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Superpotentials in IIA compactifications with general fluxes

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Abstract

We derive the effective $N = 1$, $D = 4$ supergravity for the seven main moduli of type IIA orientifolds with D6-branes, compactified on $T^6/(Z_2 \times Z_2)$ in the presence of general fluxes. We illustrate and apply a general method that relates the $N = 1$ effective Kähler potential and superpotential to a consistent truncation of gauged $N = 4$ supergravity. We identify the correspondence between various admissible fluxes, $N = 4$ gaugings and $N = 1$ superpotential terms. We construct explicit examples with different features: in particular, new IIA no-scale models and a model which admits a supersymmetric AdS_4 vacuum with all seven main moduli stabilized.

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1. Introduction

Compactifications of superstrings and M-theory[3] may lead to four-dimensional vacua with exact or spontaneously broken supersymmetries. The pattern of residual and broken supersymmetries strongly depends on the set of moduli fields predicted by the compactification geometry and on the detailed dynamics of these moduli. Even for the phenomenologically attractive compactifications with spontaneously broken $N = 1$ only, information on the dynamics of moduli is provided by the much larger symmetry of the underlying $D = 10$ string theories, with sixteen or thirty-two supercharges. Similarly, in the effective $D = 4$ low-energy supergravity theory, this information on moduli dynamics is encoded in the underlying $N \geqslant 4$ supersymmetry. Thus, the Kähler potential of the $N = 1$ effective supergravity follows from the scalar sigma-model induced by $N = 4$ auxiliary field and gauge-fixing equations. And the $N = 1$ superpotential for the moduli and matter fields is directly related to the $N = 4$ supergravity [2–4] gauging [5], which in turn corresponds to a specific flux structure of the underlying ten-dimensional string theory or eleven-dimensional M-theory.

The generation of a scalar potential for the moduli fields is a crucial ingredient in supersymmetry breaking and in the determination of a stable $D = 4$ background geometry, if any. It is also essential to reduce the number of massless scalars and/or undetermined parameters in the low-energy effective theory. Besides the curvature of the internal space itself, there are several well-known sources for a scalar potential in the compactified ten-dimensional (or eleven-dimensional) theory.

A first source is the Scherk–Schwarz mechanism [6], and its generalization to superstrings via freely acting orbifolds [7]. The relevant fluxes are the geometrical ones, associated with the internal spin connection ω_3. Some of the corresponding effective theories are no-scale supergravity models [8], with broken supersymmetry in a flat $D = 4$ background. However, the gravitino and the other masses generated in this way are proportional (modulo quantized charges) to the inverse length scale of the compactified space, $m \propto R^{-1}$. Therefore, to have supersymmetry breaking and/or preserving TeV scale masses, we need a very large internal dimension, $R \sim 10^{15} l_P$, where l_P is the (four-dimensional) Planck length.

A second source is nonzero “fluxes” of antisymmetric tensor fields, as first identified long ago for the three-form H_3 of the heterotic theory [9]. There is an extensive recent literature [10] on orientifolds of the IIB theory in the presence of three-form fluxes. For instance, simultaneous and suitably aligned NS–NS (NS = Neveu–Schwarz) and R–R (R = Ramond) 3-form fluxes, H_3 and F_3, can lead to no-scale supergravities, but now $m \propto l_P^2 R^{-3}$: as a result, TeV scale supersymmetry breaking and/or preserving masses can be obtained for $R \sim 10^5 l_P$. The richer flux content of the IIA theory has been studied to a lesser extent [11,12].

Both sources, geometric and antisymmetric tensor fluxes, can be combined, as originally examined in the heterotic theory by Kaloper and Myers [13].

[3] For an introduction, see, e.g., [1].

In this paper, we use the method of supergravity gaugings to describe in general terms the generation of moduli superpotentials in a specific compactification scheme, defined as follows. We consider compactifications of superstring theories on the orbifold $T^6/(Z_2 \times Z_2)$, combined for type-II strings with a compatible orientifold projection to reduce supersymmetry to four supercharges. The moduli spectrum includes then seven chiral multiplets from the closed string sector, and the orbifold has a natural permutation symmetry in the three two-tori (T^2) defined by the action of $Z_2 \times Z_2$ on the six-torus T^6. We then construct the gaugings associated to general flux structures respecting this "plane-interchange" permutation symmetry (this assumption could be eventually relaxed, leading to a wider spectrum of possibilities). We include the fluxes generated by all antisymmetric tensor fields (NS–NS and R–R), and also geometrical fluxes associated to components of the internal spin connection, as in Scherk–Schwarz compactifications. We analyze here in detail the case of IIA strings (with D6-branes), since it offers the broadest choice of fluxes and breaking patterns. We establish the dictionary relating fluxes, gauging structure constants and superpotential terms, and the consistency conditions applying on gauging and flux coefficients. This general formulation allows us to study examples with selected phenomenological properties. We find in particular that gaugings and fluxes exist in IIA compactifications, such that all seven moduli are stabilized in a vacuum with $N = 1$, $D = 4$ anti-de Sitter (AdS_4) supersymmetry. Other superstring theories and more general compactification schemes will be considered in a longer, companion paper [14].

This paper is organized as follows. The general method for obtaining $N = 1$ superpotentials from $N = 4$ gaugings, already anticipated in [15,16], is studied and applied to our specific compactification scheme in Section 2. The familiar example of the heterotic theory is then used to define the relation between fluxes and superpotentials, and the consistency conditions for a gauging (Section 3). We then turn to the general study of fluxes in type IIA compactifications (Section 4) and to the study of some selected examples (Section 5). We conclude in Section 6.

2. $N = 1$ superpotentials from $N = 4$ gaugings

The Lagrangian density describing the coupling of vector multiplets to $N = 4$, $D = 4$ supergravity [5] depends on two sets of numbers. The *structure constants* ${f_{ST}}^R$ define the gauge algebra, and the *duality phases* δ_R specify the duality-covariant coupling of each gauge field to the supergravity dilaton S. With n vector multiplets, the gauge group is a $(6 + n)$-dimensional subgroup of the natural $SO(6, n)$ symmetry, inherited from the superconformal origin of the Abelian theory. The structure constants must leave the $SO(6, n)$ metric η_{RS} invariant, a condition which implies antisymmetry of $f_{STR} \equiv {f_{ST}}^U \eta_{UR}$. Notice that η_{RU} is not in general the Cartan metric of the gauge group.[4] With the $SU(1, 1)/U(1)$ Kähler potential

$$K(S, \bar{S}) = -\ln(S + \bar{S}), \tag{1}$$

[4] The $SO(6, n)$ metric has six eigenvalues -1 and n eigenvalues $+1$.

the S-dependent $N=4$ (superconformal) gauge kinetic terms read

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}\sum_{R,S}\eta_{RS}S_{\delta_R}F^{R-}_{\mu\nu}F^{\mu\nu S-} + \text{h.c.} + \cdots, \tag{2}$$

where $F^{R\pm}_{\mu\nu} = F^R_{\mu\nu} \pm i\tilde{F}^R_{\mu\nu}$, and

$$S_{\delta_R} = \frac{\cos\delta_R S - i\sin\delta_R}{-i\sin\delta_R S + \cos\delta_R}. \tag{3}$$

Further gauge kinetic terms, depending on the scalars in the vector multiplets, arise from the elimination of superconformal auxiliary fields [5]. They also depend on the duality phases through the same S_{δ_R}. The duality phases δ_R must respect the structure of the gauge algebra and discretization of the $SU(1,1)$ S-duality group implies that only two choices of phases are allowed

$$\delta_R = 0 \quad \leftrightarrow \quad S_{\delta_R} = S, \quad \text{and} \quad \delta_R = \pi/2 \quad \leftrightarrow \quad S_{\delta_R} = 1/S, \tag{4}$$

commonly associated with perturbative and nonperturbative sectors, respectively.

