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D terms from D-branes, gauge invariance and moduli stabilization in flux compactifications

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Abstract: We elucidate the structure of D terms in $\mathcal{N}=1$ orientifold compactifications with fluxes. As a case study, we consider a simple orbifold of the type-IIA theory with D6-branes at angles, O6-planes and general NSNS, RR and Scherk-Schwarz geometrical fluxes. We examine in detail the emergence of D terms, in their standard supergravity form, from an appropriate limit of the D-brane action. We derive the consistency conditions on gauged symmetries and general fluxes coming from brane-localized Bianchi identities, and their relation with the Freed-Witten anomaly. We extend our results to other $\mathcal{N}=1$ compactifications and to non-geometrical fluxes. Finally, we discuss the possible rôle of $U(1)$ D terms in the stabilization of the untwisted moduli from the closed string sector.

Keywords: Flux compactifications, Compactification and String Models, Supergravity Models, D-branes.

Contents

1. Introduction

Orientifold compactifications of type-II superstrings with branes and fluxes (for some recent reviews and references, see e.g. [1 − 3]) offer new possibilities not only for obtaining the Standard Model spectrum and renormalizable interactions, but also for addressing long-standing problems such as moduli stabilization and supersymmetry breaking. We focus here on the last two problems. The ultimate goal is to find a string vacuum with spontaneously broken supersymmetry on an approximately flat four-dimensional background, with tiny positive vacuum energy density and all moduli stabilized. Eventually, we would also like to understand how 'our' vacuum may be selected among other possible ones. We are still far from this ambitious goal, but important progress is being made.

The phenomenologically relevant constructions under better theoretical control are those in which the effective potential for the light modes is generated at the classical level by fluxes, and can be described by an effective $\mathcal{N}=1$, $D=4$ supergravity, obtained from the underlying higher-dimensional theory via generalized dimensional reduction. It may well be that perturbative and non-perturbative quantum corrections play a crucial rôle in the complete resolution of the problems, but it is interesting to explore how far we can go at this classical level.

A particularly relevant set of fields, whose dynamics is crucial for the above-mentioned problems, are the closed string moduli associated with the spin-0 fluctuations of the dilaton, of the metric and of the p-form potentials in the Neveu-Schwarz Neveu-Schwarz (NSNS) and Ramond-Ramond (RR) sectors. In $\mathcal{N} = 1$ compactifications, these moduli can be assigned to chiral supermultiplets. For the sake of illustration, we concentrate here on the $T^6/(Z_2 \times Z_2)$ orbifold supplemented by a suitable orientifold, but our results apply also to other $\mathcal{N} = 1$ compactifications. In the case under consideration, there are seven 'main' moduli, denoted by

$$S = s + i\sigma \,, \qquad T_A = t_A + i\tau_A \,, \qquad U_A = u_A + i\nu_A \,, \tag{1.1}$$

where $A = 1, 2, 3$ corresponds to the three factorized 2-tori defined by the orbifold. Here and in the following, we call *main* moduli the seven complex scalars in eq. (1.1), *geometrical* moduli their real parts, *axions* their imaginary parts. When the remaining scalar excitations, living for example on branes, at brane intersections or at the orbifold fixed points, are consistently set to zero, the seven main moduli in eq. (1.1) can be described by the Kähler potential

$$K = -\log\left(s\, t_1\, t_2\, t_3\, u_1\, u_2\, u_3\right) \,. \tag{1.2}$$

However, the identification of the real and imaginary parts in eq. (1.1), in terms of the spin-0 fluctuations of the ten-dimensional bosonic fields, is model-dependent.

So far, the greatest effort went into identifying the effective superpotential W for the main moduli as a function of the fluxes, and the consistency conditions on the latter: since K is given by eq. (1.2), the knowledge of W completely determines the F-term contribution to the scalar potential for the main moduli. A general result, in the approximation of negligible warping and in the field basis of eqs. (1.1) and (1.2), is that W is a polynomial, at most of degree one in each of the seven main moduli, with specified relative phases between the different monomials: this property may be ascribed to the underlying $\mathcal{N} = 4$ supergravity structure [4].

In type-IIB orientifolds [5] with O3/O7-planes, the only available fluxes surviving the orbifold and orientifold projections are those of the NSNS and RR 3-form field strengths. As a result, dilaton and complex-structure moduli, which in the standard notation correspond to (S, U_1, U_2, U_3) in eq. (1.1), can be stabilized on a flat background with spontaneously broken supersymmetry, but the Kähler moduli, corresponding to (T_1, T_2, T_3) in eq. (1.1), remain as complex classical flat directions, as in no-scale models [6]. Similarly, type-IIB orientifolds with O5/O9-planes give, under suitable field redefinitions, the same effective theory for the main moduli as heterotic orbifolds, thus full stabilization of the main moduli is impossible. To go further, perturbative or non-perturbative quantum corrections must be advocated.

In the type-IIA theory, a richer spectrum of possibilities emerges, thanks to the richer structure of NSNS, RR and geometrical fluxes surviving the orbifold and orientifold projections [4, 7 − 13]. As in the type-IIB case, we can obtain Minkowski vacua with unbroken or spontaneously broken $\mathcal{N} = 1$ supersymmetry, but there are always some residual classical flat directions for the geometrical moduli and their axionic superpartners. In contrast with the type-IIB case, however, we can also obtain AdS_4 vacua with all the geometrical moduli stabilized, as explicitly shown in [11] for the orbifold $T^6/(Z_2 \times Z_2)$ and later discussed for other compactifications in [12, 13].

All the above results were derived ignoring the effects of the gauge fields localized on D-branes, and the associated D-term contributions to the scalar potential, under the assumption that D-branes would preserve $\mathcal{N} = 1$ supersymmetry[1]. As will be clear in the following, this is certainly consistent for the identification of supersymmetry-preserving vacua. However, a more careful investigation of brane-localized gauge fields and the associated D terms is needed to answer a number of questions. We expect additional constraints on fluxes, originating from the generalized Bianchi identities (BI) for the gauge fields localized on D-branes. We also expect that D terms contribute to the masses of geometrical moduli. If so, they could in principle remove the residual flat directions for the geometrical moduli that are typical of Minkowski vacua (as suggested for example in [23]), and perhaps decouple the masses of the geometrical moduli from the scale of the cosmological constant in the case of stable AdS_4 vacua. Also, we would like to check explicitly whether D terms are allowed to relax to zero for the field configurations corresponding to supersymmetry-breaking vacua of the F-term potential. Finally, we would like to know whether D terms can play a rôle in the generation of metastable de-Sitter vacua: a possible mechanism in the latter directions was recently proposed in [24] within a simple toy model, but no explicit string realization of it exists so far.

The main goal of the present paper is to examine the issues described above (with the exception of the last one, whose clarification may require a more general theoretical framework). We stress that, as will be explained in more detail later, $U(1)$ D terms always give a non-vanishing contribution to the bulk moduli spectrum, in all compactifications where chiral matter fields arise from brane intersections (or from magnetized branes in the mirror description). Exploring the structure of D terms is thus a mandatory step for the study of semi-realistic flux compactifications.

We now anticipate the main results of the paper. We explicitly derive, via dimensional reduction of the D6-brane action, the expression for D terms in O6 orientifolds of the type-IIA theory, compactified on the $T^6/(Z_2 \times Z_2)$ orbifold (consistently neglecting matter fields). We show that it agrees with the expected supergravity formula as long as a specific bound on the D term and the gauge coupling is satisfied. We extend the results to other $\mathcal{N} = 1$ string compactifications, of the type-IIB theory with O3/O7 or O9/O5 orientifolds, and of the heterotic theory. We show that D terms are compatible with fluxes only when the BI for the localized gauge fields are satisfied, and we find the general form for these constraints. We extend the discussion to the case of non-geometrical flux compactifications, finding the corresponding modifications to the localized BI and to the effective superpotentials. We

[1] In type-IIB constructions, we can generate supersymmetry-breaking configurations by considering brane-localized magnetic fluxes [14]. Making use of T-duality, this is equivalent to type-IIA constructions with D6-branes at angles [15]. $U(1)$ D terms for $\mathcal{N} = 1$ type-II compactifications and their dependence on the geometrical moduli were previously discussed in [16 – 21], and the possible rôle of magnetic fluxes in the stabilization of the bulk moduli in [19, 20, 22].

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show that gauge anomalies cancel when both bulk and localized BI are satisfied. We clarify the rôle of the $U(1)$ D terms in the problem of bulk moduli stabilization, both in AdS_4 and in Minkowski vacua.

The plan of the paper is the following. We complete this introduction by recalling some basic facts about F and D terms in $\mathcal{N}=1$, $D=4$ supergravity, and by summarizing the results of [11] for the effective F-term potential and superpotential for the main moduli, in a simple but very interesting class of $\mathcal{N}=1$ type-IIA compactifications. We then discuss in section 2 the structure of the effective potential for the main moduli originating from a stack of N D6-branes. We identify, in the appropriate limit, the F-term and D-term contributions, and the consistency conditions that must hold for the effective theory to be a standard $\mathcal{N}=1$, $D=4$ supergravity. Since we restrict our attention to the functional dependence on the main moduli, setting all other scalar fields to zero, the relevant D terms are those associated with the $U(1)$ gauge symmetries in the decompositions $U(N) \to U(1) \times SU(N)$, which act as shifts on the four RR axions. In section 3 we discuss the localized BI for the gauge fields living on the D6-branes, the constraints they put on gauged symmetries and fluxes, and their intriguing relation with the Freed-Witten anomaly [25]. We also derive some consequences of the localized BI that may be relevant in other contexts, such as the discussion of non-geometrical fluxes [4, 26] and of gauge anomaly cancellation. In section 4 we comment on the extensions of our results to other $\mathcal{N}=1$ compactifications, not only of the type-IIA theory but also of the type-IIB and heterotic theories. We then discuss in section 5 the rôle of $U(1)$ D terms in the problem of bulk moduli stabilization. We explain how D terms can generate positive contributions to the squared masses of some geometrical moduli that, albeit stabilized on a supersymmetric AdS_4 vacuum, have negative squared masses from the F-term potential. Finally, we describe some difficulties in moduli stabilization that arise on Minkowski flux vacua, with exact or spontaneously broken supersymmetry. We conclude in section 6 with a brief summary of our results and some comments on the prospects for future work. Finally, in the appendix we display some of our formulae for flux compactifications of the type-IIA theory on the $T^6/(Z_2 \times Z_2)$ orbifold and O6 orientifold, in a more explicit form that can be useful for model building.

