknowledgments. The authors acknowledge Cariparo Foundation (Eccellenza project) and Ministero Istruzione Universita Ricerca (PRIN project) for partial support. The authors thank Lara Benfatto, Gianluca Bertaina, Giacomo Bighin, Sergio Caprara, Luca Dell'Anna, Pieralberto Marchetti, Pierbiagio Pieri, Adriaan Schakel, and Jacques Tempere for enlightening discussions.

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