The n vector multiplets contain scalars in the representation **6** of the R-symmetry group $SU(4)$. They live [3,4] on the coset $SO(6,n)/[SO(6)\times SO(n)]$:

$$\phi^R_{ij} = -\phi^R_{ji} = \frac{1}{2}\epsilon_{ijkl}\phi^{klR}, \qquad \phi^{ijR} = \left(\phi^R_{ij}\right)^* \\ (i,j,\ldots = 1,\ldots,4,\ R = 1,\ldots,6+n). \tag{5}$$

The structure of the sigma-model is dictated by the field equation of an auxiliary scalar, which leads to the constraint

$$\eta_{RS}\phi^R_{ij}\phi^{klS} = \frac{1}{12}\left(\delta^k_i\delta^l_j - \delta^l_i\delta^k_j\right)\eta_{RS}\phi^R_{mn}\phi^{mnS}, \tag{6}$$

and by the Poincaré gauge-fixing condition

$$\eta_{RS}\phi^R_{ij}\phi^{ijS} \equiv \phi^R_{ij}\phi^{ij}_R = -6. \tag{7}$$

These two conditions eliminate twenty-one scalar fields, and the local $SU(4)$ symmetry can be used to eliminate another fifteen. The remaining $6n$ physical scalars live on the announced coset.

As usual, gauging supergravity also generates a scalar potential, and gravitino mass terms $-(1/2)\mathcal{M}_{3/2}{}^{ij}\bar{\psi}_{\mu i}\sigma^{\mu\nu}\psi_{\nu j} + \text{h.c.}$, with[5]

$$\mathcal{M}_{3/2}{}^{ij} = -\frac{4}{3}\varphi^*_{(R)}f_{RST}\phi^{ikR}\phi^S_{kl}\phi^{ljT}, \tag{8}$$

and

$$\varphi^*_{(R)} = \sqrt{\frac{2}{S+\bar{S}}}(\cos\delta_R - iS\sin\delta_R). \tag{9}$$

[5] For $N=4$, $D=4$ supergravity, we mostly follow the conventions of [17], unless otherwise stated, and set the $D=4$ Planck mass equal to one.

To reduce supersymmetry to $N = 1$, we use a $Z_2 \times Z_2$ truncation, as in string orbifolds with the same discrete point group. This truncation leads to a moduli sector with seven chiral multiplets S, T_A, U_A $(A = 1, 2, 3)$, for all string compactifications and compatible orientifolds and D-brane systems. We can also include an arbitrary number of matter multiplets, generically denoted by Z_A^I $(I = 1, \ldots, n_A)$. The $N = 4$ sigma-model reduces to the Kähler manifold

$$M_{Z_2\times Z_2} = \frac{SU(1,1)}{U(1)} \times \prod_{A=1}^{3} \frac{SO(2, 2+n_A)}{SO(2) \times SO(2+n_A)}. \tag{10}$$

Since

$$\frac{SO(2,2)}{SO(2) \times SO(2)} = \frac{SU(1,1)}{U(1)} \times \frac{SU(1,1)}{U(1)}, \tag{11}$$

in the absence of further Z_A^I fields each complex modulus is associated to an $SU(1,1)/U(1)$ structure. In the Lagrangian, the truncation is performed by first rewriting the fields in an $SU(3)$ basis,

$$\phi^{RA} \equiv \phi^{RA4}, \qquad \phi_A^R = \left(\phi^{RA}\right)^* = \frac{1}{2}\epsilon_{ABC}\phi^{RBC}. \tag{12}$$

The three $SU(3)$ nonsinglet gravitino multiplets are then truncated, and the remaining $N = 1$ gravitino mass term reads

$$m_{3/2} = -\frac{4}{3}\varphi^*_{(R)} f_{RST}\epsilon_{ABC}\phi^{RA}\phi^{SB}\phi^{TC}. \tag{13}$$

This simple formula still depends on the constrained $N = 4$ scalar fields ϕ^{RA}. However, once written in terms of the unconstrained fields, the expression of the gravitino mass term will considerably change (see below). These constrained states are truncated to $N = 1$ multiplets according to the $Z_2 \times Z_2$ action on the $SU(3)$ and $SO(6,n)$ indices A and R, as in the sigma model truncation (10). Since our goal is to work with a fixed set of well-defined moduli and matter fields (T_A, U_A, Z_A^I), and to study various classes of gaugings of these multiplets, the next step is to solve the truncated constraints (6) and (7). We then introduce three sets of $4 + n_A$ complex scalars that we denote by

$$\sigma_A^1,\ \sigma_A^2,\ \rho_A^1,\ \rho_A^2,\ \chi_A^I, \quad A = 1, 2, 3,\ I = 1, \ldots, n_A. \tag{14}$$

The truncated, $SO(2, 2 + n_A)$-invariant constraints, which for $\eta_{RS} = \mathrm{diag}(-1_6, 1_n)$ read

$$\begin{aligned} &\left|\sigma_A^1\right|^2 + \left|\sigma_A^2\right|^2 - \left|\rho_A^1\right|^2 - \left|\rho_A^2\right|^2 - \sum_I \left|\chi_A^I\right|^2 = 1/2, \\ &\left(\sigma_A^1\right)^2 + \left(\sigma_A^2\right)^2 - \left(\rho_A^1\right)^2 - \left(\rho_A^2\right)^2 - \sum_I \left(\chi_A^I\right)^2 = 0, \end{aligned} \tag{15}$$

are then solved in this basis by:

$$\sigma_A^1 = \frac{1}{2}\frac{1 + T_A U_A - (Z_A^I)^2}{[Y(T_A, U_A, Z_A^I)]^{1/2}}, \qquad \sigma_A^2 = \frac{i}{2}\frac{T_A + U_A}{[Y(T_A, U_A, Z_A^I)]^{1/2}},$$

$$\rho_A^1 = \frac{1}{2}\frac{1 - T_A U_A + (Z_A^I)^2}{[Y(T_A, U_A, Z_A^I)]^{1/2}}, \qquad \rho_A^2 = \frac{i}{2}\frac{T_A - U_A}{[Y(T_A, U_A, Z_A^I)]^{1/2}},$$
$$\chi_A^I = \frac{i Z_A^I}{[Y(T_A, U_A, Z_A^I)]^{1/2}}. \tag{16}$$

These expressions depend on the real quantity

$$Y\left(T, U, Z^I\right) = (T + \bar{T})(U + \bar{U}) - \sum_I \left(Z^I + \bar{Z}^I\right)^2. \tag{17}$$

As expected, the constraints eliminate six complex scalar fields.

The above equations allow to rewrite the scalar potential and the gravitino mass term as functions of the $N = 1$ complex scalars, the structure constants and the duality phases. The Kähler potential and the superpotential can then be obtained by separating the holomorphic part in the $N = 1$ gravitino mass term, using the relation $m_{3/2} = e^{K/2} W$. The resulting Kähler potential is

$$K = -\ln(S + \bar{S}) - \sum_{A=1}^{3} \ln Y\left(T_A, U_A, Z_A^I\right), \tag{18}$$

while the superpotential is simply

$$W = \frac{4}{3}\sqrt{2}[\cos\delta_R - i\sin\delta_R S]\left[\prod_{A=1}^{3} Y\left(T_A, U_A, Z_A^I\right)\right]^{1/2}$$
$$\times f_{RST}\epsilon_{ABC}\phi^{RA}\phi^{SB}\phi^{TC}. \tag{19}$$

It is a holomorphic function of (S, T_A, U_A, Z_A^I), once the $N = 4$ scalars from the vector multiplets have been truncated to $N = 1$ and replaced by the solutions (16).

In this paper, we discard all matter fields Z_A^I for simplicity. However, many of the features encountered in the restricted cases studied here remain true with all matter fields included. Removing the Z_A^I fields, the generic superpotential is then a polynomial in the moduli fields with maximal degree seven. In particular, each monomial is of order zero or one in each of the seven moduli S, T_A, U_A. The superpotential can then have up to $2^7 = 128$ real parameters, which are structure constants and duality phases of the underlying $N = 4$ algebra.[6] These numbers will be identified with various fluxes of compactified string theories.