1.1 F and D terms in $\mathcal{N}=1$, $D=4$ supergravity

We recall here some basic facts about the F- and D-term contributions to the scalar potential, in a generic $\mathcal{N}=1$, $D=4$ supergravity with chiral multiplets $\phi^i \sim (z^i, \psi^i)$ and vector multiplets $V^a \sim (\lambda^a, A^a_\mu)$. Up to two derivatives in the bosonic fields, the gauge–invariant supergravity action is completely determined by three ingredients (see, e.g., [27]). The first is the real and gauge-invariant Kähler function G, which can be written in terms of a real Kähler potential K and a holomorphic superpotential W as

$$G = K + \log |W|^2 \,. \tag{1.3}$$

The second is the holomorphic gauge kinetic function f_{ab}, which transforms as a symmetric product of adjoint representations, plus a possible imaginary shift associated with anomaly cancellation. Generalized Chern-Simons terms may also be needed [28], but they will not play any rôle in the situations discussed in this paper and will be neglected. Then the generalized kinetic terms for the vector fields can be written as

$$-\frac{1}{4}\,\tilde{e}_4\;\mathrm{Re}\,f_{ab}\;F^aF^b+\frac{1}{2}\,\mathrm{Im}\,f_{ab}\;F^a\wedge F^b\,. \tag{1.4}$$

The third ingredient are the holomorphic Killing vectors $X_a = X_a^i(z)(\partial/\partial z^i)$, which generate the analytic isometries of the Kähler manifold for the scalar fields that are gauged by the vector fields. In the following it will suffice to think of G, f_{ab} and X_a as functions of the complex scalars z^i rather than the superfields ϕ^i. The gauge transformation laws and covariant derivatives for the scalars in the chiral multiplets read

$$\delta z^i = X_a^i\,\epsilon^a\,,\qquad D_\mu z^i = \partial_\mu z^i - A_\mu^a X_a^i\,, \tag{1.5}$$

where ϵ^a are real parameters. The scalar potential is

$$V = V_F + V_D = e^G\left(G^iG_i - 3\right) + \frac{1}{2}D_aD^a\,, \tag{1.6}$$

where $G_i = \partial G/\partial z^i$, scalar field indices are raised with the inverse Kähler metric $G^{i\overline{k}}$, gauge indices are raised with $[(Ref)^{-1}]^{ab}$, and D_a are the Killing potentials, real solutions of the complex Killing equations:

$$X_a^i = -i\,G^{i\overline{k}}\,\frac{\partial D_a}{\partial \overline{z}^{\overline{k}}}\,. \tag{1.7}$$

The general solution to the Killing equation for D_a, compatible with gauge invariance, is

$$D_a = i\,G_i\,X_a^i = i\,K_i\,X_a^i + i\,\frac{W_i}{W}\,X_a^i\,. \tag{1.8}$$

It is not restrictive to assume that K is gauge-invariant. If W is also gauge-invariant[2],

$$W_i\,X_a^i = 0\,, \tag{1.9}$$

eq. (1.8) reduces to

$$D_a = i\,K_i\,X_a^i\,. \tag{1.10}$$

For a linearly realized gauge symmetry, $i\,K_i\,X_a^i = -K_i\,(T_a)^i{}_k z^k$, and we recover the familiar expression of [29] for the D terms. For a $U(1)$ axionic shift symmetry,

$$X_a^i = i\,q_a^i\,, \tag{1.11}$$

where q_a^i is a real constant. We thus obtain what are usually called, with a slight abuse of language, field-dependent Fayet-Iliopoulos (FI) terms: they will play a crucial rôle in the rest of this paper.

We stress here two known consequences of eq. (1.8), which shows that D terms are actually proportional to F terms, $F_i = e^{G/2}\,G_i$. First, and in contrast with the rigid case, there cannot be pure D breaking of supergravity, unless the gravitino mass vanishes

[2] For $U(1)$ factors, W can be gauge-invariant up to a phase. This corresponds to gauging the R symmetry and leads to constant Fayet-Iliopoulos terms in D_a, whose possible phenomenological relevance was discussed in [24]. Since they play no rôle for the present paper, we will neglect here this possibility.

and the D-term contribution to the vacuum energy is uncanceled, as in the (unrealistic) limit of global supersymmetry. Second, if V_F admits a supersymmetric AdS_4 vacuum configuration, $\langle G_i \rangle = 0$ $(\forall i)$ and $\langle e^G \rangle \neq 0$, such configuration automatically minimizes V_D at zero, and D terms cannot be used to raise the vacuum energy from negative to positive or zero. Moreover, the gauge invariance of W, eq. (1.9), puts severe constraints on the simultaneous presence of flux-induced superpotentials and D terms: this point will be discussed in detail in section 3.

1.2 F-term potential and superpotential for the main moduli

In the following, we will mostly concentrate on type-IIA compactifications on the chosen orbifold $T^6/(Z_2 \times Z_2)$, with the orientifold projection $\Omega(-1)^{F_L} I_3$, where Ω is the world-sheet parity operator, $(-1)^{F_L}$ is the space-time fermion number for left-movers, and I_3 acts as a parity on three of the six internal coordinates. For the main moduli of these compactifications, we know by now the effective superpotential and the full F-term contributions to the effective potential [4, 11], as well as the consistency conditions on the fluxes coming from the BI of the local symmetries gauged by bulk fields [11]. To set the stage for our discussion of D terms and localized BI, we briefly recall the main results of [11].

We begin with the bosonic field content of $D = 10$ type–IIA supergravity. In the NSNS sector, we have the (string-frame) metric g_{MN}, the 2-form potential B and the dilaton Φ. In the RR sector, we have the $(2k+1)$-form potentials $C^{(2k+1)}$, $(k = 0, \ldots, 4)$, whose degrees of freedom are not all independent, being related by Poincaré duality.

For definiteness, we take the action of the orbifold projection on the internal coordinates x^r $(r = 5, \ldots, 10)$ of the factorized 6-torus $T^6 = T^2 \times T^2 \times T^2$ to be

$$Z_2 \; : \; (z^1, z^2, z^3) \to (-z^1, -z^2, z^3) \,, \qquad Z_2' \; : \; (z^1, z^2, z^3) \to (z^1, -z^2, -z^3) \,, \tag{1.12}$$

where

$$z^1 = x^5 + i\, x^6 \,, \qquad z^2 = x^7 + i\, x^8 \,, \qquad z^3 = x^9 + i\, x^{10} \,, \tag{1.13}$$

and the action of the I_3 orientifold involution to be

$$I_3 \; : \; (z^1, z^2, z^3) \to (-\overline{z}^1, -\overline{z}^2, -\overline{z}^3) \,. \tag{1.14}$$

The intrinsic parities of the bosonic fields with respect to the orientifold projection are: $+1$ for g_{MN}, Φ and the RR p-form potentials with $p = 3, 7$; -1 for B and the RR p-form potentials with $p = 1, 5, 9$.

Before proceeding, we introduce some notation that will be used in the rest of the paper. For the chosen toroidal orbifold, there are eight independent 3-cycles on the factorized 3-torus, four even and four odd under the orientifold projection. Since each 3-cycle π can be identified by a set of topological wrapping numbers,

$$\pi = (m_1, n_1) \otimes (m_2, n_2) \otimes (m_3, n_3) \,, \tag{1.15}$$

we can define the following basis for even and odd 3-cycles:

$$\begin{cases} \alpha^0 = (1,0) \otimes (1,0) \otimes (1,0) \\ \alpha^1 = (1,0) \otimes (0,1) \otimes (0,1) \\ \alpha^2 = (0,1) \otimes (1,0) \otimes (0,1) \\ \alpha^3 = (0,1) \otimes (0,1) \otimes (1,0) \end{cases}, \qquad \begin{cases} \beta_0 = (0,1) \otimes (0,1) \otimes (0,1) \\ \beta_1 = (0,1) \otimes (1,0) \otimes (1,0) \\ \beta_2 = (1,0) \otimes (0,1) \otimes (1,0) \\ \beta_3 = (1,0) \otimes (1,0) \otimes (0,1) \end{cases}. \tag{1.16}$$

Equivalently, we can define the dual basis $([\alpha]^I, [\beta]_I)$ $(I = 0,1,2,3)$ for the cohomological 3-forms associated with the (even, odd) 3-cycles, normalized as

$$\int [\alpha]^I \wedge [\beta]_J = -\delta^I_J \,. \tag{1.17}$$

In the basis of eq. (1.16), π of eq. (1.15) and its dual 3-form read

$$\pi = p_I\, \alpha^I + q^I\, \beta_I \,, \qquad [\pi] = p_I\, [\alpha]^I - q^I\, [\beta]_I \,, \tag{1.18}$$

where

$$\begin{cases} p_0 = m_1 m_2 m_3 \\ p_1 = m_1 n_2 n_3 \\ p_2 = n_1 m_2 n_3 \\ p_3 = n_1 n_2 m_3 \end{cases}, \qquad \begin{cases} q^0 = n_1 n_2 n_3 \\ q^1 = n_1 m_2 m_3 \\ q^2 = m_1 n_2 m_3 \\ q^3 = m_1 m_2 n_3 \end{cases}. \tag{1.19}$$

The action of I_3 is compatible with the presence of O6-planes with internal coordinates $(6,8,10)$, wrapping the 3-cycle $\pi_{06} = \alpha^0$. The action of the orbifold group induces additional O6-planes in the three directions $[(6,7,9),\ (5,8,9),\ (5,7,10)]$, corresponding to $\pi'_{06} = -\alpha^A$ $(A = 1,2,3)$, respectively. A stack of parallel D6-branes wrapping a generic (factorizable) 3-cycle π will be characterized by eq. (1.15) or eq. (1.18). Later in this paper, we will also consider their mirror D6-branes with respect to the $O6$-plane π_{O6}, wrapping the 3-cycle:

$$\pi' = p_I\, \alpha^I - q^I\, \beta_I \,. \tag{1.20}$$

The imaginary parts of the seven main moduli are associated with the scalar components of the NS-NS 2-form potential and of the R-R 3-form potential surviving the chosen orbifold and orientifold projections. In the notation of [11]:

$$B_{56|78|910} = \tau_{1|2|3} \,, \qquad C^{(3)}_{6810} = \sigma \,, \qquad C^{(3)}_{679|589|5710} = -\nu_{1|2|3} \,. \tag{1.21}$$

The real parts of the seven main moduli are associated with the invariant components of the dilaton and the metric, traditionally decomposed as

$$e^{-2\Phi} = \frac{\hat{s}}{t_1 t_2 t_3} \,, \qquad g_{MN} = \mathrm{blockdiag}\left(\hat{s}^{-1}\, \widetilde{g}_{\mu\nu},\ t_1\, \hat{u}_1,\ \frac{t_1}{\hat{u}_1},\ t_2\, \hat{u}_2,\ \frac{t_2}{\hat{u}_2},\ t_3\, \hat{u}_3,\ \frac{t_3}{\hat{u}_3} \right) \,, \tag{1.22}$$

where $\widetilde{g}_{\mu\nu}$ is the metric in the $D = 4$ Einstein frame. However, the correct complexification of eq. (1.1) is achieved only after the following field redefinition:

$$s = \sqrt{\frac{\hat{s}}{\hat{u}_1\, \hat{u}_2\, \hat{u}_3}} \,, \quad u_1 = \sqrt{\frac{\hat{s}\ \hat{u}_2\, \hat{u}_3}{\hat{u}_1}} \,, \quad u_2 = \sqrt{\frac{\hat{s}\ \hat{u}_1\, \hat{u}_3}{\hat{u}_2}} \,, \quad u_3 = \sqrt{\frac{\hat{s}\ \hat{u}_1\, \hat{u}_2}{\hat{u}_3}} \,. \tag{1.23}$$

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Such redefinition, which can be inferred by computing the kinetic terms for the main moduli via dimensional reduction, is crucial for writing the effective $\mathcal{N}=1$ supergravity in the standard form, and will play an important rôle also in the discussion of the D terms[3]. Summarizing, the metric $\widetilde{g}_{\mu\nu}$ and the seven main moduli are the only bosonic bulk fields surviving the orbifold and orientifold projections. There are no surviving zero modes for the bulk vector fields coming from the ten-dimensional metric and from the p-form potentials in the NSNS and RR sectors.

We now list the various (constant) fluxes allowed by the chosen orbifold and orientifold projections. For the Scherk-Schwarz [30] geometrical fluxes ω, we have twelve independent components:

$$(\omega_{68}{}^{10}\,,\,\omega_{106}{}^{8}\,,\,\omega_{810}{}^{6})\,;\quad(\omega_{57}{}^{10}\,,\,\omega_{95}{}^{8}\,,\,\omega_{79}{}^{6})\,;\quad(\omega_{58}{}^{9}\,,\,\omega_{89}{}^{5}\,,\,\omega_{67}{}^{9}\,,\,\omega_{96}{}^{7}\,,\,\omega_{105}{}^{7}\,,\,\omega_{710}{}^{5})\,. \tag{1.24}$$

In the following, we will adopt the conventions of [11] for the contraction of ω with any p-form. For the NSNS 3-form field strength $\overline{H}$, we have four independent components:

$$\overline{H}_{579}\,;\quad(\overline{H}_{5810}\,,\,\overline{H}_{6710}\,,\,\overline{H}_{689})\,. \tag{1.25}$$

They can also be decomposed in the basis of eq. (1.16). Finally, for the field strengths in the RR sector we have eight independent components:

$$\overline{G}^{(0)}\,;\quad(\overline{G}^{(2)}_{56}\,,\,\overline{G}^{(2)}_{78}\,,\,\overline{G}^{(2)}_{910})\,;\quad(\overline{G}^{(4)}_{5678}\,,\,\overline{G}^{(4)}_{78910}\,,\,\overline{G}^{(4)}_{56910})\,;\quad\overline{G}^{(6)}_{5678910}\,. \tag{1.26}$$

The above fluxes must satisfy generalized BI associated with the local symmetries gauged by bulk fields, taking into account the possible existence of localized sources such as D6-branes and O6-planes. The integrability conditions of such BI provide the following non-trivial constraints on the allowed fluxes:

$$\omega\,\omega=0\,, \tag{1.27}$$

$$\frac{1}{2}\left(\omega\,\overline{G}^{(2)}+\overline{H}\,\overline{G}^{(0)}\right)=\sum_a N_a\,\mu_a\,[\pi_a]\,, \tag{1.28}$$

where in the last equation the index a runs over the stacks of D6-branes and the O6-planes, N_a is the number of branes in each stack ($N_a=2^3$ for O6-planes), and μ_a is the RR charge[4].

The effective superpotential is, in compact geometrical form:

$$W=\frac{1}{4}\int_{\mathcal{M}_6}\overline{\mathbf{G}}\,e^{iJ^c}-i\,\left(\overline{H}-i\omega\,J^c\right)\wedge\Omega^c\,, \tag{1.29}$$

where we have grouped the RR fluxes into the formal sum $\overline{\mathbf{G}}=\sum_{p=even}\overline{G}^{(p)}$ and, in our conventions, J^c and Ω^c read

$$\begin{aligned}J^c&=J+i\,B\,,\qquad J=\frac{i}{2}\sum_{A=1}^{3}dz^A\wedge d\overline{z}^A\,,\\ \Omega^c&=\mathrm{Re}\left(i\,e^{-\Phi}\,\Omega\right)+i\,C^{(3)}\,,\qquad\Omega=dz^1\wedge dz^2\wedge dz^3\,.\end{aligned} \tag{1.30}$$

[3]Analogous redefinitions, involving the $(\hat{s},t_1,t_2,t_3)$ fields and leaving the $(\hat{u}_1,\hat{u}_2,\hat{u}_3)$ fields untouched, must be also performed in the case of type-IIB orientifolds.

[4]In our normalization ($\kappa_{10}^2=1$), $T_a=\mu_a=1/2$ for the D6-branes and their images, which should be counted separately, and $T_a=\mu_a=-2$ for the O6-planes.

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In particular, in our case:

$$J^c_{56|78|910} = T_{1|2|3}\,, \qquad \Omega^c_{6810} = S\,, \quad \Omega^c_{679|589|5710} = -U_{1|2|3}\,. \tag{1.31}$$

As discussed in [11], the contributions to the effective potential coming from localized sources and the integrability conditions of eq. (1.28) are crucial for establishing the exact correspondence between the potential obtained via generalized dimensional reduction and the standard $\mathcal{N}=1$ formula for the F-term potential,

$$V_F = e^K \left[\sum_{i=1}^{7} |W_i(\varphi) + K_i W(\varphi)|^2 - 3|W(\varphi)|^2 \right], \qquad \varphi^{1,\ldots,7} = (S, T_1, T_2, T_3, U_1, U_2, U_3)\,, \tag{1.32}$$

where $W_i \equiv \partial W/\partial\varphi^i$, evaluated for the superpotential W of eq. (1.29), and $K_i \equiv \partial K/\partial\varphi^i$.

2. D terms from D6-branes

In this section we will derive the effective potential for the main moduli generated by a stack of N D6-branes wrapping a generic factorizable 3-cycle π, as in eq. (1.15), in the chosen class of type-IIA compactifications. The generalization to other $\mathcal{N}=1$ string compactifications is discussed in section 4. If the D6-branes are parallel to the O6-plane defined by π_{O6}, and we use the index α for the D6-brane internal space, the index $\hat\alpha$ for the orthogonal space, the embedding of the D6-brane world volume (x^α) into the ten-dimensional space (X^M) can be defined by

$$X^M = x^\alpha\, \delta^M_\alpha + \phi^{\hat\alpha}(x^\alpha)\, \delta^M_{\hat\alpha}\,, \tag{2.1}$$

where $\alpha = \mu, 6, 8, 10$, $\hat\alpha = 5, 7, 9$, and the $\phi^{\hat\alpha}$ describe the brane fluctuations in the transverse directions. If instead the D6-branes wrap a generic 3-cycle π, their embedding is described by:

$$X'^M = \Lambda^M{}_N\, X^N\,, \tag{2.2}$$

where $\Lambda^M{}_N$ is the rotation matrix (acting trivially on the four non-compact dimensions):

$$\Lambda^M{}_N = \mathbf{I}_4 \bigotimes_{A=1}^{3} \begin{pmatrix} m_A & n_A \\ -n_A & m_A \end{pmatrix}. \tag{2.3}$$

We recall that the localized action for a stack of N parallel D6-branes is $S_{DBI} + S_{WZ}$, where

$$S_{DBI} = -N\, T_6 \int_{\mathbb{R}^4\times\pi} d^7x\; e^{-\Phi} \sqrt{-\det\left(g_{\alpha\beta} + B_{\alpha\beta} + F_{\alpha\beta}\right)} \tag{2.4}$$

is the Dirac-Born-Infeld (DBI) action and

$$S_{WZ} = N\,\mu_6 \int_{\mathbb{R}^4\times\pi} \sum_{n=odd} A^{(n)}\; e^F\,, \qquad \left[A^{(n)} = e^B\, C^{(n)}\right], \tag{2.5}$$

is the Wess-Zumino (WZ) action. In the above equations we kept only the contribution from the $U(1)$ factor, in the decomposition $U(N) \to U(1) \times SU(N)$ of the gauge group.

To derive the D6-brane contributions to the effective potential for the seven main moduli, it is sufficient to consider the tension term in the DBI action, keeping the dependence on the bulk metric and antisymmetric tensor but setting all the brane fluctuations to zero. Making use of eq. (1.22), we find:

$$V_6 = N\,T_6\,\frac{e_7'}{\tilde{e}_4}\,e^{-\Phi} = N\,T_6\,\frac{1}{\hat{s}^2}\sqrt{\hat{s}\prod_{A=1}^{3}\left(\frac{m_A^2}{\hat{u}_A}+n_A^2\hat{u}_A\right)}\,, \tag{2.6}$$

where $e_7' = \sqrt{-\det g'_{\alpha\beta}}$ is the siebenbein determinant computed in the rotated coordinate system. Making use of the field redefinitions of eq. (1.23), we observe that, in the conventions of eq. (1.30), and after setting

$$\widetilde{\Omega}_\pi = \int_\pi i\,e^{-\Phi}\,\Omega\,, \tag{2.7}$$

we can write

$$\mathrm{Re}\,\widetilde{\Omega}_\pi = m_1 m_2 m_3 s - \sum_{A=1}^{3} m_A n_B n_C u_A = p_0\,s - \sum_{A=1}^{3} p_A\,u_A\,, \tag{2.8}$$

$$\mathrm{Im}\,\widetilde{\Omega}_\pi = \sqrt{s u_1 u_2 u_3}\left(\frac{n_1 n_2 n_3}{s} - \sum_{A=1}^{3}\frac{n_A m_B m_C}{u_A}\right) = \sqrt{s u_1 u_2 u_3}\left(\frac{q^0}{s} - \sum_{A=1}^{3}\frac{q^A}{u_A}\right)\,, \tag{2.9}$$

with $A \neq B \neq C = 1,2,3$. Then we obtain:

$$V_6 = \frac{N\,T_6}{s u_1 u_2 u_3}\sqrt{(\mathrm{Re}\,\widetilde{\Omega}_\pi)^2 + (\mathrm{Im}\,\widetilde{\Omega}_\pi)^2}\,. \tag{2.10}$$

To recover the standard form of the supergravity potential, it is useful to decompose V_6 as

$$V_6 = V_{6F} + V_D\,, \tag{2.11}$$

where

$$V_{6F} = \frac{N\,T_6}{s u_1 u_2 u_3}\,\mathrm{Re}\,\widetilde{\Omega}_\pi\,, \tag{2.12}$$

$$V_D = \frac{N\,T_6}{s u_1 u_2 u_3}\left(\sqrt{(\mathrm{Re}\,\widetilde{\Omega}_\pi)^2 + (\mathrm{Im}\,\widetilde{\Omega}_\pi)^2} - \mathrm{Re}\,\widetilde{\Omega}_\pi\right)\,. \tag{2.13}$$

This decomposition was already perfomed in [18]. The contribution V_{6F} was a crucial ingredient, in the generalized dimensional reduction of [11], for reconstructing the full F-term contribution to the scalar potential, associated with the $\mathcal{N}=1$ superpotential of eq. (1.29) and the Kähler potential of eq. (1.2), after making use of the integrability conditions of eq. (1.28). Notice in fact that, for supersymmetric D6-branes,

$$\mathrm{Im}\,\widetilde{\Omega}_\pi = 0\,, \qquad \mathrm{Re}\,\widetilde{\Omega}_\pi > 0\,, \tag{2.14}$$

$V_D = 0$ and no other contribution to the scalar potential, except V_{6F}, arises from the D6-brane action.

We will now show that V_D can be identified, in an appropriate limit, with the $U(1)$ D-term part of the supergravity potential.