The structure constants $f_{RS}{}^T$ gauge a subalgebra of $SO(6,6)$, with dimension equal or less than twelve and compatible with the $Z_2 \times Z_2$ truncation. They verify Jacobi identities. The gauging structure constants with lower indices $f_{RST} = f_{RS}{}^Q \eta_{QT}$ are fully antisymmetric for consistency of the gauging. The truncation to $N = 1$ provides further information. The residual Poincaré gauge fixing conditions solved by T_A and U_A are invariant under $SO(3)$ rotations of the plane index A and $SO(2,2)$ rotations inside each plane.

[6] The $N = 1$ truncation of the scalar fields ϕ^{RA} associates to each fixed value of $A = 1, 2, 3$ only four values of the index R, the four directions in each of the three $SO(2,2)$. Hence f_{RST} includes $4^3 = 64$ real numbers.

This means that structure constants can be classified using the $SO(2,2) \times SO(3)$ subgroup of $SO(6,6)$ with embedding $\mathbf{12} = (\mathbf{4}, \mathbf{3})$. One can rewrite the gauge algebra in this embedding by defining generators T_{Aa} ($A = 1, 2, 3,\ a = 1, \ldots, 4$), and commutation relations

$$[T_{Aa}, T_{Bb}] = f_{AaBb}{}^{Cc} T_{Cc}. \tag{20}$$

The antisymmetric gauging structure constants are then

$$f_{AaBbCc} = f_{AaBb}{}^{Cd} \eta_{cd}, \tag{21}$$

where η_{cd} is the $SO(2,2)$ metric. The $Z_2 \times Z_2$ orbifold projection[7] leads naturally to define a "plane-interchange symmetry" in the moduli sector. Our purpose here is to study a particular class of gaugings which respect this plane-interchange symmetry. Ref. [14] will analyse more general gauging structures. The structure constants for these particular gaugings read

$$\begin{aligned} &f_{Aa_1Bb_2Cc_3} = \Lambda_{a_1b_2c_3} \epsilon_{ABC} \quad (a_1, b_2, c_3 = 1, \ldots, 4), \\ &f_{Aa_1Bb_2}{}^{Cc_3} = \Lambda_{a_1b_2}{}^{c_3} \epsilon_{ABC}, \quad \Lambda_{a_1b_2}{}^{c_3} = \eta^{c_3d_3} \Lambda_{a_1b_2d_3}. \end{aligned} \tag{22}$$

Each index a_1, b_2, c_3 is an $SO(2,2)$ index, and there are in principle $4^3 = 64$ possible combinations, as for the number of possible superpotential terms constructed with U_A and T_A and the rule that each term is either linear or independent of each modulus ($2^6 = 64$). Each $SO(2,2)$ index refers to a specific complex plane of the $N = 1$ truncation: a_1 to the first plane, b_2 to the second, c_3 to the third. Antisymmetry of the gauging structure constants f_{RST} implies full symmetry of $\Lambda_{a_1b_2c_3}$: this reduces the number of independent structure constants to 20, which is also the number of combinations of superpotential terms left invariant by any permutation of the plane index. The Jacobi identities verified by the structure constants $f_{AaBb}{}^{Cc}$ translate into a simple cyclicity property:

$$\eta^{df} \Lambda_{abd} \Lambda_{cfe} = \eta^{df} \Lambda_{bcd} \Lambda_{afe} = \eta^{df} \Lambda_{cad} \Lambda_{bfe}, \quad \forall a, b, c, e. \tag{23}$$

Eq. (23) and symmetry of Λ_{abc} are the conditions applying to an $N = 4$ gauging respecting the plane-interchange symmetry.

There are two commonly used bases for $SO(6,6)$. Firstly, the natural basis in which the Cartan metric is diagonal, as in Eq. (15). Secondly, the S/A basis defined by

$$\begin{aligned} ds^2 &= \sum_{i=1}^{6} \left[dx^{i+} dx^{i+} - dx^{i-} dx^{i-}\right] = \sum_{i=1}^{6} \left(dx^{i+} + dx^{i-}\right)\left(dx^{i+} - dx^{i-}\right) \\ &\equiv \sum_{i=1}^{6} dx^{is} dx^{ia}. \end{aligned} \tag{24}$$

The Cartan metric in the S/A basis is off-diagonal,

$$\eta = \frac{1}{2} \begin{pmatrix} 0_6 & I_6 \\ I_6 & 0_6 \end{pmatrix}. \tag{25}$$

[7] And the $SO(3)$ invariance of the constraints.

The analysis of the consistency conditions (23) is much simpler in the S/A basis, especially in view of the solutions (16) of the Poincaré constraints. The procedure to analyse a gauging and obtain the corresponding $N=1$ superpotential is as follows. To begin with, select a symmetric set of constants $\Lambda_{a_1b_2c_3}$ that solve the cyclicity equations (23) in the S/A basis. Then, compute the resulting $N=1$ superpotential, in two steps. Firstly, use the following correspondence between the indices (a_1, b_2, c_3) and the twelve directions in $SO(6,6)$:

$$\begin{aligned} a_1 &= (1,2,3,4) \quad \leftrightarrow \quad (5_S, 6_S, 5_A, 6_A), \\ b_2 &= (1,2,3,4) \quad \leftrightarrow \quad (7_S, 8_S, 7_A, 8_A), \\ c_3 &= (1,2,3,4) \quad \leftrightarrow \quad (9_S, 10_S, 9_A, 10_A). \end{aligned} \tag{26}$$

Secondly, use the solution of the Poincaré constraints, in the form

$$\begin{aligned} (5_S, 7_S, 9_S) &\quad \rightarrow \quad 1/\sqrt{(T_A+\bar{T}_A)(U_A+\bar{U}_A)}, \quad A=1,2,3, \\ (6_S, 8_S, 10_S) &\quad \rightarrow \quad iT_A/\sqrt{(T_A+\bar{T}_A)(U_A+\bar{U}_A)}, \quad A=1,2,3, \\ (5_A, 7_A, 9_A) &\quad \rightarrow \quad T_AU_A/\sqrt{(T_A+\bar{T}_A)(U_A+\bar{U}_A)}, \quad A=1,2,3, \\ (6_A, 8_A, 10_A) &\quad \rightarrow \quad iU_A/\sqrt{(T_A+\bar{T}_A)(U_A+\bar{U}_A)}, \quad A=1,2,3. \end{aligned} \tag{27}$$

For each compactified string theory, the allowed fluxes will determine the set of allowed $\Lambda_{a_1b_2c_3}$ and the cyclicity equations (23) will impose the consistency relations between various fluxes. This method can be used to generate all superpotentials from fluxes verifying plane-interchange symmetry. Without invoking this symmetry, the analysis of a gauging would be similar, but with a set of nonzero gauging structure constants submitted to more complicated Jacobi identities, instead of the simple relations (23).

With our seven moduli fields, Kähler potential (18) and superpotential (19), the $N=1$ supergravity scalar potential simplifies to

$$e^{-K}V = \sum_{i=1}^{7} \left|W - W_i(z_i+\bar{z}_i)\right|^2 - 3|W|^2, \tag{28}$$

where $z_i = S, T_A, U_A$ and $W_i = (\partial W)/(\partial z_i)$. Each quantity $[W - W_i(z_i+\bar{z}_i)]$ is simply the superpotential W with the corresponding field z_i replaced by $-\bar{z}_i$.

3. Heterotic fluxes

Before moving to the discussion of the IIA theory, we recall some known results for $N=1$ compactifications of the heterotic theory on the $T^6/(Z_2\times Z_2)$ orbifold. This will be useful to establish some notation and to illustrate our general method in a familiar case.

We begin with the identification of the seven main moduli. Conventionally, we split the space–time indices as $M = [\mu = 0,1,2,3;\ i = 5,6,7,8,9,10]$, and we take one Z_2 acting on the coordinates $x^{5,6,7,8}$, the other Z_2 on the coordinates $x^{7,8,9,10}$. This naturally defines three complex planes $A = 1,2,3$: $i_1 = 5,6$, $i_2 = 7,8$, $i_3 = 9,10$. We follow the conventions of [1] unless otherwise stated.