We begin by observing that the universal $U(1)$ associated with the brane stack acts as a shift symmetry on the four RR axions $(\sigma, \nu_1, \nu_2, \nu_3)$ defined in eq. (1.21). To see this, it is sufficient to show that the kinetic terms for these four axions get suitably covariantized. The origin of this phenomenon is the WZ part of the D6-brane action, which contains the term

$$N\,\mu_6 \int_{\mathbb{R}^4\times\pi} A^{(5)} \wedge F\,, \tag{2.15}$$

where $F = dA$ is the 2-form field strength for the localized vector bosons. The term of eq. (2.15) is the only one linear in A_μ, and with one derivative, that can contribute to the BI for $G^{(4)}$. The latter can be easily constructed from the dual formulation of [11] and reads:

$$\frac{\delta}{\delta A^{(5)}}\left[\int \frac{1}{2} A^{(5)} \wedge dG^{(4)} - N\,\mu_6 \int_{\mathbb{R}^4\times\pi} A^{(5)} \wedge F\right] = 0\,. \tag{2.16}$$

Its solution gives the covariant derivatives for the main moduli, whose only non-trivial components are[5]:

$$\begin{cases} G^{(4)}_{\mu 6810} = \partial_\mu \sigma - 2\,N\,\mu_6\,n_1\,n_2\,n_3\,A_\mu = \partial_\mu\sigma - 2\,N\,\mu_6\,q^0\,A_\mu \\ G^{(4)}_{\mu 679} = -\partial_\mu \nu_1 - 2\,N\,\mu_6\,n_1\,m_2\,m_3\,A_\mu = -\partial_\mu\nu_1 - 2\,N\,\mu_6\,q^1\,A_\mu \\ G^{(4)}_{\mu 589} = -\partial_\mu \nu_2 - 2\,N\,\mu_6\,m_1\,n_2\,m_3\,A_\mu = -\partial_\mu\nu_2 - 2\,N\,\mu_6\,q^2\,A_\mu \\ G^{(4)}_{\mu 5710} = -\partial_\mu \nu_3 - 2\,N\,\mu_6\,m_1\,m_2\,n_3\,A_\mu = -\partial_\mu\nu_3 - 2\,N\,\mu_6\,q^3\,A_\mu \end{cases}. \tag{2.17}$$

Notice that the first line in eq. (2.17) comes actually from a higher-derivative term, since each n_A $(A = 1, 2, 3)$ in the expression for q^0 can be seen as the flux for the 1-form $d\phi$. This is just the analog of the Green-Schwarz [31] higher-derivative term, $\int B \wedge F \wedge F \wedge F \wedge F$, which gauges the shift symmetry of the universal axion in the heterotic theory. Thanks to the gauging, flat axionic directions of the effective potential involving the σ and ν_A fields can be removed, since the corresponding axions get absorbed into massive $U(1)$ vector fields, generalizing a known result for the heterotic string [32]. As noticed in [13], this mechanism allows to remove completely the residual axionic flat directions of the supersymmetric AdS_4 vacua, with all geometrical moduli stabilized, found in [11, 13]. From the covariant derivatives of eq. (2.17) we can extract the corresponding $U(1)$ Killing vectors:

$$\begin{aligned} i\,X^S &= -2\,N\,\mu_6\,n_1\,n_2\,n_3 = -2\,N\,\mu_6\,q^0\,, \\ i\,X^{U_A} &= 2\,N\,\mu_6\,n_A\,m_B\,m_C = 2\,N\,\mu_6\,q^A\,, \end{aligned} \tag{2.18}$$

where $A \neq B \neq C = 1, 2, 3$.

We can also look at the terms of the effective action quadratic in the vector field strengths, recalling that their standard form in $\mathcal{N} = 1$ supergravity is given by eq. (1.4).

[5] Notice that, strictly speaking, there are two sets of vector bosons, the first associated with the stack of branes and the second associated with the mirror stack, and that the gauge field appearing in eq. (2.17) is the antisymmetric combination of the corresponding $U(1)$ vectors, since the orthogonal combination is truncated away by the orientifold projection.

Reducing the DBI action, we get:

$$-\frac{1}{4}\tilde{e}_4\, N\, T_6 \sqrt{(\mathrm{Re}\,\widetilde{\Omega}_\pi)^2+(\mathrm{Im}\,\widetilde{\Omega}_\pi)^2}\, F^2 = -\frac{1}{4}\,\tilde{e}_4 \left(N\, T_6\, \mathrm{Re}\,\widetilde{\Omega}_\pi + s u_1 u_2 u_3\, V_D\right)\, F^2\,. \qquad (2.19)$$

Analogously, reducing the WZ action, we get:

$$\frac{1}{2} N\,\mu_6\,\left(m_1 m_2 m_3\,\sigma - \sum_{A=1}^{3} m_A n_B n_C\,\nu_A\right) F\wedge F = \frac{1}{2} N\,\mu_6\,\left(p_0\,\sigma - \sum_{A=1}^{3} p_A\,\nu_A\right) F\wedge F\,. \qquad (2.20)$$

At first sight the effective theory, as derived above from dimensional reduction, does not seem to match the general structure of $\mathcal{N}=1$ supergravity described in subsection 1.1. A disturbing fact is that the coefficient of F^2 in eq. (2.19) cannot be seen as the real part of a holomorphic function f, as in eq. (1.4), whose imaginary part would be fixed by eq. (2.20). Also, the V_D contribution to the potential is not of the form dictated by eqs. (1.6), (1.8) and (2.18). The reason of this apparent discrepancy is the fact that the DBI action includes higher-derivative terms. In the case of generic angles, i.e. of D6-branes wrapping a generic 3-cycle, the spontaneous breaking of $\mathcal{N}=1$ supersymmetry cannot be described within the standard 2-derivative formulation of $\mathcal{N}=1$ supergravity[6]. Anyway, the 2-derivative approximation was implicitly assumed when writing down the $D=10$ bulk effective action and reducing it to four dimensions. We must then look for a suitable limit in which higher-derivative terms coming from the DBI action can be neglected, and the D-brane contributions to the effective potential and to the kinetic terms for the gauge fields can be put in the standard $\mathcal{N}=1$ supergravity form.

Exploiting the gauge-invariance of the superpotential,[7] eq. (1.9), we can directly compute the D term through eq. (1.10), using the Kähler potential of eq. (1.2) and the Killing vectors of eq. (2.18):

$$D = N\,\mu_6\,\left(\frac{n_1 n_2 n_3}{s} - \sum_{A=1}^{3}\frac{n_A m_B m_C}{u_A}\right) = N\,\mu_6\,\left(\frac{q^0}{s} - \sum_{A=1}^{3}\frac{q^A}{u_A}\right)\,. \qquad (2.21)$$

Therefore eqs. (2.9) and (2.21) imply the identification

$$\mathrm{Im}\,\widetilde{\Omega}_\pi = \frac{\sqrt{s u_1 u_2 u_3}\; D}{N\, T_6}\,, \qquad (2.22)$$

and allow us to rewrite eq. (2.13) as

$$V_D = \frac{1}{2}\frac{1}{N\, T_6\;\mathrm{Re}\,\widetilde{\Omega}_\pi}\, D^2\, \frac{2}{\sqrt{1+\left(\frac{\mathrm{Im}\,\widetilde{\Omega}_\pi}{\mathrm{Re}\,\widetilde{\Omega}_\pi}\right)^2}+1}\,. \qquad (2.23)$$

For eq. (2.23) to be compatible with the standard formula of $\mathcal{N}=1$ supergravity, eq. (1.6), we must require that

$$\left|\frac{\mathrm{Im}\,\widetilde{\Omega}_\pi}{\mathrm{Re}\,\widetilde{\Omega}_\pi}\right| \ll 1\,. \qquad (2.24)$$

[6]For a discussion on how to include higher-derivative terms into the supergravity formalism, see [33].

[7]In section 3 we will show how this condition is guaranteed by localized Bianchi identities.

In the limit of eq. (2.24), also the generalized kinetic terms for the $U(1)$ vector field, eqs. (2.19) and (2.20), assume their standard $\mathcal{N}=1$ supergravity form, eq. (1.4), with:

$$\begin{aligned} f &= N\,T_6\ \left(m_1 m_2 m_3 S - m_1 n_2 n_3 U_1 - n_1 m_2 n_3 U_2 - n_1 n_2 m_3 U_3\right) \\ &= N\,T_6\ \left(p_0\,S - \sum_{A=1}^{3} p_A\,U_A\right), \end{aligned} \tag{2.25}$$

in agreement with the results of [17] in the special case of supersymmetric D6-branes. Now, observing that

$$\mathrm{Re}\,\widetilde{\Omega}_\pi = \frac{\mathrm{Re}\,f}{N\,T_6}\,, \tag{2.26}$$

and making use of eqs. (2.8) and (2.22), the limit in eq. (2.24) can also be rewritten as:

$$\frac{\hat{s}}{\mathrm{Re}\,f}\,D = g^2\,D\,Vol_6\,e^{-2\Phi} \ll 1\,, \qquad \text{or} \qquad g^2\,D\,M_P^2 \ll M_s^2\,, \tag{2.27}$$

where $Vol_6 = t_1\,t_2\,t_3$ is the volume of the internal manifold (in string units), $g^2 = (\mathrm{Re}\,f)^{-1}$, and we have reintroduced explicitly the string mass scale M_s and the four-dimensional Planck mass M_P in the second expression.

Besides the condition in eq. (2.24) we implicitly assumed that $\mathrm{Re}\,\widetilde{\Omega}_\pi > 0$ throughout the derivation of the effective action. This condition guarantees the softness of the supersymmetry breaking. In fact, if $\mathrm{Re}\,\widetilde{\Omega}_\pi \leq 0$ the tension term could not be reabsorbed into the F-term potential nor be interpreted as a D term. We illustrate the various situations in figure 1. Notice that the supersymmetric case of eq. (2.14) is actually disconnected (i.e. there is no soft limit) from the $\mathrm{Re}\,\widetilde{\Omega}_\pi < 0$ region, the origin being a singular point (vanishing D-brane volume).

We conclude this section with some general comments. We recall that, given the generic factorizable 3-cycle π of eq. (1.15), we can always associate to it an angle θ_A $(A = 1,2,3)$ in each factorized 2-torus:

$$\tan\theta_A = \frac{n_A}{m_A}\hat{u}_A = \frac{n_A}{m_A}\sqrt{\frac{u_B u_C}{u_A\,s}}\,, \qquad (A \neq B \neq C = 1,2,3)\,. \tag{2.28}$$

Brane configurations preserving $\mathcal{N}=1$ supersymmetry are defined by [15]:

$$\theta_1 + \theta_2 + \theta_3 = 0\,, \qquad (\mathrm{mod}\ 2\pi)\,, \tag{2.29}$$

or, equivalently:

$$\sum_{A=1}^{3} \tan\theta_A = \prod_{A=1}^{3} \tan\theta_A\,. \tag{2.30}$$

Notice that the above conditions correspond to the vanishing of $\mathrm{Im}\,\widetilde{\Omega}_\pi$ and therefore of V_D, as expected for supersymmetry-preserving configurations.