If we neglect the $E_8 \times E_8$ or $SO(32)$ gauge bosons (which would generate multiplets of Z_A^I type in the $D = 4$ theory, and would allow for additional fluxes associated with the internal components of their two-form field strengths), the bosonic fields of the $D = 10$ heterotic theory are just the universal ones of the NS–NS sector: the string-frame metric g_{MN}, the dilaton Φ and the two-form potential B_{MN}. Their $Z_2 \times Z_2$ invariant components can be decomposed as

$$e^{-2\Phi} = s(t_1 t_2 t_3)^{-1}, \qquad g_{\mu\nu} = s^{-1} \tilde{g}_{\mu\nu}, \tag{29}$$

$$g_{i_A j_A} = \frac{t_A}{u_A} \begin{pmatrix} u_A^2 + \nu_A^2 & \nu_A \\ \nu_A & 1 \end{pmatrix} \quad (A = 1, 2, 3), \tag{30}$$

$$B_{\mu\nu} \leftrightarrow \sigma, \qquad B_{56} = \tau_1, \qquad B_{78} = \tau_2, \qquad B_{910} = \tau_3, \tag{31}$$

where $\tilde{g}_{\mu\nu}$ is the metric in the $D = 4$ Einstein frame, and the symbol $\leftrightarrow$ indicates the four-dimensional duality transformation relating a two-form potential with an axionic pseudoscalar. Neglecting the dependence of the fields on the internal coordinates, absorbing an integration constant in the $D = 4$ Planck mass, conventionally set to unity, and making the identifications

$$S = s + i\sigma, \qquad T_A = t_A + i\tau_A, \qquad U_A = u_A + i\nu_A \quad (A = 1, 2, 3), \tag{32}$$

we obtain $D = 4$ kinetic terms described precisely by the Kähler potential of Eq. (18), for the case $Z_A^I = 0$ we have chosen to study

$$K = -\ln(S + \bar{S}) - \sum_{A=1}^{3} \ln(T_A + \bar{T}_A) - \sum_{A=1}^{3} \ln(U_A + \bar{U}_A). \tag{33}$$

In view of what follows, we stress that the kinetic terms of the seven main moduli are invariant under both $O(7)$ rotations and $SU(1,1) \times [SO(2,2)]^3$ duality transformations.

We now summarize the different allowed fluxes, and identify the associated $N = 1$ superpotentials with the method illustrated in the previous section.

3.1. $\tilde{H}_3$ heterotic fluxes

As first recognized in [9], possible fluxes in the heterotic theory are those of the modified NS–NS three-form $\tilde{H}_3 = dB_2 + \cdots$, where the dots stand for the gauge and Lorenz Chern–Simons terms. There are eight independent real fluxes, invariant under the $Z_2 \times Z_2$ orbifold projection:

$$\tilde{H}_{579}, \quad \tilde{H}_{679}, \quad \tilde{H}_{589}, \quad \tilde{H}_{689}, \quad \tilde{H}_{5710}, \quad \tilde{H}_{6710}, \quad \tilde{H}_{5810}, \quad \tilde{H}_{6810}. \tag{34}$$

The corresponding potential for the seven main moduli can be explicitly computed by dimensional reduction. Its generic structure is $V_{H_3} = e^K \prod_{A=1}^3 f_A(\nu_A, u_A^2 + \nu_A^2)$, where each f_A is a polynomial of at most degree one in its arguments. This is sufficient to deduce, similarly to what happens in IIB theories [10], the corresponding $N = 1$ effective superpotential $W_{H_3}(U)$, which carries no dependence on the S and T moduli. It is immediate to check that $\tilde{H}_3$ fluxes correspond to $N = 4$ gaugings for any choice of the parameters in (34). Leaving aside a systematic discussion, we just observe that, under the assumption of plane-interchange symmetry, there are four independent parameters, associated with four different structures in $W_{H_3}(U)$:

$$\begin{aligned}
&\tilde{H}_{579} \equiv \Lambda_{111} \quad \leftrightarrow \quad 1,\\
&\tilde{H}_{679} = \tilde{H}_{589} = \tilde{H}_{5710} \equiv \Lambda_{114} \quad \leftrightarrow \quad i(U_1 + U_2 + U_3),\\
&\tilde{H}_{689} = \tilde{H}_{5810} = \tilde{H}_{6710} \equiv \Lambda_{144} \quad \leftrightarrow \quad -(U_1U_2 + U_2U_3 + U_1U_3),\\
&\tilde{H}_{6810} \equiv \Lambda_{444} \quad \leftrightarrow \quad -iU_1U_2U_3.
\end{aligned} \tag{35}$$

3.2. Geometrical heterotic fluxes

The possible fluxes also include some geometrical ones, associated with the internal components of the spin connection ω_3, and corresponding to coordinate-dependent compactifications [6]. These fluxes are characterized by real constants with one upper curved index and two lower antisymmetric curved indices:

$$f^i{}_{jk} = -f^i{}_{kj}. \tag{36}$$

These constants must satisfy the Jacobi identities of a Lie group, $f^i{}_{jk} f^k{}_{lm} + f^i{}_{lk} f^k{}_{mj} + f^i{}_{mk} f^k{}_{jl} = 0$, and the additional consistency condition $f^i{}_{ik} = 0$. The corresponding $D = 4$ potential in the heterotic theory can be easily calculated from the formulae in [6].

In agreement with the $Z_2 \times Z_2$ orbifold projection, we must assume here that

$$f^{i_A}{}_{i_B i_C} = 0 \quad \text{for } A = B \text{ or } A = C \text{ or } B = C, \tag{37}$$

which satisfies automatically the consistency condition $f^i{}_{ik} = 0$. Geometrical fluxes are then described by 24 real parameters

$$C_{i_A i_B i_C} \equiv f^{i_A}{}_{i_B i_C}, \quad \big[(ABC) = (123), (231), (312)\big], \tag{38}$$

subject only to the Jacobi identities. Leaving aside a general discussion, we assume here plane-interchange symmetry, to reduce the number of independent parameters. Inspection of the resulting scalar potential singles out six different possible structures in the effective superpotential $W_\omega(T, U)$, always linear in the T moduli and independent of S:

$$\begin{aligned}
&C_{679} = C_{895} = C_{1057} \equiv \Lambda_{112} \quad \leftrightarrow \quad i\ (T_1 + T_2 + T_3),\\
&C_{579} = C_{957} = C_{795} \equiv \Lambda_{113} \quad \leftrightarrow \quad (T_1U_1 + T_2U_2 + T_3U_3),\\
&C_{6810} = C_{8106} = C_{1068} \equiv \Lambda_{244} \quad \leftrightarrow \quad -i\ (T_1U_2U_3 + T_2U_1U_3 + T_3U_1U_2),\\
&C_{5810} = C_{7106} = C_{968} \equiv \Lambda_{344} \quad \leftrightarrow \quad -(T_1 + T_2 + T_3)U_1U_2U_3,\\
&C_{896} = C_{1067} = C_{689} = C_{1058} = C_{6710} = C_{8105} \equiv \Lambda_{124} \quad \leftrightarrow\\
&\quad -(T_1U_2 + T_1U_3 + T_2U_1 + T_2U_3 + T_3U_1 + T_3U_2),\\
&C_{589} = C_{796} = C_{7105} = C_{958} = C_{5710} = C_{967} \equiv \Lambda_{134} \quad \leftrightarrow\\
&\quad i(T_1U_1U_2 + T_2U_2U_3 + T_3U_3U_1 + T_1U_1U_3 + T_2U_2U_1 + T_3U_3U_2).
\end{aligned} \tag{39}$$

In this case the Jacobi identities (23) impose some nontrivial constraints:

$$\Lambda_{112}\Lambda_{344} = \Lambda_{124}\Lambda_{134},$$

$$\Lambda_{113}\Lambda_{344} + \Lambda_{244}\Lambda_{134} = \Lambda_{344}\Lambda_{124} + \Lambda_{134}^2,$$
$$\Lambda_{112}\Lambda_{244} + \Lambda_{113}\Lambda_{124} = \Lambda_{112}\Lambda_{134} + \Lambda_{124}^2. \tag{40}$$

3.3. Combined heterotic fluxes

The combination of $\tilde{H}_3$ and ω_3 fluxes in the $T^6/(Z_2 \times Z_2)$ orbifold of the heterotic string is then described, under the assumption of plane-interchange symmetry, by the ten real parameters of Eqs. (35) and (39). According to Eq. (23), consistency with an underlying $N = 4$ gauging amounts to requiring the Jacobi identities (cyclicity conditions) of Eq. (40) and the additional condition

$$\Lambda_{111}\Lambda_{344} + \Lambda_{112}\Lambda_{444} + \Lambda_{113}\Lambda_{144} + \Lambda_{114}\Lambda_{244} = 2\Lambda_{144}\Lambda_{124} + 2\Lambda_{114}\Lambda_{134}. \tag{41}$$

The corresponding effective $N = 1$ superpotential would read

$$\begin{aligned} W = {} & \Lambda_{111} + i\Lambda_{114}(U_1 + U_2 + U_3) - \Lambda_{144}(U_1U_2 + U_2U_3 + U_1U_3) \\ & - i\Lambda_{444}U_1U_2U_3 + i\Lambda_{112}(T_1 + T_2 + T_3) + \Lambda_{113}(T_1U_1 + T_2U_2 + T_3U_3) \\ & - i\Lambda_{244}(T_1U_2U_3 + T_2U_1U_3 + T_3U_1U_2) - \Lambda_{344}(T_1 + T_2 + T_3)U_1U_2U_3 \\ & - \Lambda_{124}(T_1U_2 + T_1U_3 + T_2U_1 + T_2U_3 + T_3U_1 + T_3U_2) \\ & + i\Lambda_{134}(T_1U_1U_2 + T_1U_1U_3 + T_2U_2U_1 + T_2U_2U_3 + T_3U_3U_1 + T_3U_3U_2). \end{aligned} \tag{42}$$

Similar superpotentials were considered, motivated by $N > 1$ gaugings but without assuming plane interchange symmetry and without establishing the precise connections with fluxes, in [18]. The connection between Scherk–Schwarz compactifications, geometrical fluxes, $N = 4$ gaugings and $N = 1$ superpotentials was also discussed in [19], without assuming plane-interchange symmetry and in a different field basis. More results at the $N = 4$ level were obtained in [20]. A general analysis of combined fluxes in toroidal compactifications of the heterotic string was given in [13]: consistent $Z_2 \times Z_2$ truncations of their results are in complete agreement with our results.

It is important to recall that, in the heterotic theory, $\tilde{H}_3$ and ω_3 fluxes, corresponding to perturbative $N = 4$ gaugings with trivial duality phases, can never generate $N = 1$ superpotentials with both constant and linear terms in S. We can then obtain, for example, no-scale models as in [7,19], but never reach the full stabilization of all seven main moduli, including S. From the point of view of $N = 4$ supergravity, of course, we could also consider nonperturbative gaugings with nontrivial duality phases, which would give rise to both kinds of allowed S-dependences in the effective $N = 1$ superpotential. We may think of these gaugings as associated to possible nonperturbative effects such as gaugino condensation.

4. Fluxes in IIA superstrings

In type IIA and IIB superstring theories compactified on $T^6/(Z_2 \times Z_2)$, to produce $N = 1$, $D = 4$ supersymmetry we must introduce consistently an additional Z_2 orientifold projection. We discuss here only the case of the IIA theory, with a specific orientifold projection compatible with D6-branes.

The bosonic fields of the IIA theory are the universal ones of the NS–NS sector, plus those of the R–R sector: a one-form A_M and a three-form A_{MNR}. Five-forms and seven-forms are related to the previous ones by ten-dimensional duality, do not carry independent degrees of freedom and do not need to be included at this stage. Nine-form potentials do not carry any propagating degree of freedom in $D=10$, even if they can play a role, as we shall see, in the classification of allowed fluxes. We consider here a specific Z_2 orientifold projection, involving the inversion of three out of the six internal coordinates and associated with D6 branes: it is not restrictive to take the odd coordinates to be $x^{5,7,9}$. The independent invariant spin-0 fields from the NS–NS sector are then

$$g_{ii}, \quad \Phi, \quad B_{56}, \quad B_{78}, \quad B_{910}, \tag{43}$$

for which we can temporarily make the heterotic decomposition of Eqs. (29)–(31), setting $\nu_A \equiv 0$ and disregarding the off-diagonal components of the internal metric. The independent invariant spin-0 fields from the R–R sector are:

$$A_{6810} = \sigma', \qquad A_{679} = -\nu_1', \qquad A_{589} = -\nu_2', \qquad A_{5710} = -\nu_3'. \tag{44}$$

Looking at the $D=4$ kinetic terms of the fields in (44), we find

$$\mathcal{L}_R \to -\frac{1}{4}\tilde{e}_4\tilde{g}^{\mu\nu}\left[\mathcal{O}_0(\partial_\mu\sigma')(\partial_\nu\sigma') + \sum_{A=1}^{3}\mathcal{O}_A(\partial_\mu\nu_A')(\partial_\nu\nu_A')\right], \tag{45}$$

where

$$\mathcal{O}_0 = \frac{u_1u_2u_3}{s}, \qquad \mathcal{O}_1 = \frac{u_1}{su_2u_3}, \qquad \mathcal{O}_2 = \frac{u_2}{su_1u_3}, \qquad \mathcal{O}_3 = \frac{u_3}{su_1u_3}. \tag{46}$$

This immediately suggests [12] the identification of the real parts (s', u_1', u_2', u_3'), associated by $N=1$ supersymmetry to the imaginary parts $(\sigma', \nu_1', \nu_2', \nu_3')$:

$$s' = \sqrt{\frac{s}{u_1u_2u_3}}, \qquad u_1' = \sqrt{\frac{su_2u_3}{u_1}}, \qquad u_2' = \sqrt{\frac{su_1u_3}{u_2}}, \qquad u_3' = \sqrt{\frac{su_1u_2}{u_3}}. \tag{47}$$

These identifications can be cross-checked by looking at the $F_{\mu\nu}F^{\mu\nu}$ and $F_{\mu\nu}\tilde{F}^{\mu\nu}$ terms in the effective four-dimensional action for the Yang–Mills vectors, generated by the Dirac–Born–Infeld and Wess–Zumino actions for the D6 branes, aligned along the (6810), (679), (589), (5710) O6-planes.

The theory under consideration exhibits a rich structure of possible invariant fluxes. As in the heterotic case, we first discuss each of them separately, then we look at a generic combination. The Jacobi identities of $N=4$ gaugings, Eq. (23), will be automatically satisfied if there are only NS–NS three-form fluxes, or only fluxes of the R–R forms: as we shall see, nontrivial constraints will arise only in the presence of geometrical fluxes or of combined fluxes.

4.1. H_3 fluxes in IIA

Only four out of the eight independent fluxes allowed for the NS–NS 3-form in the heterotic case, Eq. (34), are also invariant with respect to the orientifold projection. In terms of $H_3 = dB_2$, they are:

$$H_{579}, \quad H_{689}, \quad H_{6710}, \quad H_{5810}. \tag{48}$$

Inspection of the four-dimensional potential obtained from dimensional reduction shows that, after moving to the IIA field basis of Eq. (47), and assuming plane-interchange symmetry, there are two independent superpotential structures:

$$\begin{aligned} &H_{579} = \Lambda'_{111} \quad \leftrightarrow \quad iS, \\ &-H_{689} = -H_{6710} = -H_{5810} = \Lambda_{114} \quad \leftrightarrow \quad i(U_1 + U_2 + U_3). \end{aligned} \tag{49}$$

Notice that, in contrast with the heterotic case, two independent $SU(1,1)$ phases are involved.