We can then distinguish among three different types of D6-brane configurations. Those wrapping 3-cycles with $q^I = 0$ $(I = 0,1,2,3)$ correspond to the four independent 3-cycles α^I of the O6-planes and are always supersymmetric as long as $\mathrm{Re}\,\widetilde{\Omega}_\pi > 0$ (i.e. $p_0 > 0$,

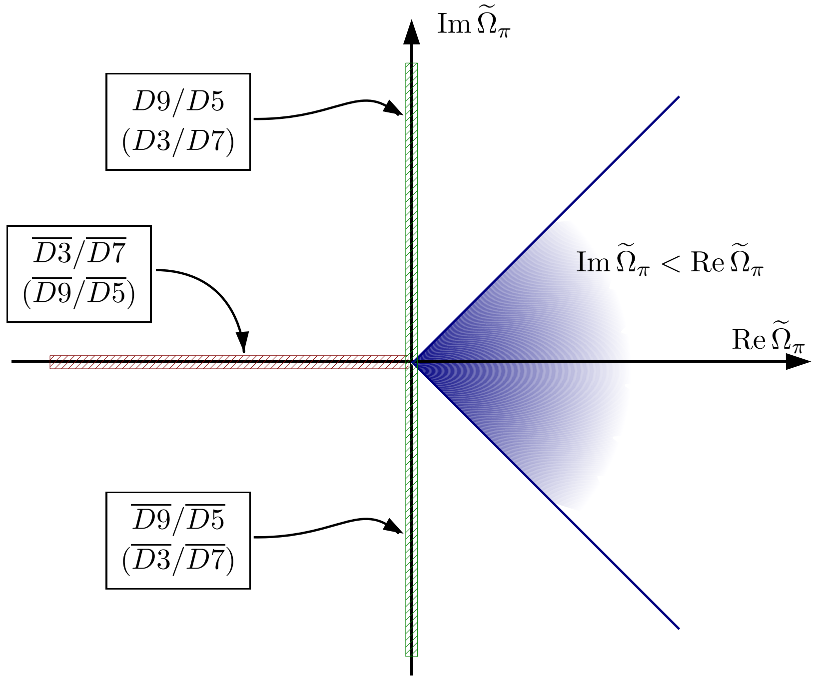

Figure 1: D6-brane configurations in the complex $\widetilde{\Omega}_\pi$ plane. Supersymmetric configurations lie on the $\mathrm{Re}\,\widetilde{\Omega}_\pi > 0$ axis. The D6-brane DBI action still allows a low energy supergravity description (see the text) in the shaded region where $\mathrm{Im}\,\widetilde{\Omega}_\pi < \mathrm{Re}\,\widetilde{\Omega}_\pi$. Outside that region supersymmetry is broken beyond the regime of validity of the effective theory. For D6-branes with either $\mathrm{Re}\,\widetilde{\Omega}_\pi = 0$ or $\mathrm{Im}\,\widetilde{\Omega}_\pi = 0$ and $\mathrm{Re}\,\widetilde{\Omega}_\pi < 0$, we indicate the corresponding brane setup in the T-dual type-IIB O3/O7 (O9/O5) orientifolds.

$p_A < 0$): the corresponding localized actions do contribute to the F-term potential but do not contribute to the D-term potential and, as we will see in the next section, are not constrained by localized BI. A more interesting class of D6-branes are those with components (p_I, q^I) along both even and odd 3-cycles. These D6-branes do contribute to D terms, can preserve $\mathcal{N} = 1$ supersymmetry only for specific values of the moduli, can produce chiral matter fields and can be used to absorb possible massless bulk axions via a supersymmetric Higgs effect. As will be discussed in the following sections, they can also play an important rôle in moduli stabilization via the associated D terms, but are strongly constrained by localized BI. Finally, we have the D6-brane configurations with $p_I = 0$ $(I = 0, 1, 2, 3)$. They do not allow the condition of eq. (2.24) to be satisfied for any value of the moduli and correspond to having a supersymmetry-breaking scale lying beyond the range of validity of the effective supergravity theory.

3. Localized Bianchi identities

In the previous section we showed how slightly decalibrated D6-branes can generate D terms depending on the dilaton and the complex structure moduli, associated to localized $U(1)$ vector multiplets that gauge some shift symmetry of the RR axions: the D-brane setup determines which axionic direction is gauged by the $U(1)$. We also stressed that $U(1)$ gauge invariance imposes some non-trivial conditions on D-brane configurations and fluxes: the effective superpotential W must be invariant under the gauged axionic $U(1)$, eq. (1.9). It was already noticed in [13] that, in the absence of geometrical fluxes, eq. (1.9) applied to the superpotential of eq. (1.29) can be understood in term of the cancellation of the Freed-Witten (FW) anomaly [25]:

$$\int_\pi \overline{H} = 0\,, \tag{3.1}$$

which can be interpreted in turn as the integrability condition for the BI of the localized field strength,

$$dF + H = 0\,, \tag{3.2}$$

evaluated on every 3-cycle π wrapped by D6-branes. Since geometrical fluxes can break some shift symmetries, we expect that in the presence of geometrical fluxes also eq. (3.1) is modified: we will now derive the appropriate modification, and relate it to the gauge invariance of the superpotential W.

In the presence of several stacks of D6-branes, the expression of eq. (2.17) for the $U(1)$-covariant derivatives of the RR axions generalizes to:

$$D_\mu\sigma = \partial_\mu\sigma - 2\sum_a N_a\,\mu_a\,q_a^0\,A_\mu^a\,, \qquad D_\mu\nu_A = \partial_\mu\nu_A + 2\sum_a N_a\,\mu_a\,q_a^A\,A_\mu^a\,, \quad (A=1,2,3)\,, \tag{3.3}$$

where the 3-cycles π_a, wrapping numbers (m_A^a, n_A^a) and four-vectors $(p_{a\,I}, q_a^I)$ are associated with the a-th stack of N_a D6-branes. Among all the $U(1)$ vectors appearing in eq. (3.3), at most four combinations can get a mass via the Stückelberg mechanism. This number, however, can be reduced in the presence of fluxes.

First we consider the case without geometrical fluxes. The $U(1)$ shift symmetries that can be gauged are only those that leave the superpotential W of eq. (1.29) invariant. Since they act on the fields as follows

$$\delta\Omega^c = i\,\delta C^{(3)} = i\,\chi\,, \tag{3.4}$$

where $\chi = \chi^I[\beta]_I$ is a constant 3-form, this means

$$\delta W = \frac{1}{4}\int \overline{H}\wedge\chi \quad \propto \quad \overline{H}_I\,\chi^I = 0\,, \tag{3.5}$$

which is a condition on the Killing vectors χ^I. Eq. (3.1) also reads

$$\int \overline{H}\wedge[\pi_a] = \overline{H}_I\,q_a^I = 0\,, \tag{3.6}$$

so that its solution q_a^I is also solution of eq. (3.5), i.e. $q_a^I \propto \chi^I$. In other words, the only D6-branes allowed by the localized BI are those satisfying eq. (3.6), which produce in eq. (3.3) a gauging compatible with the symmetries of the superpotential, eq. (3.5). In the absence of geometrical fluxes ω, only the flux $\overline{H}$ contributes to the FW anomaly cancellation condition, thus three axionic shift symmetries are always present and at least three independent $U(1)$ are needed if we want to absorb the corresponding axions via a supersymmetric Higgs effect.

We now discuss the condition that guarantees gauge invariance of the effective theory in the presence of geometrical fluxes. The constraint in eq. (3.5) now becomes:

$$\delta W = \frac{1}{4} \int (\overline{H} - i\omega J^c) \wedge \chi = 0\,, \tag{3.7}$$

and must be satisfied for every choice of the 2-form J^c [see eq. (1.31)]. Eq. (3.7) gives now four constraints that in general are independent. The first one is just eq. (3.5), while the other ones, using the fact that $J^c \wedge \chi = 0$, can be rewritten as

$$\int J \wedge \omega\,\chi = 0\,, \tag{3.8}$$

for every value of J in eq. (1.30), or, equivalently, as

$$\int_\gamma \omega\,\chi = 0\,, \tag{3.9}$$

for every holomorphic 4-cycle γ (5678, 78910, 56910).

Summarizing, eq. (3.7) gives four linear equations in the four variables χ^I. If these equations are independent, there are no Killing vectors. In this case, all RR axions are stabilized by fluxes and no shift symmetry can be gauged. This also means that D6-branes can only wrap $q_a^I = 0$ cycles, i.e. supersymmetric cycles, with identically vanishing D terms. In this case, however, no chiral fermions arise from D-brane intersections. When two or more equations are dependent, then at least one axionic symmetry is left and D terms are allowed. This is the case for the stable supersymmetric AdS_4 vacua found in [11], where all D terms vanish on the vacuum because of supersymmetry.

By looking at the localized BI of eq. (3.2), it is not clear how to derive the constraints on D6-branes needed to satisfy gauge invariance, eq. (3.7). The authors of ref. [13] proposed to extend the FW anomaly cancellation conditions of eq. (3.1) to

$$\int_\pi \overline{H} - i\omega J^c = 0\,, \tag{3.10}$$

evaluated on the vacuum. We now give a proof of this condition, using T-duality, directly from the localized BI of eq. (3.2). What we find, however, is a stronger condition: eq. (3.10) must be satisfied for any J^c, to ensure gauge invariance as in eq. (3.7).

We start by considering the localized BI in the type-IIB O9-orientifold with (magnetized) D9-branes on twisted tori:

$$dF + \omega F + H = 0\,. \tag{3.11}$$

The integrability condition reads:

$$\omega \overline{F} + \overline{H} = 0\,, \tag{3.12}$$

evaluated on every invariant 3-cycle. Because of the orientifold $\overline{H} = 0$, and without loss of generality we can rewrite this constraint as

$$\int_\Sigma \omega\,\overline{F} = \int (\omega\overline{F}) \wedge [\Sigma] = 0\,, \tag{3.13}$$

for every 3-cycle Σ. We thus have eight independent constraints, one for each independent 3-cycle (α^I, β_I). We now perform a T-dualization along the three directions of the torus that map the type-IIB O9-plane into the type-IIA O6-plane lying on the 3-cycle α^0. Under this T-duality, magnetized D9-branes map into D6-branes at angles. In particular, the magnetic fluxes in each 2-torus T^2_A will determine the D6-brane embedding via the relation

$$\int_{T^2_A} \overline{F} = \frac{n_A}{m_A}\,. \tag{3.14}$$

Therefore, T-duality maps $\overline{F}$ of eq. (3.13) into pull-backs (or pull-forwards) of type-IIA. In the NSNS sector, T-duality mixes geometrical fluxes ω with $\overline{H}$-fluxes and, in general, also with non-geometrical fluxes (see [4, 26]). Out of the eight constraints of eq. (3.13), four (corresponding to $\Sigma = \beta_0$ and $\Sigma = \alpha^A$) give constraints on the non-geometrical fluxes of type-IIA, and will be discussed below in subsection 3.1. We discuss now the other four constraints. In the case $\Sigma = \alpha^0$, T-duality transforms the components of ω of type-IIB into $\overline{H}$ of type-IIA, and eq. (3.13) into the constraint of eq. (3.1):

$$\int_\pi \overline{H} = 0\,, \tag{3.15}$$

where π is a D6-brane with wrapping numbers determined by eq. (3.14). Notice that not all the components of eq. (3.15) can be recovered from eq. (3.13), in fact the component $\overline{H}_0$ is lacking. The reason is that this component is T-dual to a non-geometrical flux of type-IIB, which we did not turn on in eq. (3.13). In the case $\Sigma = \beta_A$, ω is mapped into itself by T-duality and eq. (3.13) into

$$\int_\gamma \omega[\pi] = 0\,, \tag{3.16}$$

for each of the three invariant 4-cycles γ. Again, one component of ω is missing in each of the three conditions of eq. (3.16), because it would be T-dual to a non-geometrical flux of type-IIB, which was kept turned off in eq. (3.13). The condition in eq. (3.16) has actually a geometrical interpretation[8]. The geometrical fluxes ω change the topology of the internal manifold by removing some of the internal cycles. Eq. (3.16) is just the condition that the 3-cycle π wrapped by the D6-brane be a true cycle, in fact:

$$0 = \int_\gamma \omega[\pi] = \int_\gamma d[\pi] = -\int_\pi d[\gamma] = -\int_{\partial\pi} [\gamma] \;\Rightarrow\; \partial\pi = \emptyset\,. \tag{3.17}$$

The 1+3 constraints of eqs. (3.15) and (3.16) can thus be rewritten as follows:

$$\int_\pi (\overline{H} - i\,\omega J^c) = 0\,, \tag{3.18}$$

[8]We thank Alessandro Tomasiello for a discussion on this point.

which, repeating the discussion given in the absence of geometrical fluxes, is precisely what we need to require that only axionic directions leaving the superpotential invariant can be gauged.