4.2. Geometrical IIA fluxes

Again, the IIA orientifold projection leaves invariant only half of the geometrical fluxes (38) that were allowed, modulo Jacobi identities, in the heterotic case:

$$\begin{aligned} &C_{5710}, C_{7105}, C_{1057}; \qquad C_{679}, C_{796}, C_{967}; \\ &C_{589}, C_{895}, C_{958}; \qquad C_{6810}, C_{8106}, C_{1068}. \end{aligned} \tag{50}$$

Inspection of the four-dimensional potential obtained from dimensional reduction shows that in this case, after moving to the IIA field basis of Eq. (47), and assuming plane-interchange symmetry, there are three independent superpotential structures:

$$\begin{aligned} &C_{679} = C_{895} = C_{1057} \equiv \Lambda'_{112} \quad \leftrightarrow \quad -S(T_1 + T_2 + T_3), \\ &C_{6810} = C_{8106} = C_{1068} \equiv \Lambda_{113} \quad \leftrightarrow \quad (T_1U_1 + T_2U_2 + T_3U_3), \\ &C_{589} = C_{796} = C_{7105} = C_{958} = C_{5710} = C_{967} \equiv \Lambda_{124} \quad \leftrightarrow \\ &\quad -(T_1U_2 + T_1U_3 + T_2U_1 + T_2U_3 + T_3U_1 + T_3U_2). \end{aligned} \tag{51}$$

Notice that also for geometrical fluxes two different $SU(1,1)$ phases appear, in contrast with the heterotic case. From Eq. (23) we can easily derive the Jacobi identities required for a consistent $N = 4$ gauging with geometrical fluxes only:

$$\Lambda_{124}(\Lambda_{124} - \Lambda_{113}) = 0. \tag{52}$$

4.3. F_0 flux

The mass parameter of massive IIA supergravity [21] can be regarded as a ten-dimensional zero-form flux F_0, dual to the ten-form field strength associated with a nine-form potential, which does not carry any propagating degree of freedom. Inspecting the F_0 contribution to the potential via dimensional reduction, and moving to IIA variables, we can identify the associated structure in the effective superpotential W:

$$F_0 = \Lambda_{222} \quad \leftrightarrow \quad -i(T_1 T_2 T_3). \tag{53}$$

4.4. F_2 fluxes

The independent F_2 fluxes invariant under the orbifold and orientifold projections are:

$$F_{56}, \quad F_{78}, \quad F_{910}. \tag{54}$$

Looking at their contributions to the potential via dimensional reduction, moving to the IIA field basis of Eq. (47), and assuming plane-interchange symmetry, we find that the corresponding structure in the effective superpotential W is

$$F_{56} = F_{78} = F_{910} = \Lambda_{122} \quad \leftrightarrow \quad -(T_1 T_2 + T_1 T_3 + T_2 T_3). \tag{55}$$

4.5. F_4 fluxes

The four-form fluxes with internal indices, invariant under the orbifold and orientifold projections, are:

$$F_{5678}, \quad F_{78910}, \quad F_{91056}. \tag{56}$$

Looking at their contribution to the potential via dimensional reduction, moving to the IIA field basis of Eq. (47), and assuming plane-interchange symmetry, we can identify the corresponding structure in the effective superpotential:

$$F_{5678} = F_{78910} = F_{56910} = \Lambda_{112} \quad \leftrightarrow \quad i(T_1 + T_2 + T_3). \tag{57}$$

4.6. F_6 flux

Among the components of the R–R four-form field strength, invariant under both the orbifold and the orientifold projections, there is also $F_{\mu\nu\rho\sigma}$, which is not associated with any $D = 4$ propagating degree of freedom, and can be related by ten-dimensional duality to a ten-dimensional six-form flux F_6. A similar flux was considered in [22] to address the cosmological constant problem. Looking at the corresponding potential terms generated by dimensional reduction, and moving to the IIA variables of Eq. (47), we can identify the corresponding structure in the effective superpotential:

$$F_6 = \Lambda_{111} \quad \leftrightarrow \quad 1. \tag{58}$$

Notice that the above flux generates a constant superpotential, not a constant potential, in the four-dimensional effective theory.

4.7. Combined IIA fluxes

Switching on simultaneously all the independent fluxes identified so far corresponds to having, as nonvanishing coefficients:

- $(\Lambda'_{111}, \Lambda'_{112})$ with $SU(1,1)$ phase factor iS;
- $(\Lambda_{111}, \Lambda_{112}, \Lambda_{122}, \Lambda_{222}, \Lambda_{113}, \Lambda_{114}, \Lambda_{124})$ with $SU(1,1)$ phase 1.

Under our simplifying assumption of plane-interchange symmetry, the Jacobi identities constraining such combined fluxes read

$$\Lambda_{222}\Lambda_{114} + \Lambda_{113}\Lambda_{122} = 2\Lambda_{122}\Lambda_{124}, \qquad \Lambda_{113}\Lambda_{124} = \Lambda_{124}^2. \tag{59}$$

Any combination of fluxes satisfying the above Jacobi identities corresponds to a $N=4$ gauging, and can be easily translated into an effective $N=1$ superpotential:

$$\begin{aligned} W = {} & \Lambda_{111} + i\Lambda'_{111}S + i\Lambda_{112}(T_1+T_2+T_3) - \Lambda'_{112}S(T_1+T_2+T_3) \\ & + i\Lambda_{114}(U_1+U_2+U_3) + \Lambda_{113}(T_1U_1+T_2U_2+T_3U_3) \\ & - \Lambda_{122}(T_1T_2+T_1T_3+T_2T_3) \\ & - \Lambda_{124}(T_1U_2+T_1U_3+T_2U_1+T_2U_3+T_3U_1+T_3U_2) - i\Lambda_{222}T_1T_2T_3. \end{aligned} \tag{60}$$

The results of Eqs. (59) and (60) provide a powerful and practical tool for analyzing, directly in the $N=1$, $D=4$ effective theory, the different vacuum structures associated with the different allowed combinations of fluxes, in the chosen orbifold and orientifold of the IIA theory. The study of a large number of examples of flux configurations [14], in heterotic and type II strings, actually shows that the effective supergravity approach based upon $N=4$ gaugings can accurately reproduce the conditions imposed by the full field equations of the ten-dimensional theories. This of course requires to include all necessary brane and orientifold plane contributions to these equations. For the combinations of fluxes leading to stable vacua of our $N=1$, $D=4$ effective theory, it would be interesting to explicitly examine the corresponding combinations of D6-branes and O6-planes required to satisfy the $D=10$ equations and Bianchi identities, and the associated tadpole cancellation conditions. This analysis goes beyond the scope of the present paper.

5. Some selected IIA examples

We present now some selected examples of admissible IIA fluxes that correspond to $N=4$ gaugings with non-trivial $SU(1,1)$ phases and give rise to physically different situations.

5.1. *Flat gaugings, no-scale models: stabilization of four moduli*

Switching on a system of $(\omega_3, H_3, F_0, F_2)$ fluxes, with nonzero parameters $(B, D > 0)$

$$\begin{aligned} &\Lambda'_{112} = -A, \qquad \Lambda_{122} = -AB, \\ &\Lambda'_{111} = C, \qquad \Lambda_{222} = -CD, \end{aligned} \tag{61}$$

the Jacobi identities (59) are automatically satisfied, and the following effective $N=1$ superpotential is generated:

$$W = A\big[S(T_1+T_2+T_3) + B(T_1T_2+T_2T_3+T_1T_3)\big] + iC[S + DT_1T_2T_3]. \tag{62}$$

It is immediate to see that this corresponds to a no-scale model. Since W does not depend on (U_1, U_2, U_3), the scalar potential is a sum of positive semi-definite terms

$$e^{-K} V = \left|W - (S + \bar{S}) W_S\right|^2 + \sum_{A=1}^{3} \left|W - (T_A + \bar{T}_A) W_{T_A}\right|^2, \tag{63}$$

and stabilization of the S and T_A moduli occurs at

$$\langle S \rangle = BT, \qquad \langle T_1 \rangle = \langle T_2 \rangle = \langle T_3 \rangle = T, \quad T = \sqrt{\frac{B}{D}}, \tag{64}$$

with $\langle V \rangle = 0$. The U_A moduli remain as complex flat directions and supersymmetry is broken in the U_A sector, since the stabilization conditions lead to $\langle W \rangle \neq 0$, with a gravitino mass

$$\left\langle m_{3/2}^2 \right\rangle \propto \frac{|9A^2 B + C^2 D|}{u_1 u_2 u_3}. \tag{65}$$

To identify the gauging associated with the above fluxes and superpotential, it is convenient to rescale the fields according to

$$S \to B^{3/2} D^{-1/2} S, \qquad T_A \to B^{1/2} D^{-1/2} T_A. \tag{66}$$

The stabilization of the rescaled fields occurs at $\langle S \rangle = \langle T_A \rangle = 1$, with a superpotential as in (62) with $B = D = 1$. The resulting group is $E_3 \times E_3$ [14], where E_3 is the three-dimensional Euclidean group, i.e., the $SO(3)$-invariant contraction of $SO(4)$ or $SO(3, 1)$.

This kind of rescalings can be applied in general, to shift the values at which the fields are stabilized. For simplicity, and without losing the full generality of the combination of fluxes, we choose that moduli are stabilized at value one in most of the following examples.

As a side remark, we notice here that the same phenomenology of the above example can be obtained without respecting the plane-interchange symmetry. As an example, we can consider as before a system of fluxes $(\omega_3, H_3, F_0, F_2)$, but this time corresponding to a superpotential:

$$W = A(ST_1 + T_2 T_3) + iB(S + T_1 T_2 T_3), \quad \text{with} \left\langle m_{3/2}^2 \right\rangle \propto \frac{A^2 + B^2}{u_1 u_2 u_3}. \tag{67}$$

The complex flat directions are (U_1, U_2, U_3), as in the first example.

We finally notice that, in the type-IIA theory, purely geometrical fluxes ω_3 are not sufficient to stabilize all moduli explicitly appearing in the corresponding superpotential W, because the latter is always quadratic in the fields. As an example to illustrate this point, based on the two-dimensional Euclidean group E_2 (the $SO(2)$-invariant contraction of $SO(3)$ or $SO(2, 1)$) and breaking the plane-interchange symmetry, we consider the superpotential

$$W = A(T_1 U_2 + T_2 U_1), \quad \text{with} \left\langle m_{3/2}^2 \right\rangle \propto \frac{A^2}{s t_3 u_3}. \tag{68}$$

This corresponds to a Z_2 freely acting orbifold (generalized Scherk–Schwarz mechanism in string theory), with complex flat directions (S, T_3, U_3). However, there are additional flat directions, because the auxiliary fields associated with (T_1, U_1, T_2, U_2) are all set to zero by requiring $\tau_1 = \tau_2 = \nu_1 = \nu_2 = 0$ and $t_1 u_2 = t_2 u_1$. In this case the spectrum has 4 massive and 3 massless "axions", 1 massive and 6 massless "dilatons".

5.2. *Gaugings with $V > 0$, cosmological models*

Examples can be easily found, in which less than four moduli are stabilized and the potential is always strictly positive-definite, leading to runaway solutions (in time).

Superpotentials with a single monomial are of course examples where no modulus gets stabilized. For instance, we can choose the fluxes $\Lambda_{111} = F_6$, $\Lambda_{222} = F_0$ or $\Lambda'_{111} = H_3$, corresponding to

$$W = F_6, \qquad W = -i F_0 T_1 T_2 T_3 \quad \text{or} \quad W = i H_3 S. \tag{69}$$

This leads to $V = 4e^K |W|^2$, with $|W| > 0$ and a gravitino mass term of the form

$$m_{3/2}^2 = \frac{1}{2^7 s t_1 t_2 t_3 u_1 u_2 u_3} \times \left\{ |F_6|^2, |F_0 T_1 T_2 T_3|^2 \text{ or } |H_3 S|^2 \right\}, \tag{70}$$

respectively.

An example where three moduli are stabilized is obtained by switching on a system of R–R fluxes (F_0, F_2, F_4, F_6), with parameters

$$\Lambda_{111} = -\Lambda_{122} = A, \qquad \Lambda_{112} = -\Lambda_{222} = B. \tag{71}$$

The Jacobi identities (59) are automatically satisfied, and the following effective $N = 1$ superpotential is generated:

$$W = A(1 + T_1 T_2 + T_2 T_3 + T_3 T_1) + i B(T_1 + T_2 + T_3 + T_1 T_2 T_3). \tag{72}$$

This choice of fluxes and superpotential is actually a gauging of $SO(1, 3)$. It is immediate to see that, since the superpotential does not depend on four of the seven main moduli (the T-moduli are stabilized at one), supersymmetry is broken and a positive-definite runaway $D = 4$ scalar potential is generated:

$$\langle V \rangle = \langle m_{3/2}^2 \rangle, \quad \text{with } \langle m_{3/2}^2 \rangle = \frac{A^2 + B^2}{8 s u_1 u_2 u_3}, \tag{73}$$

possibly leading to time-dependent vacua of cosmological interest.

5.3. *Gaugings with $V < 0$, stabilization of all moduli*

We now look at situations where more than four moduli are stabilized, leading to negative-definite potentials once the stabilized moduli are set to their appropriate values.

We begin with a gauging of E_3 with fluxes $\Lambda_{113} = -\omega_3$ (geometric) and $\Lambda_{111} = F_6$ (R–R six-form), with $\omega_3, F_6 > 0$. The R–R six-form corresponds to the $SO(3)$ directions in E_3 while ω_3 corresponds to the translations. The superpotential reads

$$W = -\omega_3 (T_1 U_1 + T_2 U_2 + T_3 U_3) + F_6. \tag{74}$$

The six equations for the nontrivial supergravity auxiliary fields are solved at $\langle\tau_A\rangle = \langle\nu_A\rangle = 0$ and $\langle t_1u_1\rangle = \langle t_2u_2\rangle = \langle t_3u_3\rangle = F_6/\omega_3$. At these values, $W = -2F_6$, and the s-dependent scalar potential and gravitino mass term read

$$V = -2e^K|W|^2 = -\frac{\omega_3^3}{16F_6 s}, \qquad m_{3/2}^2 = -\frac{1}{2}V. \tag{75}$$

At the string level, this is the well-known NS five-brane solution plus linear dilaton, in the near-horizon limit. The original gauging is $SU(2)$, combined with translations, which emerge as free actions at the level of the world-sheet conformal field theory. It is remarkable that this E_3 algebra remains visible at the supergravity level. It is also interesting that, if we allow extra fluxes, induced by the presence of fundamental-string sources, we can reach AdS_3 background solutions with stabilization of the dilaton. All moduli are therefore stabilized. This has been studied recently at the string level [23].

Using all fluxes admissible in IIA, $Z_2 \times Z_2$ strings, we can obtain the stabilization of all moduli in AdS_4 space–time geometry. Switching on all fluxes (ω_3, H_3, F_0, F_2, F_4, F_6), with parameters

$$-\frac{1}{9}\Lambda_{111} = -\frac{1}{2}\Lambda'_{112} = \frac{1}{6}\Lambda_{113} = \Lambda_{122} = A, \tag{76}$$

$$\frac{1}{2}\Lambda'_{111} = -\frac{1}{3}\Lambda_{112} = \frac{1}{2}\Lambda_{114} = -\frac{1}{5}\Lambda_{222} = B, \tag{77}$$

the Jacobi identities (59) are satisfied for

$$6A^2 = 10B^2, \tag{78}$$

and the following effective $N = 1$ superpotential is generated:

$$\begin{aligned} W = {} & A\big[2S(T_1+T_2+T_3) - (T_1T_2+T_2T_3+T_3T_1) + 6(T_1U_1+T_2U_2+T_3U_3) - 9\big] \\ & + iB\big[2S + 5T_1T_2T_3 + 2(U_1+U_2+U_3) - 3(T_1+T_2+T_3)\big]. \end{aligned} \tag{79}$$

Notice that condition (78) relates the terms with even and odd powers of the fields in the superpotential, thus its sign ambiguity is irrelevant. The superpotential (79) leads to a supersymmetric vacuum at $\langle S\rangle = \langle T_A\rangle = \langle U_A\rangle = 1$ ($A = 1, 2, 3$). Since at this point $\langle W\rangle = 4(3A + iB) \neq 0$, implying $\langle V\rangle = -3m_{3/2}^2 < 0$, this vacuum has a stable AdS_4 geometry with all seven main moduli frozen.