We can now understand how $U(1)$ D terms for the (s, u_1, u_2, u_3) moduli are strongly constrained by localized BI. Gauge invariance of the flux-induced superpotential implies

$$D_a = -2\, N_a\, \mu_6\, (q_a^0\, K_S - q_a^A K_{U_A})\,, \tag{3.19}$$

with strong constraints linking the fluxes and the q^I, coming from localized BI. For instance, in the case of generic geometrical fluxes only one axionic shift symmetry is preserved by W, thus the vector q_a^I is determined up to a normalization factor, and the form of the D term is completely specified by the bulk fluxes, without need of making explicit reference to D-brane data!

3.1 Consistency conditions for non-geometrical fluxes

We showed above that, by applying T-duality to the localized BI in the type-IIB theory, we get, besides the constraints for $\overline{H}$ and ω, additional consistency relations connecting the D-brane setup with non-geometrical fluxes. The latter show up when T-duality is applied to the usual NSNS 3-form fluxes, as follows [26]:

$$\overline{H}_{mnr} \overset{\mathcal{T}_r}{\longleftrightarrow} \omega_{mn}{}^{r} \overset{\mathcal{T}_n}{\longleftrightarrow} Q_m{}^{nr} \overset{\mathcal{T}_m}{\longleftrightarrow} R^{mnr}\,, \tag{3.20}$$

where $\mathcal{T}_{r,m,n}$ indicates a T-dualization in the (r, m, n) internal direction.

Consider now the cases $\Sigma = \beta_0$ and $\Sigma = \alpha^A$ in eq. (3.13), which were neglected before. They transform ω into R and Q fluxes, respectively, and map eq. (3.13) into the 1+3 conditions

$$\begin{aligned} \Sigma = \beta_0 :& \qquad R\,[\pi] = 0\,, \\ \Sigma = \alpha^A :& \qquad \int [\gamma] \wedge Q\,[\pi] = 0\,, \quad (\forall\, \gamma)\,, \end{aligned} \tag{3.21}$$

where $R\,[\pi]$ and $Q\,[\pi]$ are a 0-form and a 2-form whose components read:

$$R\,[\pi] = R^{mnr}\,[\pi]_{mnr}\,, \qquad (Q\,[\pi])_{mn} = Q_{[m}{}^{rs}\,[\pi]_{rsn]}\,. \tag{3.22}$$

More generally, when applied to a p-form X, R and Q decrease its rank by 3 and 1, respectively. In our notation:

$$\begin{aligned} (R\,X)_{r_1\cdots r_{p-3}} &= R^{s_1\,s_2\,s_3} X_{s_1\,s_2\,s_3\,r_1\cdots r_{p-3}}\,, \\ (Q\,X)_{r_1\cdots r_{p-1}} &= Q_{r_1}{}^{s_1\,s_2} X_{s_1\,s_2\,r_2\cdots r_{p-1}} + \text{permut.} \end{aligned} \tag{3.23}$$

Notice also that, for consistency, the intrinsic orientifold parities for the NSNS fluxes must be chosen to be:

$$\overline{H}\ (-)\,, \qquad \omega\ (+)\,, \qquad Q\ (-)\,, \qquad R\ (+)\,. \tag{3.24}$$

Eqs. (3.15), (3.16) and (3.21) can also be rewritten in a compact form as

$$\int e^{iJ}(\overline{H}+\omega+Q+R)[\pi]$$
$$=\int \overline{H}\wedge[\pi]+i\,J\wedge\omega[\pi]-\frac{1}{2}J\wedge J\wedge Q[\pi]-\frac{i}{3!}\,J\wedge J\wedge J\,R[\pi]=0\,,\quad (\forall\,J)\,. \tag{3.25}$$

The constraints of eq. (3.21) are consistency conditions for the simultaneous presence of D-branes and non-geometrical fluxes, and must be added to the constraints from the bulk BI found in [26] when these type of fluxes are considered. Still they guarantee that the modified non-geometrical superpotential:

$$W=\frac{1}{4}\int e^{iJ^c}\left(\overline{\mathbf{G}}-i\,(\overline{H}+\omega+Q+R)\Omega^c\right) \tag{3.26}$$

be invariant under the axionic shift symmetries gauged by the D6-branes.

Analogously, we can start from eq. (3.25) and T-dualize it back to type-IIB, to extend eq. (3.13) to non-geometrical fluxes. This case will be discussed further in section 4.

3.2 Generalized Bianchi identities and gauge anomaly cancellation

Localized BI are important not only to guarantee the tree-level gauge invariance of the superpotential but, together with the bulk BI, they also play a crucial rôle in the cancellation of the gauge anomalies. To see how anomaly cancellation works in the presence of fluxes[9], we can generalize the standard proof (see e.g. [2, 34]) valid in the absence of fluxes.

In toroidal compactifications, the cubic $SU(N_a)^3$ anomaly $\mathcal{A}_a$ is proportional to the number of chiral fermions charged with respect to $SU(N_a)$, i.e. to the sum of all D-brane intersections with the D-brane stack a, namely:

$$\mathcal{A}_a=\sum_b I_{ab}N_b\,, \tag{3.27}$$

where the intersection number,

$$I_{ab}=\int[\pi_a]\wedge[\pi_b]=p_{a\,I}\,q_b^I-q_a^I\,p_{b\,I}\,, \tag{3.28}$$

counts the number of intersections with the b-th stack of D-branes. Notice that, in order to have chiral matter, i.e. $I_{ab}\neq 0$, at least one stack of D-branes must have $q^I\neq 0$, thus a non-trivial D term.

In constructions involving orbifolds and orientifolds, eq. (3.27) is modified into:

$$\mathcal{A}_a=\sum_b \mu_b I_{ab}N_b\,, \tag{3.29}$$

where the sum runs over both D-branes and O-planes. This is due to the contributions from non-fundamental representations induced by the O-planes (see e.g. [34]). By using the bulk BI in the presence of fluxes, eq. (1.28), the anomaly reads

$$\mathcal{A}_a=\int[\pi_a]\wedge\sum_b\mu_bN_b[\pi_b]=\frac{1}{2}\int[\pi_a]\wedge\left(\omega\overline{G}^{(2)}+\overline{G}^{(0)}\overline{H}\right)\,, \tag{3.30}$$

[9] We thank Angel Uranga for discussions on this point.

or equivalently

$$\mathcal{A}_a \propto \int_{\pi_a} \left(\omega\,\overline{G}^{(2)} + \overline{G}^{(0)}\overline{H}\right) \,, \tag{3.31}$$

which vanishes because of the localized BI (3.18).

After having imposed localized and bulk BI as above, also mixed $U(1)$-$SU(N_a)^2$ and $U(1)^3$ gauge anomalies cancel via the generalized Green-Schwarz mechanism [34, 2, 3], arising from the gauging of the shift symmetries discussed before.

4. Extensions to other $\mathcal{N} = 1$ compactifications

The results obtained in the previous sections were derived explicitly in a particular example, the $T^6/(Z_2 \times Z_2)$ orbifold of the type-IIA O6 orientifold. However, they are also valid in more general compactifications. This is due to the fact that the structure of D terms and BI is mainly determined by gauge invariance and supersymmetry.

An example of the previous statement is the superpotential of eq. (1.29). Derived explicitly in the $T^6/(Z_2 \times Z_2)$ orbifold case [11], it seems to hold for all known geometrical $\mathcal{N} = 1$ type-IIA compactifications [4, 10 − 13]. The details of each specific compactification only affect the number of active moduli inside Ω^c and J^c, and the allowed components for the fluxes $\overline{\mathbf{G}}$, $\overline{H}$ and ω. Analogously, the general formulae for the bulk (eq. (1.28)) and localized (eqs. (3.15) and (3.16)) BI apply to every $\mathcal{N} = 1$ compactification of the type-IIA theory on the O6 orientifold. The particular flux and O6-plane content are, however, model-dependent. Finally, from supersymmetry and gauge invariance, we know that D terms are determined just by the Killing vectors and by the Kähler potential (eq. (1.10)). In particular, the former are fixed by the RR-couplings of eq. (2.15). When the latter reduces to a form analogous to eq. (1.2), as in [4, 10 − 13], then the functional dependence of the D terms on the closed string moduli is fixed to be the one of eq. (2.21).

In this way, the connection between BI, gauge invariance of the superpotential, anomaly cancellation and D terms easily extends beyond the specific orbifold and orientifold explicitly considered in the previous sections. The same arguments can be used to show that the extensions of the localized BI (eq. (3.25)) and of the superpotential (eq. (3.26)) to non-geometrical fluxes must hold in general.

The discussion can also be extended to type-IIB compactifications. D terms have already been computed, in compactifications with magnetized branes, in a number of cases [19 − 21]. Since the derivation is analogous to the type-IIA case, we will not repeat it here. We just give the results, focusing on the intriguing connection between gauge invariance and localized BI.

In the type-IIB case, the localized BI can be obtained by mirror symmetry from our eq. (3.25), and read:

$$\overline{H} + \omega\,\overline{F} + \frac{1}{2} Q\,(\overline{F} \wedge \overline{F}) + \frac{1}{3!} R\,(\overline{F} \wedge \overline{F} \wedge \overline{F}) = 0\,, \tag{4.1}$$

evaluated on every 3-cycle wrapped by a D-brane. The indices of Q- and R-fluxes are saturated as described in eq. (3.23). Without non-geometrical fluxes, eq. (4.1) reduces to eq. (3.12). Moreover, only part of the terms of eq. (4.1) survive the orientifold projections. In particular, in O9/O5 compactifications $\overline{H}$ and Q vanish, while in O3/O7 compactifications ω and R vanish.