The educated reader might feel uncomfortable with condition (78), which seems to imply noninteger flux numbers. This is a consequence of our choice for presenting the model, with $S = T_A = U_A = 1$ at the minimum. One can recover integer flux numbers by rescaling appropriately the moduli. A possible choice (among many others) is the following:

$$(S, T_A, U_A) \to b(S, T_A, U_A), \qquad b = \frac{B}{A} = \sqrt{\frac{3}{5}}. \tag{80}$$

With that choice

$$
\begin{aligned}
W = {} & N\big[2S(T_1+T_2+T_3) - (T_1T_2+T_2T_3+T_3T_1) \\
& + 6(T_1U_1+T_2U_2+T_3U_3) - 15\big] \\
& + iN\big[2S + 3T_1T_2T_3 + 2(U_1+U_2+U_3) - 3(T_1+T_2+T_3)\big],
\end{aligned}
\tag{81}
$$

where $N = (3/5)A$.

We should emphasize here that this is the *only* known example of a complete stabilization of the moduli, reached in IIA by switching on fundamental fluxes (NS or R). We should also stress that this cannot happen in the heterotic string, because of the absence of S-dependence in the general flux-induced superpotential. Such a dependence could however be introduced under the assumption of gaugino condensation. In type IIB with D3-branes, the orientifold projection that accompanies the $Z_2 \times Z_2$ orbifold projection eliminates the ω_3 fluxes, thus the T moduli are not present in the superpotential and cannot be stabilized by fluxes. The case of D9-branes (open string) is similar to the heterotic case, whereas the D7-brane set-up is not captured by the $Z_2 \times Z_2$ orbifold projection used here. The heterotic approach à la Horava–Witten is under investigation [14], whereas F-theory on Calabi–Yau four-folds can introduce exponential dependences in the superpotential [24] and stabilize the T moduli.

6. Conclusions and outlook

In this paper we proposed a novel, bottom-up approach for studying the infrared physics of superstring compactifications that preserve an exact or spontaneously broken $N = 1$ supersymmetry. The approach is in principle applicable to all ten-dimensional superstring theories and M-theory, and is based on the powerful constraints of the gauged $N = 4$ supergravity underlying all these compactifications. Since in $N = 4$, $D = 4$ supergravity the manifold of the scalar fields is unique, once the number of vector fields is given, our approach allows to identify unambiguously the Kähler potential of the $N = 1$, $D = 4$ effective theory. The various systems of fluxes allowed in the different superstring theories are then used to determine, without solving the ten-dimensional equations of motion and Bianchi identities, the structure constants and duality phases that specify the gauging of the $N = 4$ theory. This in turn can be used to identify the superpotential of the resulting $N = 1$ theory. The search for the possible vacuum structures corresponding to the different systems of fluxes can then be performed in a very powerful and elegant formalism, by looking at the potential and auxiliary fields of the effective $N = 1$, $D = 4$ theory.

To be specific, we applied our strategy to situations where the reduction from $N = 4$ to $N = 1$ is achieved by a $Z_2 \times Z_2$ orbifold projection. In the present work, we kept for clarity the six $N = 4$ vector-multiplet geometrical moduli. Thus, the moduli sector of the resulting $N = 1$ theory (after the projection) contained seven distinguished chiral multiplets: S, T_A, U_A $(A = 1, 2, 3)$. In the heterotic theory, the $Z_2 \times Z_2$ projection is enough to reduce the initial $(N = 4))$ supersymmetry to $N = 1$. For describing type-II theories, and in particular the type-IIA compactifications on which we focused for this paper, an extra Z_2 orientifold projection is needed: we chose the one acting as a parity on three of the six internal coordinates, associated with $N = 1$ compactifications of the IIA theory with D6-branes and O6-planes. Type IIB can be treated similarly, by introducing a Z_2 orientifold projection associated with either D3- or D9-branes.

A major geometrical difference exists, however, which makes the type IIA compactifications far more interesting. In type IIB, the Calabi–Yau smooth manifold resolution of the orbifold holds even in the presence of H_3 and F_3 fluxes [10]. This explains why most of the literature deals with this kind of type IIB constructions. However, even in this case, the ω_3 geometrical fluxes are incompatible with the $Z_2 \times Z_2$ orbifold projection (and its smooth Calabi–Yau resolution). The situation for type IIA is even more exotic, since no well-defined mathematical framework has yet been unraveled for understanding the "deformed Calabi–Yau" geometry.

Nevertheless, this is by no means an obstruction to us, when the situation is considered from the conformal field theory perspective of string theory. Many examples exist that demonstrate this: asymmetric orbifolds, fermionic constructions, twisted Gepner constructions, supersymmetric compactifications on manifolds with torsion, etc. Furthermore, these exact models, as well as the whole procedure we have developed so far for dealing with fluxes, point towards the existence of a generalized mirror symmetry, despite the absence of a Calabi–Yau geometrical interpretation for type IIA. A manifestation of that symmetry emerges, for instance, when a nontrivial ω_3 flux is switched on. In the (freely-acting) orbifold limit, a mirror-like $U \leftrightarrow T$ duality relates the type IIA to the type IIB side.

The gauging approach we propose here relies on the rich but constrained $N = 4$ structure. It enables us to bypass the above geometrical difficulties and to organize the fluxes in a systematic way. Indeed, the gauging procedure goes along with a set of structure constants and duality phases, where the former must satisfy Jacobi identities and antisymmetry conditions. These all enter the superpotential, which in turn determines the scalar potential. In the above framework, it is possible to list exhaustively the various choices for the structure constants and duality phases. The choice of the $Z_2 \times Z_2$ projection plays an important role, since it naturally induces an interchange symmetry among the three planes, and simplifies considerably the implementation of the full antisymmetry of f_{RST}. Releasing the assumption of plane-interchange symmetry, changing the orbifold and/or orientifold projections preserving $N = 1$ supersymmetry, moving to type-IIB or type-I superstring theories, all goes beyond the scope of the present paper, and is postponed to [14].

For each of the possible choices of the structure constants (this is equivalent to choosing the subgroup of $SO(6,6)$ that is gauged) and duality phases, one can readily analyze the issue of moduli stabilization. Furthermore, and this is the core of the present paper, one can trace back the origin of the structure constants and duality phases in terms of fluxes in the underlying fundamental theory in ten dimensions. Although our main motivation was the analysis of the yet not unraveled type IIA, we applied our technique to the heterotic case, where we clarified the case where geometrical and NS–NS three-form fluxes are combined. Our pattern allows to reproduce systematically the various examples available in the literature, such as the no-scale models.

As far as type IIA is concerned, more possibilities exist, thanks to the presence of R–R fluxes, besides the geometric and NS–NS ones: F_0, F_2, F_4 and even F_6. They can be introduced one by one, or in combination, provided the Jacobi identities are still satisfied. A specific combination exists, which generates a solution where *all* seven moduli are stabilized, in an AdS_4 geometry. This is typical of type IIA and cannot happen in heterotic, where the allowed geometrical and three-form fluxes cannot create an S-dependence in the superpotential. More examples can be displayed with partial moduli stabilization: domain-wall solutions, runaway solutions, no-scale models. . .

There are various directions that would be worth exploring, besides those already mentioned above. Among them, the detailed correspondence of the ten-dimensional equations of motion, Bianchi identities and tadpole cancellation conditions, with the equations and consistency conditions in the effective gauged four-dimensional supergravity theory. Also, the inclusion in our formalism of the scalar and vector fields associated to brane excitations, and the exploration of the new systems of fluxes associated with their field strengths.

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