In the O9/O5 case, D9-branes fill the whole ten-dimensional space and, as expected, the localized BI correspond to the bulk BI of the type-I or heterotic theory [35]:

$$\omega \overline{F} = 0 \,. \tag{4.2}$$

This equation is precisely what is needed for the effective superpotentials of the type-IIB O9 and heterotic [36, 4] theories,

$$\begin{aligned} W_{IIB(O9)} &\propto \int \left(\overline{G}^{(3)} - i\omega J^c\right) \wedge \Omega^c \,, \\ W_{Het} &\propto \int \left(\overline{H} - i\omega J^c\right) \wedge \Omega^c \,, \end{aligned} \tag{4.3}$$

to be gauge invariant under the shift symmetries

$$J^c \to J^c + X \,. \tag{4.4}$$

In fact, the Killing vectors $X = i\overline{F}$ are associated to the gauging of the shift symmetry produced by the coupling

$$\int A^{(6)} \wedge \overline{F} \wedge F \,, \tag{4.5}$$

where $A^{(6)}$ is the RR 6-form $C^{(6)}$ dual to $C^{(2)} = \operatorname{Im} J^c$ in the type-IIB O9 theory, or the NSNS 6-form $B^{(6)}$ dual to $B^{(2)} = \operatorname{Im} J^c$ in the heterotic case. In both cases, geometrical fluxes do not produce a potential for the universal axion $\operatorname{Im} S = C^{(6)}$ $(B^{(6)})$ of the type-IIB (heterotic) theory, which is gauged by the Green-Schwarz term

$$\int A^{(2)} \wedge \overline{F} \wedge \overline{F} \wedge \overline{F} \wedge F \,. \tag{4.6}$$

On the other hand, the corresponding Killing vector $\overline{F} \wedge \overline{F} \wedge \overline{F}$ is constrained by the non-geometrical R-fluxes, which in fact induce superpotentials for the chiral superfield containing the axion:

$$\begin{aligned} W_{IIB(O9)} &\propto \int (\overline{G}^{(3)} - i\,\omega\, J^c - i\, R\, \tilde{S}) \wedge \Omega^c \,, \qquad \tilde{S} \doteq \star_{(6)} S \,, \\ W_{Het} &\propto \int (\overline{H} - i\,\omega\, J^c - i\, R\, \tilde{S}) \wedge \Omega^c \,. \end{aligned} \tag{4.7}$$

When interpreted in terms of an underlying gauged $\mathcal{N} = 4$ supergravity, these superpotentials exhibit a non-trivial de Roo-Wagemans phase [37]. In the general case, with both ω and R fluxes turned on, eq. (4.1) guarantees that the combination of axions gauged by fluxes is associated to a flat direction of the superpotential.

As in the type-IIA case, D terms can be derived taking the DBI action and performing the same supergravity limit of eq. (2.27). The result is just the mirror symmetric of the type-IIA result of eq. (2.21), where the wrapping numbers are replaced by magnetic fluxes via eq. (3.14) and the U and T moduli are exchanged, i.e.

$$D \propto \left(\frac{n_1 \, n_2 \, n_3}{s} - \sum_{A=1}^{3} \frac{n_A m_B m_C}{t_A} \right) = m_1 m_2 m_3 \left(\frac{\prod_{A=1}^{3} \overline{F}_A}{s} - \sum_{A=1}^{3} \frac{\overline{F}_A}{t_A} \right) . \tag{4.8}$$

The linear terms in $\overline{F}$ can also be derived by generalized dimensional reduction, using the standard heterotic action. The cubic term, however, corresponds to higher-derivative contributions, as the associated Green-Schwarz term of eq. (4.6).

In the case of type-IIB O3/O7 orientifolds, only $\overline{H}$ and Q fluxes can be turned on. Without non-geometrical fluxes, the effective superpotential [38],

$$\int (\overline{G}^{(3)} - i \, S \, \overline{H}) \wedge \Omega^c \, , \tag{4.9}$$

produces only a potential for the dilaton S and the complex structure moduli U, while the Kähler moduli T remain undetermined. Accordingly, the localized BI of eq. (4.1) require the vanishing of the $\overline{H}$ flux over all possible 3-cycles wrapped by a D-brane. This condition is actually satisfied trivially by all branes but magnetized D9-branes, which in fact gauge the shift symmetry of the axion $\mathrm{Im}\, S = C^{(0)}$ through the coupling

$$\int C^{(8)} \wedge F \, . \tag{4.10}$$

This means that magnetized D9-branes cannot coexist with $\overline{H}$-fluxes, unless also non-geometrical fluxes are considered (see below). As before, non-geometrical fluxes modify both the superpotential, which now reads

$$\int (\overline{G}^{(3)} - i \, S \, \overline{H} - i Q \tilde{J}^c) \wedge \Omega^c \, , \qquad \tilde{J}^c \doteq \star_{(6)} J^c \, , \tag{4.11}$$

and the localized BI [see eq. (4.1)]. This produces a potential for the T moduli in J^c, which is however compatible with the gauging of the shift symmetries associated to J^c and generated by magnetized D9- and D7-branes via the couplings

$$\begin{aligned} D9 : & \int C^{(4)} \wedge \overline{F} \wedge \overline{F} \wedge F \, , \\ D7 : & \int C^{(4)} \wedge \overline{F} \wedge F \, . \end{aligned} \tag{4.12}$$

Again, in the presence of both $\overline{H}$ and Q fluxes the localized BI guarantee that the axionic directions gauged by magnetized D-branes are associated to shift symmetries of the effective superpotential. Finally, D terms have the form:

$$D \propto \left(\frac{m_1 m_2 m_3}{s} - \sum_{A} \frac{m_A n_B n_C}{t_A} \right) , \tag{4.13}$$

with m's and n's exchanged with respect to the former cases because of T-duality. The first contribution comes from D9-branes [which must be magnetized in order to satisfy eq. (2.27)], while the others come from magnetized D7- and D9-branes.

Notice that also in the type-IIB case, after the appropriate field redefinitions, the D terms are homogeneous functions of (s, t_1, t_2, t_3). Mechanisms for moduli stabilization via D terms that rely on a different functional dependence on the geometrical moduli cannot be consistently implemented, at least as long as matter fields are neglected.

Summarizing, we can see that, independently of the choice of the ten-dimensional theory and of the details of the compactification, there is a precise correspondence between gauge invariance of the effective flux superpotential and localized BI. This translates into strong consistency conditions for the coexistence of fluxes and D terms. Remarkably, the same connection holds also in compactifications with non-geometrical fluxes.

In analogy with the type-IIA case discussed in section 3.2, also in the type-IIB case gauge chiral anomalies cancel automatically when both bulk and localized BI are satisfied. The proof can be derived along the line of section 3.2 and holds also when non-geometrical fluxes are turned on.

All these results can have important consequences for string model-building. For instance, in O3/O7 type-IIB compactifications on the $T^6/(Z_2 \times Z_2)$ orbifold, it is a standard lore to add magnetized D9-branes, to avoid the positivity bounds on the bulk BI coming from flux quantization. However, according to the discussion above, these types of branes are incompatible with $\overline{H}$ fluxes in geometrical compactifications[10].

5. D terms and moduli stabilization

In the previous sections we showed how D terms arising from D-branes can be embedded, under some well-defined consistency conditions, into the standard formalism of $\mathcal{N} = 1$, $D = 4$ supergravity. We now comment on the rôle of $U(1)$ D terms, with their specific dependence on the geometrical moduli, for the problem of bulk moduli stabilization in string compactifications with fluxes. For definiteness we refer, as before, to the $T^6/(Z_2 \times Z_2)$ orbifold and the $(-1)^{F_L}\Omega I_3$ orientifold of the type-IIA theory, but the results have more general validity. For the sake of clarity, and for an easier use of our results for model building, we collect some of the formulae of sections 2 and 3, translated into a more explicit notation, in the appendix.

A property of the stable AdS_4 supersymmetric vacua found in the flux compactifications of [11, 13] is the existence of modes with negative squared masses. Their presence is not a problem, because they satisfy the Breitenlohner-Freedman bound [39] that controls stability in AdS spaces. These "AdS tachyons" are usually given by the linear combinations of geometrical moduli "parallel" to the linear combinations of axions gauged by the $U(1)$ D6-brane vectors. As explained in section 2, this dependence has a precise relation

[10]This inconsistency was already noticed in [20], where it was proposed to overcome the problem by adding fractional D5-branes, acting as sources for the localized BI. This modification, however, cannot be introduced without additional modifications to the model, because of the connection between gauge invariance of the superpotential, anomaly cancellation, bulk and localized BI.

with the functional dependence of the $U(1)$ D terms on the geometrical moduli. In this case, therefore, there can be a twofold rôle played by the $U(1)$ gauge interactions associated with D6-branes and by the corresponding D terms, for the problem of bulk moduli stabilization. On the one hand, the gauged axionic flat directions of the F-term potential are removed via the Stückelberg mechanism: a supersymmetric Higgs effect takes place and the axion provides the longitudinal degree of freedom of the massive $U(1)$ vector. On the other hand, D terms can provide positive-definite contributions to the tachyonic squared masses and, if allowed by flux quantization and by the full set of BI, they may even push these masses to positive values. This may happen also in the case of non-supersymmetric AdS_4 vacua, but it is more easily discussed in the case of supersymmetric AdS_4 vacua. For these solutions, in fact, D terms vanish on the vacuum, because of supersymmetry, and the D-term contribution to the mass matrix of the geometrical moduli can be easily computed. In general, working in the complex basis of the seven main moduli, and taking into account that the D-term potential depends only on the geometrical moduli and not on the axions, we can write for the normalized mass matrix:

$$M^2_{kl} = M^2_{\overline{kl}} = M^2_{k\overline{l}} = K^{1/2}_{\overline{m}k} K^{1/2}_{\overline{l}n} [(Re\, f)^{-1}]^{ab} X^m_a \overline{X}^{\overline{n}}_b \,. \tag{5.1}$$

In the representative case under consideration, and around a vacuum with $\langle D\rangle = 0$, where $\langle s\rangle = s_0$ and $\langle u_A\rangle = u_{A\,0}$ $(A = 1, 2, 3)$, with $q^0_a/s_0 = \sum_{A=1}^3 (q^A_a/u_{A\,0})$, the D-term contribution to the (normalized) mass matrix for the fields (s, u_1, u_2, u_3) reads:

$$\mathcal{M}^2_{(D)} = \frac{N\,T_6}{p_0 s_0 - \sum_{A=1}^3 p_A u_{A\,0}} \begin{pmatrix} \frac{(q^0)^2}{s_0^2} & -\frac{q^0\,q^A}{s_0\,u_{A\,0}} \\ -\frac{q^0\,q^A}{s_0\,u_{A\,0}} & -\frac{q^A\,q^B}{u_{A\,0}\,u_{B\,0}} \end{pmatrix} . \tag{5.2}$$

The only non-vanishing eigenvalue is:

$$m^2_{(D)} = \frac{N\,T_6}{p_0 s_0 - \sum_{A=1}^3 p_A u_{A\,0}} \left(\frac{(q^0)^2}{s_0^2} + \sum_{A=1}^3 \frac{(q^A)^2}{u^2_{A\,0}} \right) , \tag{5.3}$$

and the corresponding eigenvector is

$$\frac{q^0}{s_0^2} s - \sum_{A=1}^3 \frac{q^A}{u^2_{A\,0}} u_A \,, \tag{5.4}$$

whose direction in the space of geometrical moduli is linked to the one of the gauged axions. We then see that, even though D terms can be consistently neglected in the search for supersymmetric vacuum configurations, they do play an important rôle in the computation of the moduli spectrum.

We finally discuss how D terms can remove some F-flat directions for the geometrical moduli in the case of Minkowski vacua. An important class of these vacua is given by no-scale models, where supersymmetry is broken for all field configurations along one or more flat directions of the scalar potential. The simplest way to obtain a no-scale model in the present context is to introduce a superpotential that does not depend on three of the seven main moduli, but depends non-trivially on the remaining four, so that the corresponding auxiliary fields can relax to zero on the vacuum. Using the index $\tilde{k}$ ($\hat{k}$) for the complex moduli that do (not) appear in the superpotential, we have in formulae: $W_{\hat{k}} \equiv 0$, so that $G^{\hat{k}}G_{\hat{k}} \equiv K^{\hat{k}}K_{\hat{k}} \equiv 3$, and $\langle G_{\tilde{k}} \rangle = 0$. There are at least three geometrical moduli that are left to be stabilized, but at most three axionic shift symmetries that can be gauged and give rise to three independent D terms: in the present class of models, the only other axionic symmetry that could be gauged is broken by the superpotential. Since D terms are homogeneous functions of the geometrical moduli, with three D terms there is no way of stabilizing three geometrical moduli to non-zero values in a Minkowski background. With the inclusion of matter fields the analysis is more delicate since, in general, the assumption of complete factorizability of the Kähler manifold must be relaxed. However, for no-scale models realized with vanishing F terms for the matter fields, the latter do not contribute to D terms along the flat directions and the previous arguments still apply. We have checked on a number of examples that the above discussion seems to extend also to no-scale superpotentials with an explicit dependence on more than four out of the seven main moduli, once the various consistency conditions are taken into account.

Another interesting class of Minkowski vacua are those where supersymmetry is preserved. As usual, $U(1)$ D terms may be added if the corresponding axionic shift symmetries are preserved by the superpotential. In a supersymmetric Minkowski background, since $\langle W \rangle = 0$ and $\langle W_i \rangle = 0$, axionic shift symmetries get complexified [40], so that the corresponding geometrical moduli are not stabilized by the F-term potential either. As discussed before, D terms could just remove these flat directions of the scalar potential. However, even though we did not perform an exhaustive search, we are not aware of any explicit example, with all the consistency conditions satisfied, where all the main moduli are stabilized in this way.

6. Conclusions and outlook

In this paper we studied the rôle of D terms in string compactifications with D-branes and fluxes, preserving an exact or spontaneously broken $\mathcal{N} = 1$ supersymmmetry in the effective field theory limit. In particular, we neglected the matter field fluctuations, and focused on the D-brane contributions to the potential for the closed string moduli, which we derived explicitly in the case of type-IIA O6 orientifold compactifications on the $T^6/(Z_2 \times Z_2)$ orbifold. We clarified under which assumptions the D-term breaking of supersymmetry can be considered soft, and how the effective D-brane actions can be embedded into the standard $\mathcal{N} = 1$ supergravity formalism. The D terms depending on the closed string moduli are associated to the $U(1)$ gaugings that act as shift symmetries on the RR axions. The latter can thus be absorbed by the brane vector fields, which become massive. The structure becomes more involved and more interesting when also bulk fluxes are considered. We showed that there is a strong connection between supersymmetry, gauge invariance, anomaly cancellation, Bianchi identities, flux superpotentials and D terms. The crucial point is the existence of constraints connecting D-branes and fluxes, which guarantee the consistency of the effective theory both at the classical and at the quantum level. We derived these

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constraints using localized Bianchi identities and T-duality, for $\mathcal{N} = 1$ compactifications with generic RR, NSNS, geometrical and also non-geometrical fluxes. These constraints dictate how D-branes can be embedded into a flux compactification. The resulting effective action is automatically consistent with the gauging associated to the brane vector fields, and has vanishing gauge anomalies. Also, D terms appear to be highly constrained by bulk fluxes, which in some cases are sufficient to completely specify their form. Our results are particularly relevant for string model building with flux compactifications, since they provide important constraints on these constructions.

We also discussed the rôle of D terms for the problem of moduli stabilization. As shown in the text, the D-term contribution to the moduli spectrum is always non-vanishing in all string compactifications where the chiral fields arise from intersecting (or magnetized) D-branes, even in the supersymmetric cases where D terms vanish on the vacuum. We pointed out that, although D terms help removing residual flat directions of the F-term scalar potential, in the chosen context there are some difficulties in reaching full stabilization of the closed string moduli in Minkowski or de-Sitter vacua. A systematic study of this aspect, however, goes beyond the purpose of the present paper.

There are various directions along which the results of the present paper could be further developed.

An important step would be the incorporation of the chiral superfields, corresponding to open string fluctuations living on branes or at brane intersections, into the effective $\mathcal{N} = 1$ supergravity. In particular, fields leaving at brane intersections are charged with respect to the gauged $U(1)$s: in the present paper they have been neglected, but a more detailed analysis of their possible impact is certainly needed, especially if we want to study supersymmetry-breaking vacua with non-vanishing vacuum expectation values for both F and D terms. Some results on the Kähler potential for these "matter" fields are already available [41]. It would be interesting to explore systematically, taking into account all available consistency constraints, whether the inclusion of matter fields can lead to new possibilities that do not seem to be realized in their absence: for example, full moduli stabilization in Minkowski or de-Sitter vacua.

Last but not least, it would be interesting to include the effects of warping in the derivation of the effective supergravity, and see whether they can introduce any qualitatively new ingredients in the discussion of supersymmetry breaking, moduli stabilization and the generation of hierarchies.

Note added. When the present paper was ready for submission, a new paper appeared [42], whose results partially overlap with our results on the effective superpotentials with non-geometrical fluxes of sections 3.1 and 4.

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A. Explicit results for the type-IIA $T^6/(Z_2 \times Z_2)$ orbifold with O6-planes

We recall here the main formulae of sections 2 and 3, valid for flux compactifications of the type-IIA theory on the $T^6/(Z_2 \times Z_2)$ orbifold and O6 orientifold, in a more explicit notation, so that they can be readily and unambiguously used for model-building.

The general form for the effective flux superpotential in $\mathcal{N}=1$ type-IIA compactifications was given in eq. (1.29). In components, and for the class of models under consideration, we can also write:

$$\begin{aligned}
4\,W = &\left(\omega_{810}{}^{6}\,T_1\,U_1 + \omega_{106}{}^{8}\,T_2\,U_2 + \omega_{68}{}^{10}\,T_3\,U_3\right) - S\left(\omega_{79}{}^{6}\,T_1 + \omega_{95}{}^{8}\,T_2 + \omega_{57}{}^{10}\,T_3\right) \\
&-\left(\omega_{89}{}^{5}\,T_1\,U_3 + \omega_{96}{}^{7}\,T_2\,U_3 + \omega_{710}{}^{5}\,T_1\,U_2 + \omega_{67}{}^{9}\,T_3\,U_2 + \omega_{105}{}^{7}\,T_2\,U_1 + \omega_{58}{}^{9}\,T_3\,U_1\right) \\
&+i\left(\overline{G}^{(4)}_{78910}\,T_1 + \overline{G}^{(4)}_{91056}\,T_2 + \overline{G}^{(4)}_{5678}\,T_3\right) - \left(\overline{G}^{(2)}_{56}\,T_2\,T_3 + \overline{G}^{(2)}_{78}\,T_1\,T_3 + \overline{G}^{(2)}_{910}\,T_1\,T_2\right) \\
&+\overline{G}^{(6)} - i\,\overline{G}^{(0)}\,T_1\,T_2\,T_3 + i\left(\overline{H}_{579}\,S - \overline{H}_{5810}\,U_1 - \overline{H}_{6710}\,U_2 - \overline{H}_{689}\,U_3\right).
\end{aligned} \tag{A.1}$$

Fluxes are constrained by the bulk and localized BI, which in the specific orbifold and orientifold under consideration give the following non-trivial relations:

$$\omega_{mn}{}^{p}\,\omega_{pr}{}^{s} = 0\,, \tag{A.2}$$

$$\begin{cases}
\frac{1}{2}\left(\omega\,\overline{G}^{(2)} + \overline{H}\,\overline{G}^{(0)}\right)_{579} = \frac{1}{2}\sum_a N_a\,p_{a\,0} - 16\,, \\
\frac{1}{2}\left(\omega\,\overline{G}^{(2)} + \overline{H}\,\overline{G}^{(0)}\right)_{5810} = \frac{1}{2}\sum_a N_a\,p_{a\,1} + 16\,, \\
\frac{1}{2}\left(\omega\,\overline{G}^{(2)} + \overline{H}\,\overline{G}^{(0)}\right)_{6710} = \frac{1}{2}\sum_a N_a\,p_{a\,2} + 16\,, \\
\frac{1}{2}\left(\omega\,\overline{G}^{(2)} + \overline{H}\,\overline{G}^{(0)}\right)_{689} = \frac{1}{2}\sum_a N_a\,p_{a\,3} + 16\,,
\end{cases} \tag{A.3}$$

$$\begin{cases}
\overline{H}_{579}\;q_a^0 + \overline{H}_{5810}\;q_a^1 + \overline{H}_{6710}\;q_a^2 + \overline{H}_{689}\;q_a^3 = 0\,, \\
-\omega_{79}{}^{6}\,q_a^0 - \omega_{810}{}^{6}\,q_a^1 + \omega_{710}{}^{5}\,q_a^2 + \omega_{89}{}^{5}\,q_a^3 = 0\,, \\
-\omega_{95}{}^{8}\,q_a^0 + \omega_{105}{}^{7}\,q_a^1 - \omega_{106}{}^{8}\,q_a^2 + \omega_{96}{}^{7}\,q_a^3 = 0\,, \\
-\omega_{57}{}^{10}\,q_a^0 + \omega_{58}{}^{9}\,q_a^1 + \omega_{67}{}^{9}\,q_a^2 - \omega_{68}{}^{10}\,q_a^3 = 0\,.
\end{cases} \tag{A.4}$$

where the sums run over the stacks of D6-branes and their images. Besides the Kähler potential and the superpotential of eqs. (1.2) and (A.1), the remaining ingredients needed to reconstruct the scalar potential of eq. (1.6) are

$$\mathrm{Re}\,f_{ab} = \delta_{ab} N_a\,T_a\;\left(p_{a\,0}\,s - p_{a\,1}\,u_1 - p_{a\,2}\,u_2 - p_{a\,3}\,u_3\right)\,, \tag{A.5}$$

$$D_a = N_a\,T_a\left(\frac{q_a^0}{s} - \frac{q_a^1}{u_1} - \frac{q_a^2}{u_2} - \frac{q_a^3}{u_3}\right)\,, \tag{A.6}$$

and the supergravity description is valid as long as eq. (2.24) is satisfied,

$$\sqrt{s u_1 u_2 u_3}\left(\frac{q_a^0}{s} - \frac{q_a^1}{u_1} - \frac{q_a^2}{u_2} - \frac{q_a^3}{u_3}\right) \ll \left(p_{a\,0}s - p_{a\,1}u_1 - p_{a\,2}u_2 - p_{a\,3}u_3\right)\,. \tag{A.7}$$

When this condition is not satisfied (large angles/D terms, anti D-branes, ...), the scale of supersymmetry breaking lies outside the range of validity of the effective field theory, which cannot be described anymore by the supergravity formalism above.

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